Problems for Chapter 18 of 'Ultra Low Power Bioelectronics'

Problem 18.1

The commonest differential CMOS oscillator at radio frequencies is the *LC*-based circuit shown in Figure P18.1. The value of *C* depends on the dc voltage across it; thus V_{TUNE} can be used to change the oscillation frequency.

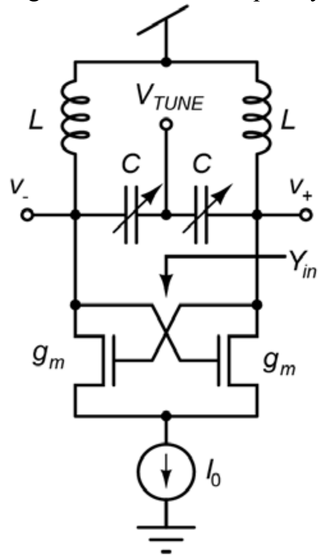

Figure P18.1: A differential *LC* oscillator.

a) Qualitatively explain the behavior of this circuit. What is the oscillation frequency?
b) Show that the differential input impedance across the drains of the cross-coupled NMOS transistors is given by

$$Y_{in} = -\frac{g_m}{2} + s\left(\frac{C_{gs}}{2} + 2C_{gd}\right)$$

Where g_m, C_{gs}, and C_{gd} are the transconductance, gate-to-source capacitance, and gate-to-drain capacitance of each transistor, respectively.
c) Based on your results in part b), derive a more accurate expression for the oscillation frequency (include the transistor capacitances).
d) Assume that the series resistance of each inductor is R_S. What is the quality factor Q of the inductor at the oscillation frequency?
e) Show that the oscillation amplitude is approximately given by

$$V_{osc} \approx (I_0 R_S) Q^2$$

f) Does an exact describing-function analysis (Chapter 9.6 and Problem 9.5 discuss such analysis) yield a similar answer?
g) Assume that the capacitance varies between C and $C + \Delta C$ as V_{TUNE} increases between 0 and V_{DD}. What is the fractional change in oscillation frequency $\Delta f_{osc} / f_{osc}$ (also referred to as the tuning range) that results?
h) Does the oscillation amplitude remain constant as V_{TUNE} varies? Explain.

Problem 18.2

Consider the oscillator circuit shown in Figure 18.2. Assume that $MOD = V_{DDH}$, and prove that Equation (18.8) must hold in order for the oscillation frequency to be exactly $f_{osc} = 1/\sqrt{L_1 C_1}$.

Problem 18.3

Consider the canonical envelope-detector circuit shown in Figure P18.3. We would like the dc output voltage to be equal to the positive envelope of the input waveform, i.e., $V_{DC} + v_{rf}$.

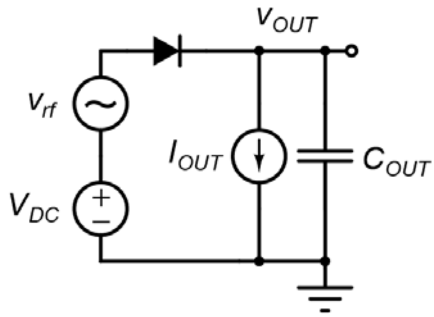

Figure P18.3: An envelope detector circuit

a) Qualitatively explain the behavior of this circuit.
b) Find $V_{OUT,0}$, the dc output voltage when the RF input is absent, i.e., when $v_{rf} = 0$.
c) A sinusoidal input voltage $v_{rf}\sin(\omega_{RF}t)$ is now applied at the input of the circuit. The resultant output voltage consists of an ac ripple superimposed on top of a dc voltage V_{OUT}. Show that

$$\Delta V_{OUT} \le \left(\frac{2\pi}{\omega_{RF}}\right)\frac{I_{OUT}}{C_{OUT}}$$

where ΔV_{OUT} is the peak-to-peak ripple.
d) Assume that I_{OUT} and C_{OUT} are chosen such that $\Delta V_{OUT} \ll \phi_t$, the thermal voltage. Show that, in this case

$$V_{OUT} = V_{OUT,0} + v_{rf}G_{ED}$$

where G_{ED}, the gain of the envelope detector, is given by

$$G_{ED} = \frac{\ln\left[\overline{\exp(n\sin(\theta))}\right]}{n}$$

Here $n = v_{rf}/\phi_t$ is the normalized RF voltage, and the line denotes averaging over one RF cycle, i.e., as θ increases from 0 to 2π. [Hint: use the fact that the dc current through the diode is independent of the input and output voltages.]
e) Use the same procedure as in part d) to find G_{ED} in the case when $v_{rf}\sin(\omega_{RF}t)$ is replaced by a square wave of the same amplitude and period, i.e., switching between $\pm v_{rf}$ with period $1/f_{RF}$.

