Chapter 10

```
In[1]:= Needs["Graphics`MultipleListPlot`"]

In[2]:= Needs["Graphics`Legend`"]
```

à Question 1

We first need to establish the equilibrium values for income and consumption. In equilibrium $Y = \overline{Y}$ for all t. Before the shock the equilibrium income and consumption are:

```
In[3]:= Solve[Ybar == 110 + 0.75 Ybar + 300, Ybar]

Out[3]= {{Ybar → 1640.}}

In[4]:= Cbar = 110 + 0.75 Ybar /. Ybar -> 1640

        General::spell1 :
         Possible spelling error: new symbol name "Cbar" is similar to existing symbol "Ybar".

Out[4]= 1340.
```

After the shock the equilibrium income and consumption are:

```
In[5]:= Solve[Ybar1 == 110 + 0.75 Ybar1 + 310, Ybar1]

Out[5]= {{Ybar1 → 1680.}}

In[6]:= Cbar1 = 110 + 0.75 Ybar1 /. Ybar1 -> 1680

        General::spell1 :
         Possible spelling error: new symbol name "Cbar1" is similar to existing symbol "Ybar1".

Out[6]= 1370.
```

For the 1-period lag we have:

$$Y_t = 420 + 0.75\, Y_{t-1}$$

and for consumption

$$C_t = 110 + 0.75\, Y_t$$

```
In[7]:= Y1[0] = 1640;
        Y1[t_] := 420 + 0.75 Y1[t - 1]

In[9]:= ylist1 = Table[{t, Y1[t]}, {t, 0, 20}];

In[10]:= clist1 = Table[{t, 110 + 0.75 Y1[t]}, {t, 0, 20}];

        General::spell1 :
         Possible spelling error: new symbol name "clist1" is similar to existing symbol "ylist1".
```

For the 2-period lag we have:

$$Y_t = 420 + 0.75\, Y_{t-2}$$

and for consumption

$$C_t = 110 + 0.75\, Y_t$$

Here we assume income is at its initial equilibrium and then rises by 10 in period 1.

```
In[11]:=
        {Y2[0] = 1640, Y2[1] = 1650};
        Y2[t_] := 420 + 0.75 Y2[t - 2]

In[13]:= ylist2 = Table[{t, Y2[t]}, {t, 0, 20}];

In[14]:= clist2 = Table[{t, 110 + 0.75 Y2[t]}, {t, 0, 20}];

        General::spell1 :
         Possible spelling error: new symbol name "clist2" is similar to existing symbol "ylist2".
```

For the 3-period lag we have:

$$Y_t = 420 + 0.75\, Y_{t-3}$$

and for consumption

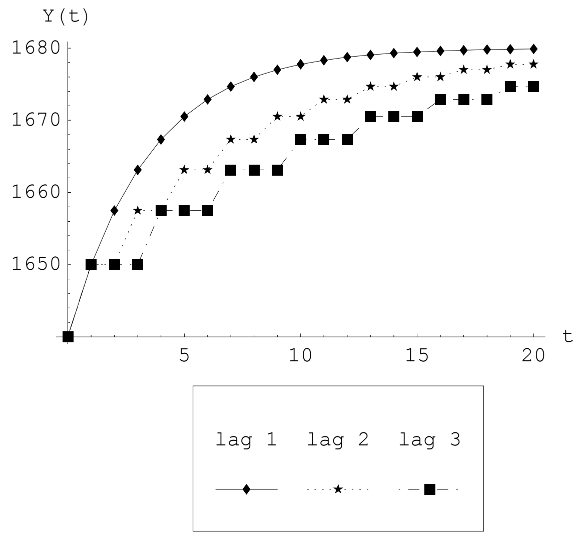
$$C_t = 110 + 0.75\, Y_t$$

Here we assume income is at its initial equilibrium and then rises by 10 in periods 1 and 2.

