Solutions to exercises in chapter 9

1. **Electrostatics**

   $\Delta E = q\Delta V$, so the charge is: $q = \Delta E/\Delta V$, or numerically: $q = 1.92 \cdot 10^{-17}$ J/600(J/C)= $1.92/6 \cdot 10^{-19}$C= $3.2 \cdot 10^{-20}$ C. This would be 0.2 elementary charges and since e is the smallest charge freely observable, this result cannot be!

2. **Electrostatics 2**

   The screening length is $\lambda = \sqrt{\epsilon\epsilon_0 k_B T/(2\rho q)}$, where ρ is the chare density of the ions and q their charge. With 0.16 M NaCl the charge is $q = e$ and the density is $\rho = 0.16 mol/L \cdot e = 0.16 \cdot 6 \cdot 10^{23} \cdot 1.6 \cdot 10^{-19}\mathrm{C}/(10^{-3}\mathrm{m}^3) = 1.6 \cdot 10^7\mathrm{C/m}^3$. Hence $2\rho q = 2 \cdot 1.6^2 \cdot 10^{-12}\mathrm{C}^2/\mathrm{m}^3 = 5 \cdot 10^{-12}\mathrm{C}^2/\mathrm{m}^3$. Which finally give the screening length as: $\lambda = \sqrt{80 \cdot 8.85 \cdot 10^{-12} C/(Vm) \cdot 4 \cdot 10^{-21} J/(5 \cdot 10^{-12} C^2/m^3)} = \sqrt{8 \cdot 8.85 \cdot 0.8 \cdot 10{-}20}m\simeq 8 \cdot 10^{-10}m= 8\mathring{A}$.

3. **Electrostatics 3**

   Nernst-Potential: $\Delta V = k_B T/q ln(c1/c2)$. The ion charge is the elementary charge, hence numerically: $\Delta V = 4 \cdot 10^{-21}\mathrm{J}/(1.6 \cdot 10^{-19}\mathrm{C}) \ln(160/20) = 4/16 10^{-1} \ln(8)\mathrm{V} = 0.025 \cdot \ln(2^3)\mathrm{V} = 0.075 \cdot \ln(2)\mathrm{V} = 0.05\mathrm{V} = 50\mathrm{mV}$.

4. **Electrostatics 4**

   a) The applied field created by the charge is $E = V/d = 7/5 \cdot 10^7$V/m. According to Gauss' law we can also write $EA = Q/(\epsilon\epsilon_0)$ or solving for ϵ: $\epsilon = Q/(A\epsilon_0) \cdot d/V = 1.6 \cdot 10^{-19} \cdot 6 \cdot 10^{15}\mathrm{C/m}^2/(8.85 \cdot 10^{-12}\mathrm{C/(Vm)}) \cdot 5/7 \cdot 10^{-7}\mathrm{m/V} = 50/(8.85 \cdot 7)10^{-11}/10^{-12} = 50/6 \simeq 8$.

   b) Everything enters linearly into a ratio, hence we add relative errors in squares: $r_\epsilon = \sqrt{r_V^2 + r_d^2 + r_n^2}$. Numerically $r_V = 0.1; r_d = 0.2; r_n = 0.1$, hence $r_\epsilon = \sqrt{6} * 0.1 \simeq 0.25$.

5. **Electrostatics 5**

   a) The E-field will point radially towards the centre of the DNA over the entire length, i.e. will have the symmetry of a cylinder. In that case we can use Gauss' law for its strength: $E \cdot A = Q/(\epsilon\epsilon_0)$, with a cylinder surface at a distance r and length L, i.e. $A = 2\pi r L$. The charge is given by the charge density time the DNA length, i.e. $Q = \sigma L$, where $\sigma = e/d$ with a base pair distance d. Taking everything together, we obtain: $E 2\pi r L = eL/(d\epsilon\epsilon_0)$ or $E = e/(2\pi r d\epsilon\epsilon_0)$. The field decreases with $1/r$ and has a prefactor: $e/(2\pi d\epsilon\epsilon_0) = 1.6 \cdot 10^{-19}\mathrm{C}/(2\pi \cdot 0.34 \cdot 10^{-9}\mathrm{m} 80 \cdot 8.85 \cdot 10^{-12}\mathrm{C/(Vm)}) = 1.6/(2\pi \cdot 0.34 \cdot 0.8 \cdot 8.85)\mathrm{V} \simeq 1/8.85\mathrm{V} \simeq 0.11$ V.