f) Plot G_{ED} as a function of n for both types of input. In each case, what is the minimum value of n that results in $G_{ED} > 1/\sqrt{2}$? Qualitatively explain the behavior of G_{ED} with n.

g) Now imagine that the RF input is amplitude modulated. In particular, assume that its envelope is given by $v_{rf}\left[1+m\sin\left(\omega_{MOD}t\right)\right]$, where $m < 1$ is the modulation depth. In general, the output voltage may now be written as

$$V_{OUT} = V_{OUT,0} + v_{rf}G_{ED}(0)\left[1+m\left(\frac{G_{ED}\left(\omega_{MOD}\right)}{G_{ED}(0)}\right)\sin\left(\omega_{MOD}t+\phi\right)\right]$$

What are the physical meanings of $G_{ED}(0)$, $G_{ED}(\omega_{MOD})$, and ϕ? Show that $G_{ED}\left(\omega_{MOD}\right)/G_{ED}(0)$ behaves as a low-pass filter, and find its 3-dB cut-off frequency when $mn \ll 1$.

h) The junction diode in Figure P18.3 is replaced by a diode-connected NMOS or PMOS transistor. How do the results of parts d) – g) change? [Hint: you do not need to do any math.]

Problem 18.4

a) Find the loop transmission $L(s)$ of the phase-locked loop shown in Figure 18.4, i.e., prove Equation (18.10). Justify any assumptions that you make.

b) Draw Bode plots of the magnitude and phase of $L(s)$ for the following parameter values[1]: $G_m / Q_{CCO} = 5$ MHz/V, $I_{CP} = 1$ μA, $C_1 = 220$ pF, $C_2 = 10$ pF, $C_3 = 10$ pF, $R_1 = 120$ kΩ, and $R_3 = 80$ kΩ.

c) Use your Bode plots to estimate the crossover frequency and phase margin of the PLL.

Problem 18.5

The error rate of the pulse-width demodulator circuit shown in Figure 18.9 is minimized if the value of $\left|V_A - V_B\right|$ at the end of the bit period is equal for '0' and '1' bits (assuming that they are equally likely).

a) Show that the minimum-error condition is satisfied if

$$\frac{\left(I_A / C_A\right)}{\left(I_B / C_B\right)} = \left(\frac{2}{\alpha_0+\alpha_1} - 1\right)$$

Where $0 < \alpha_0 < 1$ and $0 < \alpha_1 < 1$ are the pulse widths (normalized by the bit period) used to signal a '0' and a '1', respectively.

b) What factors set the range of data rates over which the pulse-width demodulator circuit functions correctly? Derive a formula for the minimum allowable data rate.

Problem 18.6

a) Consider an ω–version of Equation (18.44) for the skin depth, i.e.,

---

[1] These values are equal to those that were actually used to implement the low-power transceiver described in this chapter.

$$d(\omega) = \frac{1}{Re\{k(\omega)\}},$$

where $\omega = 2\pi f$. When $\omega \ll (\sigma / \epsilon)$ in Figure 18.17 (a), show that Equation (18.44) reduces to

$$d(\omega) \approx \sqrt{\frac{2}{\omega\mu\sigma}}$$

b) The frequency below which the approximation derived in part a) is valid is given by $f_c = \sigma / (2\pi\epsilon)$. Calculate f_c for the following materials: Sea water, with $\sigma = 4.8$ S/m and $\epsilon_r \approx 78$ at 300K, and air, with $\sigma \approx 0.5 \times 10^{-14}$ S/m and $\epsilon_r \approx 1$. Note that ϵ_r denotes the *relative* permittivity of a dielectric material.

Problem 18.7

Consider the simplified impedance-modulation communication system shown in Figure 18.14. The efficiency η of the power amplifier is defined as the ratio of the RF power generated at node v_1 to the DC power supplied by V_{DDp}.

a) Calculate η for arbitrary values of the input RF voltage v_{in} under the assumption that the transistor is always 'on' (carrying non-zero current). Your result should be the same as Equation (18.14).
b) What is the maximum possible value of η? Under what conditions is it achieved?
c) Can η be increased beyond the maximum value calculated in part b) by relaxing some of your prior assumptions? Explain, preferably including relevant waveforms.

Problem 18.8

Consider the modified impedance modulation data link shown in Figure P18.8. Instead of generating an open or short circuit across C_2, data bits add or remove a small capacitor of value $2\Delta C$ that is connected in parallel with a fixed capacitor of value $(C_2 - \Delta C)$. Assume that $L_1C_1 = L_2C_2$, as in the original data link, i.e., that the external and internal resonators both resonate at $\omega_0 = 1/\sqrt{L_1C_1} = 1/\sqrt{L_2C_2}$ when $\Delta C = 0$.