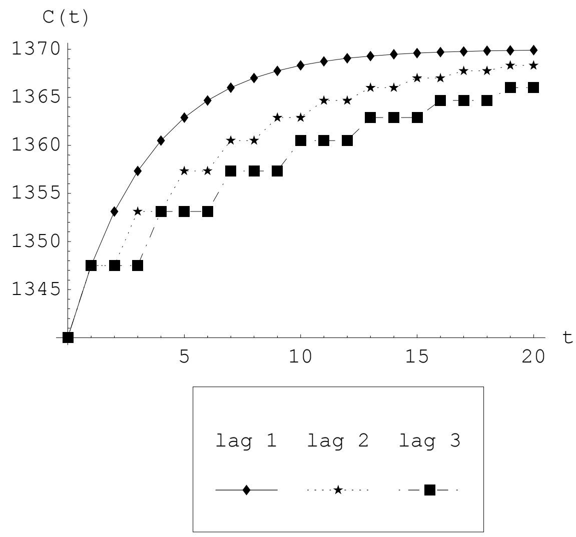
```
In[15]:=
        {Y3[0] = 1640, Y3[1] = 1650, Y3[2] = 1650};
        Y3[t_] := 420 + 0.75 Y3[t - 3]

In[17]:= ylist3 = Table[{t, Y3[t]}, {t, 0, 20}];

In[18]:= clist3 = Table[{t, 110 + 0.75 Y3[t]}, {t, 0, 20}];

        General::spell1 :
         Possible spelling error: new symbol name "clist3" is similar to existing symbol "ylist3".
```

Since we wish to plot the three paths for Y and the three paths for C on the same plot, we need to load the MultipleListPlot command. In addition, in order to clearly distinguish them we need to include a legend. This requires the Legend command to be called.

```
In[19]:= MultipleListPlot[ylist1, ylist2, ylist3, PlotJoined -> True,
           AxesLabel -> {"t", "Y(t)"}, PlotLegend -> {"lag 1", "lag 2", "lag 3"},
         LegendPosition -> {-0.35, -1.2}, LegendOrientation -> Horizontal,
           LegendSize -> {1, .5}];
```




```
In[20]:= MultipleListPlot[clist1, clist2, clist3,
           PlotJoined -> True, AxesLabel -> {"t", "C(t)"},
           PlotLegend -> {"lag 1", "lag 2", "lag 3"}, LegendPosition -> {-0.35, -1.2},
           LegendOrientation -> Horizontal, LegendSize -> {1, .5}];
```



à Question 2

With the new investment function the macroeconomic system is represented by the following set of equations:

$c_t = 110 + 0.75\,(y_t - \text{tax}_t)$
$\text{tax}_t = -80 + 0.2\,y_t$
$i_t = 320 - 4\,r_{t-1}$
$g_t = 330 + s$
$e_t = c_t + i_t + g_t$
$y_t = e_{t-1}$
$m_t^d = 20 + 0.25\,y_t - 10\,r_t$
$m_t^s = 470$
$m_t^d = m_t^s$

where s is the shock.

Consider first the IS-curve.

```
In[21]:= Clear[e, y, r, s]

In[22]:= Simplify[
          Solve[e[t] == 110 + 0.75 (y[t] + 80 - 0.2 y[t]) + 320 - 4 r[t - 1] + 330 + s, e[t]]]

Out[22]= {{e[t] → 820. + 1. s - 4. r[-1 + t] + 0.6 y[t]}}
```

Lagging this one period to obtain e[t-1], we can then solve for y[t] in terms of lagged y[t] and lagged r[t], giving the IS-curve.

```
In[23]:= Simplify[Solve[y[t] == 820 + s - 4 r[t - 2] + 0.6 y[t - 1], y[t]]]

Out[23]= {{y[t] → 820. + 1. s - 4. r[-2 + t] + 0.6 y[-1 + t]}}
```

Obtaining the LM-curve from the money market equations, we get:

```
In[24]:= Simplify[Solve[20 + 0.25 y[t] - 10 r[t] == 470, y[t]]]

Out[24]= {{y[t] → 1800. + 40. r[t]}}
```

From this equation we immediately have that,

$r(t-2) = -45 + 0.025\,y(t-2)$

and subsituting this into the IS-curve we obtain,

```
In[25]:= Simplify[Solve[y[t] == 820 + s - 4 (-45 + 0.025 y[t - 2]) + 0.6 y[t - 1], y[t]]]

Out[25]= {{y[t] → 1000. + 1. s - 0.1 y[-2 + t] + 0.6 y[-1 + t]}}
```

