# Solutions to exercises in chapter 10

1. **Static magnetic fields**

   Far away, only the sum of the currents matters, hence $B = \mu_0 I/(2\pi r)$

2. **Static magnetic fields 2**

   For one mole, the total magnetic moment is $M = N_A \cdot \mu = 6 \cdot 9 \cdot 10^{-24} \cdot 10^{23}$J/T $= 5.4$ J/T. This dipole moment can be associated to a field $B = \mu_0 M/(2\pi r^3) = 4\pi \cdot 10^{-7}$Vs/(Am)$\cdot$5.4V$\cdot$As/T/$(2\pi 10^{-6}$m$^3) = 1.08$V$^2$s$^2$/(Tm$^4$) $\simeq 1$(Vs/m$^2$)$^2$/(Vs/m$^2$) $= 1$ T

3. **Static magnetic fields 3**

   a) Ampere's law says: $\int \vec{B} \cdot d\vec{s} = \mu_0 I_{enclosed}$. For a coil, we have: $I_{enclosed} = NlI/L$, where $l$ is the length of the enclosing path. Choosing the path such that it runs parallel to the coil inside and then moved perpendicularly outside to return only far away, where the field is zero, we have $\int \vec{B} \cdot d\vec{s} = Bl$ (the scalar product is zero on the perpendicular parts of the path). This gives for the field of the coil: $|\vec{B}| = \mu_0 NI/L$ or numerically: $|\vec{B}| = 4\pi \cdot 10^{-7}$Vs/(Am)$500 \cdot 0.1$A/0.25m $= 8\pi \cdot 10^{-5}$Vs/m$^2 \simeq 2.5 \cdot 10^{-4}$T.

   b) Using relative errors we have: $r_B^2 = r_N^2 + r_I^2 + r_L^2$. With $r_N = 0.01$ and $r_L = 0.004$ much smaller than that of the current, $r_I = 0.1$, they can be neglected and we have $r_B \simeq r_I = 0.1$.

4. **Coulomb-force vs. Lorentz-force**

   a) The charge per unit length of the beam is $Q/l = \lambda$. According to Gauss' law we therefore have for the $\vec{E}$-field of a linear chain of charges at a distance $r$: $|\vec{E}| = \frac{Q}{2\pi r l \epsilon_0} = \frac{\lambda}{2\pi r \epsilon_0}$. The distance of the beams is $d$, hence the field of one beam at the place of the other is:

   $|\vec{E}| = \frac{\lambda}{2\pi d \epsilon_0}$

   This gives for the Coulomb force: $|\vec{F}_C| = q|\vec{E}| = \frac{\lambda q}{2\pi d \epsilon_0}$.

   b) The Lorentz force is given by $\vec{F}_L = q\vec{v} \times \vec{B}$. Using Ampere's law to obtain the field $\vec{B}$ we have: $\int \vec{B} \cdot d\vec{s} = \mu_0 I$. For a straight conductor, this gives for the field at a distance $r$: $|\vec{B}| = \frac{\mu_0 I}{2\pi r}$ (pointing azimuthally). Therefore we have for the Lorentz force: $|\vec{F}_L| = \frac{\mu_0 I q v}{2\pi r}$, or setting $r = d$ as the two beams are a distance $d$ apart. $|\vec{F}_L| = \frac{\mu_0 I q v}{2\pi d}$. Finally, we have to relate the current with the flow speed of the charges. The flowing charge is $q(t) = Qvt/l = \lambda vt$, which gives the current $I = \frac{dq(t)}{dt} = \lambda v$. Inserting this into the Lorentz force:

   $|\vec{F}_L| = \frac{\mu_0 \lambda q v^2}{2\pi d}$.

   For the ratio of the two forces we obtain:

   $\frac{|\vec{F}_L|}{|\vec{F}_C|} = \frac{\mu_0 \lambda q v^2}{2\pi d} \frac{2\pi d \epsilon_0}{\lambda q} = \mu_0 \epsilon_0 v^2 = v^2/c^2$

   c) As only the speed enters here (squared), the relative error of this ratio is given by twice the relative error of the speed.

5. **Lorentz-force**

   The maximum Lorentz force is if speed and field are perpendicular and given by $F = qvB$, hence $v = F/(qB) = 3 \cdot 10^{-12}$N/(30Vs/m$^2 \cdot 1.6 \cdot 10^{-19}$As) $= 10^{-12}$N/$1.6 \cdot 10^{-18}$Js/m$^2 = 10^6$/1.6N/(Ns/m)$\simeq 6 \cdot 10^5$ m/s.