   b) The binding energy can be obtained from the electrostatic potential energy $E = Q_1 Q_2/(4\pi\epsilon\epsilon_0 r)$, where we are given $r = 1$nm and $Q_2 = e = 1.6 \cdot 10^{-19}$ C as the charge of the histone. The charge of the DNA we obtain from, $Q_1 = e \cdot 2\pi(R + r)/d = e\pi 12\mathrm{nm}/0.34$ nm $= 370/3.4\ e \simeq 110e$. With $\epsilon_{water} = 80$ we get: $E = 110e^2/(80 \cdot 10^{-10}\mathrm{As/(Vm)} \cdot 10^{-9}\mathrm{m} = 11/8 1.6^2 \cdot 10^{-19} \cdot 10^{-19}/10^{-19}\mathrm{VC} \simeq 3.5 \cdot 10^{-19}$ J. For comparison the bending energy from the exercise in chapter 8 was $E_{bend} = 2.5 \cdot 10^{-19}$ J.

6. **van der Waals interaction, dipoles**

   a) The pre-factor in van der Waals is given by $M = \alpha P^2/(4\pi^2\epsilon_0^2)$, where $P = qd$ is the fluctuating dipole moment. This then gives the polarisability of: $\alpha = M * 4\pi^2\epsilon_0^2/P^2$. From the values given, we have $P = 1.6 \cdot 10^{-19}$ C$\cdot 2 \cdot 10^{-10}$m $= 3.2 \cdot 10^{-29}$Cm and $2\pi\epsilon_0 = 5 \cdot 10^{-11}$ As/Vm. We therefore obtain: $\alpha = 1.6 \cdot 10^{-77}\mathrm{Jm}^6 \cdot 25 \cdot 10^{-22}\mathrm{C}^2/(\mathrm{V}^2\mathrm{m}^2)/(3.2^2 \cdot 10^{-58}\mathrm{C}^2\mathrm{m}^2) = 4 \cdot 10^{-41}\mathrm{C}^2\mathrm{m}^2/\mathrm{J} = 2.5 \cdot 10^{-2}\mathrm{C}^2\mathring{A}^2/(\mathrm{eV})$.

   b) We have a power law with the size entering quadratically and M linearly. Hence $r_\alpha = \sqrt{4r_r^2 + r_M^2}$. Numerically $r_r = 0.1$ and $r_M = 0.06$, so $r_\alpha = \sqrt{0.04 + 0.0036} = \sqrt{0.0436} \simeq 0.21$

7. **Dipoles**

   a) The dipole moment is $p = q \cdot d$, thus, the average charge per atom is: $q = p/d$. Numerically: $q = 3.4 \cdot 10^{-30}$ Cm/10^{-10}m$= 3.4 \cdot 10^{-20}$ C or only about 0.2e. The effective average charge of the atom is due to the fact that the electrons are at slightly different positions on average. Therefore, this does not have to correspond to a multiple of the elementary charge.

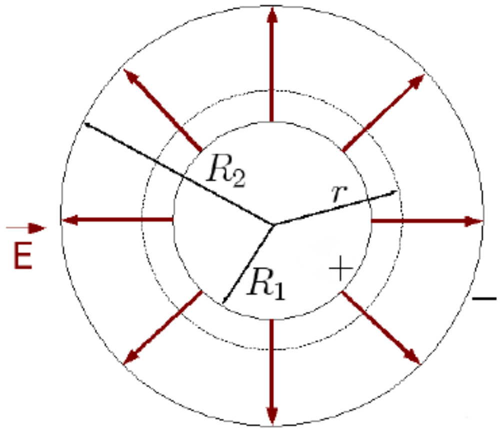
b) $r_q^2 = r_p^2 + r_d^2$ with $r_d = 0.1$ and $r_p \simeq 0.06$ we obtain: $r_q = \sqrt{1 + 0.6^2} \cdot 0.1 = \sqrt{1.36} \cdot 0.1 \simeq 0.12$.