We have, then, the difference equation for income

$y_t = 1000 + s + 0.6\,y_{t-1} - 0.1\,y_{t-2}$

The initial equilibrium has $s = 0$, and y is the same for all periods. Hence

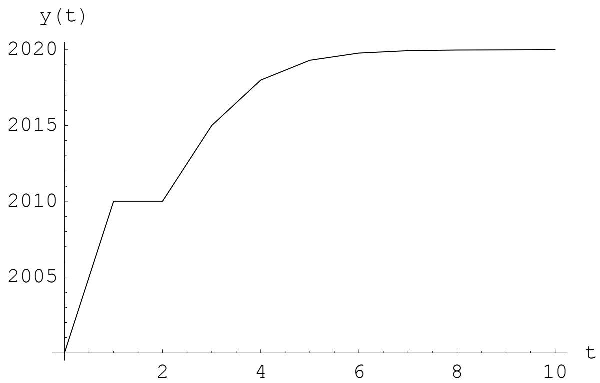
```
In[26]:= Clear[y, ybar, soly]

         General::spell1 :
          Possible spelling error: new symbol name "ybar" is similar to existing symbol "Ybar".

In[27]:= Solve[ybar == 1000 + 0.6 ybar - 0.1 ybar, ybar]

Out[27]= {{ybar → 2000.}}
```

With a shock s = 10, then the new equilibrium is

```
In[28]:= Solve[ybar == 1010 + 0.6 ybar - 0.1 ybar, ybar]

Out[28]= {{ybar → 2020.}}

In[29]:= Needs["DiscreteMath`RSolve`"]

In[30]:= soly = RSolve[{y[t] == 1010 + 0.6 y[t - 1] - 0.1 y[t - 2],
            y[0] == 2000, y[1] == 2010, y[2] == 2010}, y[t], t]

         RowReduce::luc : Result for RowReduce of badly conditioned matrix
            {{0., 0., 1., -2000.}, ≪4≫, {1., 0., 0., -2010.}} may contain significant numerical errors.

Out[30]= {}
```

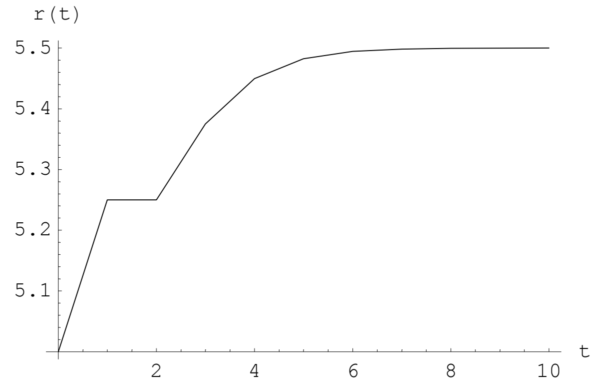
Mathematica refuses to give a solution for this difference equation. We can, however, list the points for y[t].

```
In[31]:= {y[0] = 2000, y[1] = 2010, y[2] = 2010};
         y[t_] := 1010 + 0.6 y[t - 1] - 0.1 y[t - 2]

In[33]:= income = Table[{t, y[t]}, {t, 0, 10}];

In[34]:= ListPlot[income, PlotJoined -> True, AxesLabel -> {"t", "y(t)"}];
```




```
In[35]:= interest = Table[{t, -45 + 0.025 y[t]}, {t, 0, 10}];
```

```
In[36]:= ListPlot[interest, PlotJoined -> True, AxesLabel -> {"t", "r(t)"}];
```



à Question 3

For this question we only need to use the data computed in question 1. The income series for each lag is contained in the series defined by Y1[t], Y2[t] and Y3[t].

```
In[37]:= Clear[Y1, Y2, Y3]

In[38]:= Y1[0] = 1640;
         Y1[t_] := 430 + 0.75 Y1[t - 1]

In[40]:= {Y2[0] = 1640, Y2[1] = 1660};
         Y2[t_] := 430 + 0.75 Y2[t - 2]

In[42]:= {Y3[0] = 1640, Y3[1] = 1660, Y3[2] = 1660};
         Y3[t_] := 430 + 0.75 Y3[t - 3]

In[44]:= y1series = Table[Y1[t], {t, 0, 20}];
         y2series = Table[Y2[t], {t, 0, 20}];
         y3series = Table[Y3[t], {t, 0, 20}];
```