6. **Lorentz-force 2**

   a) The Lorentz force is $F = qvB \cdot \sin(\alpha)$. For a circular trajectory, this would correspond to the centripetal force, i.e. $F = mv^2/r$. This gives the radius of curvature of the trajectory as $r = mv/(qB \cdot \sin\alpha)$. The charge here is the total charge of an erythrocyte, i.e. $q = N_{Hb} \cdot 4 \cdot 2 \cdot e = 3 \cdot 10^8 \cdot 8 \cdot 1.6 \cdot 10^{-19}$C$= 3.9 \cdot 10^{-10}$C. Numerically this gives: $r = 1.5 \cdot 10^{-13}$kg$\cdot$0.2(m/s)/(3.9 $\cdot$

$10^{-10}$C$\cdot$10T$\sqrt{3}/2) = 1.5 \cdot 10^{-3}$kg$\cdot$m/s0.1/($\sqrt{3} \cdot 10$CVs/m$^2$) $= 1.5 \cdot 10^{-5}$kgm/s/($\sqrt{3}$Js/m$^2$) $= 0.8 \cdot 10^{-5}$(kgm/s)/(kg/s) $= 8 \cdot 10^{-6}$m$= 8\mu$m.

b) With relative errors we have $r_r^2 = r_q^2 + r_m^2 + r_v^2$. Numerically $r_r^2 = 1/9+1/9+1/16 = 2/9+1/16 == 59/225$ and hence $r_r = \sqrt{59/225} \simeq 7.5/15 = 0.5$

7. **Mass spectrometer**

   For given $U, B$ and $r$ we have a mass of $m = qr^2B^2/(2U)$. If only $r$ has a significant uncertainty, the error in $m$ is given by $\sigma_m/m = 2\sigma_r/r$. So if we are given the uncertainty $\sigma_r$, the radius of curvature needs to be: $r = 2\sigma_r m/\sigma_m$. If we want to be able to distinguish $C^{12}$ and $C^{14}$, we also know the relative uncertainty in $m$ that we at least need to have, namely $\sigma_m/m = 1/13$, because for an atomic mass of 13 we need to have a resolution of better than plus or minus 1. Therefore the radius of curvature need to be: $r = 26\sigma_r = 26$ mm.

8. **Nuclear magnetic resonance**

   a) We are using a length of 20 cm rather than 120 as written in the text, as the 1 is a typo. The field difference is $\Delta B = dB/dx\Delta x =$ 20cm$\cdot 0.5 \cdot 10^{-4}$T/cm$= 10^{-3}$T. The frequency difference is directly given by the field difference, $\Delta\omega = \gamma_p \Delta B = \gamma_p dB/dx \Delta x = 10^{-3}$T$\cdot 42 \cdot 10^6$Hz/T$= 1000 \cdot 42$Hz$= 42$ kHz. This is the differnce in frequency relative to the main frequency of $\omega_L = 5 \cdot 42 \cdot 10^6$Hz $= 210$ MHz.

   b) For a given gradient $dB/dx$, the uncertainty in $B$, $\sigma_B$, is directly given by the one in $x$, $\sigma_x$, through $\sigma_B = dB/dx\sigma_x$. Hence the spatial resolution is given by $\sigma_x = (dB/dx)^{-1}\sigma_B = (dB/dx)^{-1}\sigma_B/B \cdot B = (dB/dx)^{-1}\sigma_\omega/\omega \cdot B = (dB/dx)^{-1}\sigma_\omega/\gamma_p$. Numerically: $\sigma_x = 2 \cdot 10^4$cm/T$\cdot$10Hz/(42$\cdot 10^6$Hz/T) $= 1/210$cm$\simeq 50\mu$ m.

9. **Nuclear magnetic resonance 2**

   For a circular current $I$ over an area $A$ we obtain a magnetic moment of $\mu_p = I \cdot A$. For the model of the proton this means $\mu_p = e\omega/(2\pi) \cdot \pi r_p^2 = e\omega r_p^2/2$. If we get the angular frequency from the angular momentum, i.e.: $\omega = \hbar/(m_p r_p^2)$ we get: $\mu_p = e\hbar/(m_p r_p^2) \cdot r_p^2/2 = e\hbar/(2m_p)$ or numerically: $\mu_p = 1.6 \cdot 10^{-19}$C$\cdot 10^{-34}$Js/($4 \cdot 10^{-27}$kg) $= 4 \cdot 10^{-27}$J$\cdot$kg$\cdot$m$\cdot$m$\cdot$s/(V$\cdot$kg$\cdot$s$^2$) $= 4 \cdot 10^{-27}$J$\cdot$m$^2$/(V$\cdot$s) $= 4 \cdot 10^{-27}$ J/T.

10. **Nuclear magnetic resonance 3**

    The potential energy is $\Delta E = \mu_p \cdot B = 1.4 \cdot 10^{-26}$J. If 10% of the protons are supposed to be aligned, we have to have $\Delta E/(k_BT) = 0.1$ (approximating the Boltzmann distribution for small ratios of $\Delta E/(k_BT)$. Hence, we need to have $k_BT = 10 \cdot \Delta E$. Numerically $k_BT = 1.4 \cdot 10^{-25}$ J. At 300 K, $k_BT = 4 \cdot 10^{-21}$ J, hence the temperature we look for is $T = 300$K$\cdot 0.35 \cdot 10^{-4} = 10$ mK.