c) $E_{pot} = -\vec{p} \cdot \vec{E} = -|\vec{p}||\vec{E}| \cos(\theta)$. For a rotation by 45° we have $\cos(\theta) = 1/\sqrt{2}$, and hence: $E_{pot} = -|\vec{p}||\vec{E}| + |\vec{p}||\vec{E}|/\sqrt{2}$. Numerically: $E_{pot} = 3.4 \cdot 10^{-30} Cm 2.5 \cdot 10^4 V/m(-1+0.7) \simeq 2.5 \cdot 10^{-26} J$.

8. **Capacitors**

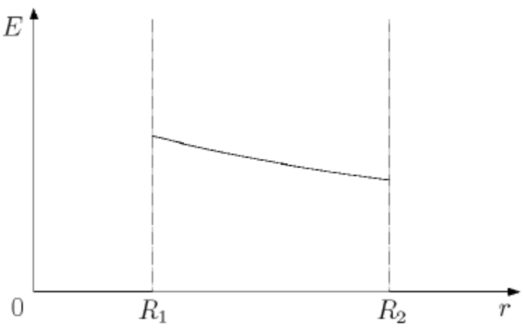
The field of a plate capacitor is $E = Q/(A\epsilon\epsilon_0)$. The electric potential then is: $V = Ed = Qd/(A\epsilon\epsilon_0)$. Therefore, the work performed on a test charge dQ is $W = \int V dQ = d/(A\epsilon\epsilon_0) \int Q dQ = Q^2 d/(2A\epsilon\epsilon_0)$. This is the energy stored in the capacitor.

9. **Capacitors 2**



a)

b) Take a cylinder of radius r with $R_1 < r < R_2$ and length L for the flux determination in Gauss'law. Then the flux of the E-field is: $\frac{Q_{innen}}{\epsilon_0} = EA = E 2\pi r L$ and hence for the field in this region: $E = \frac{Q_{innen}}{2\pi\epsilon_0 r L}$. total charge enclosed by a cylinder in these cases is zero.



c) With $V = -\int \vec{E} \cdot d\vec{r}$ and the result from b) we get: $V = -\int_{R_1}^{R_2} \frac{Q}{2\pi\epsilon_0 r L} dr = -\frac{Q}{2\pi\epsilon_0 L} \int_{R_1}^{R_2} dr/r = \frac{Q}{2\pi\epsilon_0 L} \ln(R_1/R_2)$.

From the definition of the capacitance $C = \frac{Q}{V}$ we finally have: $C = \frac{2\pi\epsilon_0 L}{\ln(R_2/R_1)}$

10. **Atomic Physics**

a) The energy difference is given by the Rydberg energy times $(1/n_1^2 - 1/n_2^2)$ with $n_1 = 10$ and $n_2 = 3$ we obtain $1/100 - 1/9 = 0.01 - 0.111 \simeq 0.1$ or in other words an energy of $\Delta E = 0.1 \cdot E_{Ryd} = 1.36 eV$.

b) For a standing wave, we have: Circumference $c = 2\pi a_B$ must be equal to a wave-length, i.e. $\lambda = 2\pi a_B \simeq \pi \mathring{A}$.

11. **Atomic Physics 2**

a) The binding energy is proportional to Z^2/n^2. For the outer electron in lead with $n = 6$ and $Z = 82$ the binding energy thus increases by a factor of $(82/6)^2 \simeq 12.5^2 \simeq 150$. Therefore $E_B \simeq 13.6eV \cdot 150 \simeq 2keV$.

b) The binding energy is linear in the mass of the particle, hence it is increased by a factor of 200: $E_B = 200 \cdot 13.6$ eV $\simeq 2.7$ keV

c) As the mass is increased by another factor of 10, the energy is correspondingly increased by another factor of ten to $E_B = 27keV$. To be more exact, we would have to take into account the fact that now two objects of equal mass are circulating around their common centre of mass, which would change the result by another factor of two.