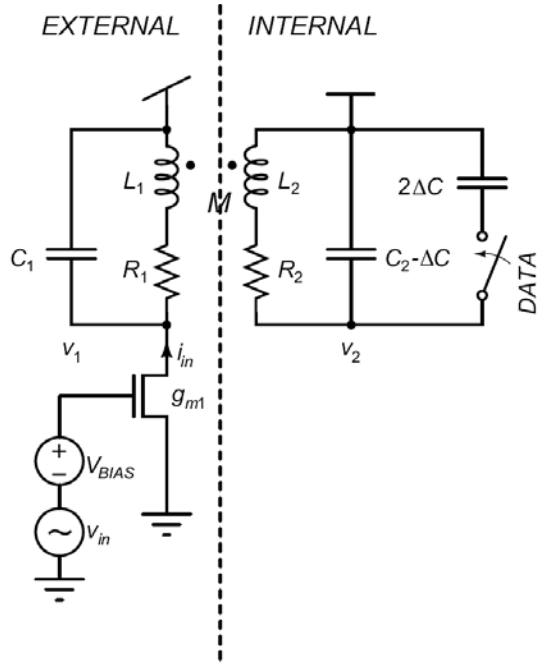

Figure P18.8: A modified impedance-modulation data link

a) Find the fractional change in the resonant frequency of the internal resonator as a function of ΔC.

b) Assume that the external resonator is driven by v_{in} at its resonance frequency ω_0. Find the impedance of the internal resonator at ω_0 as a function of ΔC.

c) Find the reflected impedance in the external resonator at ω_0 as a function of ΔC. What are the fractional changes in the effective resistance and inductance of the external resonator caused by data transmitted from the internal resonator?

d) Show that the data transmitted from the external resonator changes the phase, but not the amplitude, of v_1. Prove that the phase modulation, i.e., the phase difference between the cases when '0' and '1' bits are being transmitted, is given by

$$\Delta\phi \approx 2Q_2\left(\frac{\Delta C}{C_2}\right)k^2 Q_1 Q_2$$

e) Compare the advantages and disadvantages of the impedance-modulation technique introduced in this problem to that described in detail in Chapter 18.

Problem 18.9

You may want to refresh your knowledge of Chapter 17 before attempting this problem. The far-field version of impedance modulation is used in radio-frequency identification (RFID) systems to transmit information. It is known as back-scatter modulation, because modulating the input impedance of an antenna changes its reflection coefficient Γ, and thus the amount of power that is reflected and re-radiated (backscattered) by the antenna.

Consider two antennas separated by a distance R in free space. The first antenna (with gain G_1) is perfectly impedance-matched and radiates a fixed amount of power P_T, while the second one (with gain G_2) transmits '0' and '1' by switching its reflection coefficient between Γ_0 and Γ_1, respectively. The wavelength of the radiation is λ.

a) Calculate the amount of power that is absorbed by the second antenna as a function of P_T, the bit being transmitted, λ, and R [Hint: begin with the Friis equation, Equation (17.29)].
b) Using your results from part a), calculate P_{BS}, the amount of power that is backscattered by the second antenna.
c) Using your results from part b), calculate P_R, the amount of backscattered power that is absorbed by the first antenna.
d) The absorbed power P_R, which you calculated in part c), introduces a reflected impedance in the first antenna. Calculate the value of this impedance. What is the modulation depth?
e) Prove that your formula for the reflected impedance is compatible with Equations (17.13) and (17.37).

Problem 18.10

The skin depth d in a conductive material decreases with frequency $\omega=2\pi f$. As a result the resistance of a wire increases at high frequencies.

a) We want to design a circular coil of value $L = 250$ nH for an impedance-modulation system. The coil has to fit within a rectangle 2 cm on a side. Calculate the radius a, and number of turns N, of the coil if 26-gauge copper wire ($R \approx 0.2\,\text{mm}$) is used. You may use the following approximate formula for L (Equations (16.6) and (16.7) in Chapter 16):

$$L \approx \mu_0 N^2 a \ln\left(\frac{a}{R}\right)$$

b) Calculate the DC series resistance R_{DC} of the coil. The electrical conductivity of copper is $\sigma = 5.96\times10^7$ S/m.
c) Calculate the skin depth d of copper at 25 MHz. Justify any assumptions that you make.
d) Consider a cylindrical wire of radius R. When $R >> d$ most of the current flows in a thin cylindrical shell of thickness d next to the surface of the wire. Find the high-frequency resistance of the wire as a function of d, R, and the resistance at DC.
e) Use the formula derived in part c) to calculate the series resistance and quality factor of the coil at 25 MHz, the center frequency of the link. By what factor has the coil resistance increased compared to its DC value?
f) Assume that the cross sectional area of a wire is kept constant. Suggest ways in which its resistance at high frequencies can be reduced.