We define the dynamic multiplier as:

$$k_t = \frac{\Delta \mathrm{Y}_t}{\Delta \mathrm{I}}$$

where $\Delta Y_t = (Y_t - \overline{Y})$ and $\overline{Y}$ is the initial equilibrium. With $\overline{Y} = 1640$ and $\Delta \mathrm{I} = 20$, then

```
In[47]:= kseries = Table[
            {t, (Y1[t] - 1640) / 20, (Y2[t] - 1640) / 20, (Y3[t] - 1640) / 20}, {t, 0, 20}];
```

```
In[48]:= TableForm[kseries, TableHeadings -> {{}, {"t", "k1(t)", "k2(t)", "k3(t)"}}]
```

Out[48]//TableForm=

t	k1(t)	k2(t)	k3(t)
0	0	0	0
1	1.	1	1
2	1.75	1.	1
3	2.3125	1.75	1.
4	2.73438	1.75	1.75
5	3.05078	2.3125	1.75
6	3.28809	2.3125	1.75
7	3.46606	2.73438	2.3125
8	3.59955	2.73438	2.3125
9	3.69966	3.05078	2.3125
10	3.77475	3.05078	2.73438
11	3.83106	3.28809	2.73438
12	3.87329	3.28809	2.73438
13	3.90497	3.46606	3.05078
14	3.92873	3.46606	3.05078
15	3.94655	3.59955	3.05078
16	3.95991	3.59955	3.28809
17	3.96993	3.69966	3.28809
18	3.97745	3.69966	3.28809
19	3.98309	3.77475	3.46606
20	3.98732	3.77475	3.46606

It can be seen that the multiplier for any period is smaller the longer the time lag.

à Question 4

(i)

Since

$$E_t = 110 + 0.75\, Y_t + 4\,(Y_t - Y_{t-1})$$

then

$$E_{t-1} = 100 + 0.75\, Y_{t-1} + 4\,(Y_{t-1} - Y_{t-2})$$

Substituting into the equilibrium condition $Y_t = E_{t-1}$, we can solve for Y[t].

```
In[49]:= Simplify[Solve[Y[t] == 110 + 0.75 Y[t - 1] + 4 (Y[t - 1] - Y[t - 2]), Y[t]]]

Out[49]= {{Y[t] → 110. - 4. Y[-2 + t] + 4.75 Y[-1 + t]}}

In[50]:= RSolve[Y[t] == 110 + 4.75 Y[t - 1] - 4 Y[t - 2], Y[t], t]

Out[50]= {{Y[t] →
           -0.25 (1. C[2] If[t ≥ -2, -16. 1.^t + 23.7722 1.09413^t - 105.022 3.65587^t, 0] +
              (1. C[1] - 5.75 C[2]) If[t ≥ -1, -16. 1.^t + 21.727 1.09413^t -
                 28.727 3.65587^t, 0] + (110. - 1. C[1] + 4.75 C[2])
               If[t ≥ 0, -16. 1.^t + 19.8578 1.09413^t - 7.85778 3.65587^t, 0])}}
```

(ii)

In this situation,

$$E_t = 110 + 0.75\,Y_t + 4\,(C_t - C_{t-1})$$
$$= 110 + 0.75\,Y_t + 4[110 + 0.75\,Y_t - (110 + 0.75\,Y_{t-1})]$$
$$= 110 + 0.75\,Y_t + 4\,(0.75)\,(Y_t - Y_{t-1})$$
$$= 110 + 3.75\,Y_t - 3\,Y_{t-1}$$