12. **Currents**

The current density multiplied by the time gives the charge density deposited on the membrane, which according to Gauss' law gives the field and hence the voltage difference: $V = j \cdot \tau \cdot d/(\epsilon\epsilon_0)$ numerically: $V = 0.8\text{A/m}^2 \cdot 10^{-3}\text{s} \cdot 5 \cdot 10^{-9}\text{m}/(7 \cdot 8.85 \cdot 10^{-12}\text{As/(Vm)}) = 4/(7 \cdot 8.85) \cdot 10^{-12}/10^{-12}\text{V} \simeq$ 60 mV.

13. **Currents 2**

With 50 channels/μ m^2 and an area of 10^6 μ m^2, we have a total of $5 \cdot 10^7$ ion channels. There's a current of $5 \cdot 10^{10}$ ions/ms or $5 \cdot 10^{13}$ ions/s through all of these channels and with one elementary charge per ion, we get a current of $5 \cdot 1.6 \cdot 10^{-6}$ C/s$= 8 \cdot 10^{-6}$A.

14. **Currents 3**

The mobility is $\mu = \pm e/(6\pi\eta r)$, so for Cl: $\mu_{Cl} = -e/(6\pi\eta r_{Cl}) = -1.6 \cdot 10^{-19}$ C $/(18 \cdot 10^{-3}$Pa s$1.5 \cdot 10^{-10}$m$) = -1.6/2.710^{-7}$m^2/(Vs)$= -6 \cdot 10^{-8}$m^2/(Vs), and for Na: $\mu_{Cl} = e/(6\pi\eta r_{Na}) = 1.6 \cdot 10^{-19}$C/$(18 \cdot 10^{-3}$Pa s $2 \cdot 10^{-10}$m$) = 1.6/3.610^{-7}$m^2/(Vs)$= 4.5 \cdot 10^{-8}$m^2/(Vs).

The conductivity is given by the charge density times the mobility, i.e. $\sigma = q_{Na}n_{Na}\mu_{Na} + q_{Cl}n_{Cl}\mu_{Cl} = en(\mu_{Na} - \mu_{Cl})$. The charge density is directly given by the ion concentration $n = 160 \cdot 10^{-3} \cdot 6 \cdot 10^{23}/10^{-3}$ m^3 $= 10^{26}m^{-3}$ multiplied by the charge, i.e. $en = 1.6 \cdot 10^{-19} \cdot 10^{26}$ C/m^3 $= 1.6 \cdot 10^7$ C/m^3. Together with the mobility we thus have: $\sigma = 1.6 \cdot 10^7 \cdot (4.5 + 6) \cdot 10^{-8}$ C/m^3·m^2/(Vs)$= 1.05 \cdot 1.6$ A/(Vm) $= 1.7$ A/(Vm), or for the resistivity $\rho = 1/\sigma = 1/1.7Vm/A \simeq 0.6\Omega m$.

15. **Currents 4**

The power is given by $P = V \cdot I$. Since we know the power and the voltage, we get the average current as $I = \frac{P}{V} = \frac{10^9 W}{5 \cdot 10^7 V} = 20A$. The charge is given by the current times the time this current is flowing, hence $Q = I \cdot t = 20\text{A} \cdot 0.2\text{s} = 4\text{C}$.

16. **Resistors**

The volume (AL), as well as the resistivity (ρ) of the clay are constant. The resistance is given by $R = \rho L/A$. Because volume is conserved, we have $LA_1 = A_2L/2$, hence $A_1 = A_2/2$ and thus $R_2 = \rho L_2/A_2 = \frac{\rho L_1/2}{2A_1} = \rho L_1/(4A_1) = R/4$.