Hence

$$E_{t-1} = 110 + 3.75\,Y_{t-1} - 3\,Y_{t-2}$$

We immediately get,

$$Y_t = 110 + 3.75\,Y_{t-1} - 3\,Y_{t-2}$$

which is different from the difference equation in (i).

```
In[51]:= RSolve[Y[t] == 110 + 3.75 Y[t - 1] - 3 Y[t - 2], Y[t], t]

Out[51]= {{Y[t] → -0.333333
            (1. C[2] If[t ≥ -2, -12. 1.^t + 23.8477 1.15693^t - 59.2852 2.59307^t, 0] +
              (1. C[1] - 4.75 C[2]) If[t ≥ -1, -12. 1.^t + 20.6129 1.15693^t -
                 22.8629 2.59307^t, 0] + (110. - 1. C[1] + 3.75 C[2])
              If[t ≥ 0, -12. 1.^t + 17.8169 1.15693^t - 8.81694 2.59307^t, 0])}}
```

à Question 5

We shall take this opportunity to introduce some other of *Mathematica*'s commands. First we shall solve the original model as a set of simultaneous equations.

```
In[52]:= Clear[y, r, c, yd, tax, i, e, md, ms]

In[53]:= sol = Simplify[Solve[{c == 110 + 0.75 yd, yd == y - tax,
              tax == -80 + 0.2 y, i == 320 - 4 r, e == c + i + 330, md == 20 + 0.25 y - 10 r,
              md == ms, ms == 470, y == e}, {y, tax, yd, c, r, i, e, md, ms}]]

Out[53]= {{y → 2000., tax → 320., yd → 1680.,
           c → 1370., r → 5., i → 300., e → 2000., md → 470., ms → 470.}}
```

Using these solutions we can compute savings and the budget deficit:

```
In[54]:= s = yd - c  /. sol

Out[54]= {310.}

In[55]:= bd = 330 - tax /. sol

Out[55]= {10.}
```

All these values conform to the figures in the first row of Table 8.2.

The original IS-LM curves and the equilibrium values for *y* and *r* can be established as folows:

```
In[56]:= Simplify[Solve[e == 110 + 0.75 (y + 80 - 0.2 y) + 320 - 4 r + 330, e]]

Out[56]= {{e → 820. - 4. r + 0.6 y}}
```

```
In[57]:= Simplify[Solve[470 == 20 + 0.25 y - 10 r, y]]

Out[57]= {{y → 1800. + 40. r}}

In[58]:= Simplify[Solve[{y == 820 - 4 r + 0.6 y, y == 1800 + 40 r}, {y, r}]]

Out[58]= {{y → 2000., r → 5.}}
```

The dynamic specification of the IS-curve and the LM-curve are:

$$y_t = 820 + 0.6\, y_{t-1} - 4\, r_{t-1}$$
$$y_t = 1800 + 40\, r_t$$

From the second equation,

```
In[59]:= Simplify[Solve[y == 1800 + 40 r, r]]

Out[59]= {{r → -45 + y/40}}
```

Hence,

$$y_t = 820 + 0.6\, y_{t-1} - 4\,(-45 + \tfrac{y_{t-1}}{40})$$

```
In[60]:= Simplify[Solve[y == 820 + 0.6 y1 - 4 (-45 + y1/40), y]]

Out[60]= {{y → 1000. + 0.5 y1}}
```

But the IS-curve is shocked by an autonomous component of 20, so income rises by 20. Thus,

$$y_t = 1020 + 0.5\, y_{t-1}$$

```
In[61]:= y[0] = 2000;
         y[t_] := 1020 + 0.5 y[t - 1]

In[63]:= Table[y[t], {t, 0, 15}]

Out[63]= {2000, 2020., 2030., 2035., 2037.5, 2038.75, 2039.38, 2039.69,
          2039.84, 2039.92, 2039.96, 2039.98, 2039.99, 2040., 2040., 2040.}

In[64]:= Table[-45 + (y[t] / 40), {t, 0, 15}]

Out[64]= {5, 5.5, 5.75, 5.875, 5.9375, 5.96875, 5.98438, 5.99219, 5.99609,
          5.99805, 5.99902, 5.99951, 5.99976, 5.99988, 5.99994, 5.99997}
```

A shift in the LM-curve arsing from ms increasing by £20mill leads to,

```
In[65]:= Simplify[Solve[490 == 20 + 0.25 y - 10 r, y]]

Out[65]= {{y → 1880. + 40. r}}
```

The new equilibrium is readily found by solving the new IS-LM intersection.

```
In[66]:= Solve[{y == 820 - 4 r + 0.6 y, y == 1880 + 40 r}, {y, r}]

Out[66]= {{y → 2016., r → 3.4}}

In[67]:= Simplify[Solve[y == 1880 + 40 r, r]]

Out[67]= {{r → -47 + y/40}}
```

In other words we have the two dynamic equations,

$$y_t = 820 - 4\, r_{t-1} + 0.6\, y_{t-1}$$

$$r_t = -47 + \frac{y_t}{40}$$

Lagging the interest rate one period and substituting into the first equation we get,

$$y_t = 820 - 4\,(-47 + \frac{y_{t-1}}{40}) + 0.6\, y_{t-1}$$

which we can solve for y_t.

```
In[68]:= Simplify[Solve[y == 820 - 4 (-47 + y1/40) + 0.6 y1, y]]

Out[68]= {{y -> 1008. + 0.5 y1}}

In[69]:= ym[0] = 2000;
         ym[t_] := 1008 + 0.5 ym[t - 1]

In[71]:= Table[ym[t], {t, 0, 15}]

Out[71]= {2000, 2008., 2012., 2014., 2015., 2015.5, 2015.75, 2015.88,
          2015.94, 2015.97, 2015.98, 2015.99, 2016., 2016., 2016., 2016.}
```

Note that the series for r begins on the new LM curve. With $y = 2000$ then,

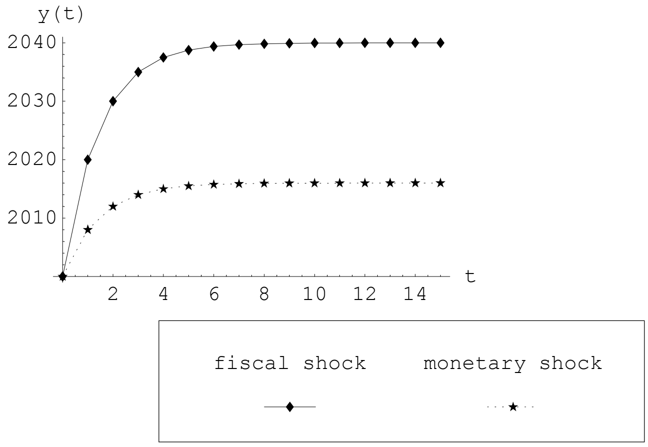
$$-47 + (2000/40) = 3$$

and the interest rate falls immediately from 5% to 3% with income remaining at 2000 and then rises gradually until it reaches 2016, i.e., the trajectory is up the new LM curve.

Here we shall finally make a comparison just between the time paths for income and the time paths for the rate of interest. All other endogenous variables can be calculated using one or more of these series.

```
In[72]:= gincome = Table[{t, y[t]}, {t, 0, 15}];

    General::spell1 :
     Possible spelling error: new symbol name "gincome" is similar to existing symbol "income".

In[73]:= mincome = Table[{t, ym[t]}, {t, 0, 15}];

    General::spell : Possible spelling error: new
       symbol name "mincome" is similar to existing symbols {gincome, income}.
```

```
In[74]:= MultipleListPlot[gincome, mincome,
           PlotJoined -> True, AxesLabel -> {"t", "y(t)"},
           PlotLegend -> {"fiscal shock", "monetary shock"},
           LegendPosition -> {-0.35, -1.2},
           LegendOrientation -> Horizontal, LegendSize -> {2, .5}];
```




```
In[75]:= ginterest = Table[-45 + (y[t] / 40), {t, 0, 15}];

General::spell1 :
 Possible spelling error: new symbol name "ginterest" is similar to existing symbol "interest".

In[76]:= minterest = Table[-47 + (ym[t] / 40), {t, 0, 15}];

General::spell : Possible spelling error: new symbol
   name "minterest" is similar to existing symbols {ginterest, interest}.

In[77]:= MultipleListPlot[ginterest, minterest,
           PlotJoined -> True, AxesLabel -> {"t", "r(t)"},
           PlotLegend -> {"fiscal shock", "monetary shock"},
           LegendPosition -> {-0.35, -1.2},
           LegendOrientation -> Horizontal, LegendSize -> {2, .5}];
```