Introduction to Part I

Three elements, introduced in Part I of the book, play a key role in the book itself.

The first is the derivation of the Maxwell–Bloch equations, which is carried out in Chapters 2–4. In particular the derivation of the field-envelope equation, discussed in Section 3.1, is explained with special care. The derivation is based, of course, on the slowly varying envelope approximation and on the paraxial approximation. In textbooks these two approximations are usually discussed separately and rather briefly. We show that the two approximations are intimately connected, and only in this way it is possible to demonstrate that, while the second-order derivatives with respect to time and to the longitudinal coordinate z can be neglected, the second-order derivatives with respect to the transverse coordinates x and y must be kept. It is also possible to demonstrate that the term $\nabla(\nabla \cdot \mathbf{E})$ in the Maxwell wave equation for the electric field can indeed be neglected. An additional bonus of this approach is that, once one has shown in detail the derivation of the envelope equation in the simplest context, it becomes straightforward to generalize it to the more complex cases of frequency dispersion in the background refractive index (Section 5.4) and of quadratic and cubic nonlinearities (Chapters 6 and 7).

The second key element is the use of a multiple limit in the parameters, which in the literature is called the mean-field limit or uniform-field limit, whereas in this book we prefer to name it more precisely the *low-transmission limit*. This plays a central role when we derive modal equations and single-mode models from sets of equations with propagation. In this way one avoids, for example, steps like the phenomenological introduction of a damping term in the field equation(s) to describe the escape of photons from the cavity, and derives the damping from the boundary conditions of the cavity in a transparent way (Chapters 12 and 14). In the book, the low-transmission limit is introduced in Section 9.2.

The last key element is the adiabatic elimination principle, which allows one to notably simplify the dynamical equations and obtain simpler pictures. This is discussed in Chapter 10.

The first chapter of the book illustrates the classic rate-equation model for the laser. This is the only case in which we discuss a phenomenological model. We do that because of the historical importance of this model and for pedagogical reasons. After Chapter 1 we turn to the semiclassical theory, and in Chapter 19 in Part II we derive this model in a rigorous way, with a clear specification of its limits of validity, in the case of class-B lasers.

Chapters 2–7 discuss the radiation–matter interaction in free propagation, whereas the following chapters of the book mainly consider the case in which the atomic medium is contained in an optical cavity. For the sake of simplicity, we consider mainly ring cavities, but in Chapter 14 and Section 22.7 we turn our attention to Fabry–Perot cavities.

The standard laser configuration is considered in Chapters 9, 15 (in this case, with inhomogeneous broadening) and 16. Chapter 16 focusses on the case of semiconductor lasers for two reasons, in addition to the technological importance of such lasers. One reason is that this gives us the chance of showing that, despite the complexity of the physics which governs the system, after a number of steps one arrives at a relatively simple model which, as shown in Section 19.3 in Part II, is identical to that which describes class-B lasers. The other reason is that in Section 30.3 of Part III we describe extensively the topic of cavity solitons in semiconductor lasers.

On the other hand, the case of passive, optically bistable systems is discussed in detail in Chapter 11, while Chapter 13 deals with kinds of light sources that represent variants of the laser, such as the laser with injected signal (Section 13.1) and the laser with saturable absorber (Section 13.2), or other sources of light such as the optical parametric oscillator (Section 13.4).

The final Chapter 17 extends the vision to systems that involve multilevel atoms, with the aim of illustrating some striking phenomena that arise from atomic coherence and quantum interference.

Introduction to Part II

In Part I of this book we studied almost exclusively stationary solutions of the models; we also mentioned in which parametric ranges they are stable or unstable, but without proving it. In this part we not only describe the technique used to assess the stability of stationary solutions but also, especially, study the temporal phenomena which originate from the instabilities themselves. As a matter of fact, the appearance of spontaneous pulsations in the form of undamped trains of pulses was first observed in masers even before the advent of the laser in 1960. However, a strong interest in temporal optical instabilities arose much later, in the early 1980s, in resonance, as it were, with the general interest in the field of chaos in nonlinear dynamical systems.

In Part II of the book we focus on the dynamical phenomena in nonlinear optical systems, of course taking the general viewpoint of nonlinear dynamical systems. In Chapter 18 we describe first the technique of linear-stability analysis and then we discuss the most relevant and general instability-related dynamical aspects in nonlinear dissipative systems such as, for example, attractors, bifurcations and routes to chaos. This provides the language to describe the results discussed in the following chapters of Part II. For reasons of space, we do not introduce technicalities such as, for instance, Lyapunov exponents.

In order to identify the instabilities, in general we prefer to use a procedure that is more direct than the Routh–Hurwitz stability criterion described in Appendix A. A more straightforward technique is to calculate the stability boundaries in the space of the parameters of the system, as described in Section 18.1 and Appendix B.

In Chapter 19 we discuss the prominent transient dynamical features in class-A and class-B lasers, as is usual in textbooks on laser physics, whereas in the following chapters of Part II we discuss temporal instabilities and their consequences. In Chapter 20 we outline a general scenario that encompasses and structures all temporal instabilities that arise in the framework of two-level systems in ring cavities. A characteristic trait of such an approach is that single-mode and multimode instabilities are treated in parallel, showing their intrinsic and profound interconnections. This notably facilitates the description of instabilities, spontaneous oscillations and optical chaos in lasers (Chapter 22 deals with homogeneously broadened lasers and Chapter 23 with inhomogeneously broadened lasers) and in passive optically bistable systems (Chapter 24). The final chapter, Chapter 25, deals with temporal instabilities in the same systems as were described in Chapter 13, i.e. a laser with an injected signal, a laser with a saturable absorber and an optical parametric oscillator. As in Part III, many experimental results are presented and discussed, in close connection with the theoretical predictions. General discussions of temporal optical instabilities are included in the special issues [211–219] and in the review articles and books [86, 130, 220–233].

Introduction to Part III

The field of optical pattern formation concerns the spatial and spatio-temporal phenomena that arise in the structure of the electromagnetic field in the planes orthogonal with respect to the direction of propagation. Most theoretical treatments of the interaction between matter and radiation introduce the plane-wave approximation, i.e. assume that the electric field is uniform in each transverse plane. In this way, the time-evolution equations depend only on one spatial variable, i.e. the longitudinal variable z which corresponds to the direction of propagation. By dropping the plane-wave approximation, one opens the door to the fascinating world of pattern formation. In the paraxial approximation this corresponds to keeping, in the time-evolution equation of the electric field, the term with the transverse Laplacian which describes diffraction of radiation; this term couples the different points of the transverse (x, y) plane, as is necessary for pattern formation. With the exception of the discussion of spatial Kerr solitons in Section 7.3, in the analysis of Maxwell–Bloch equations, or of other nonlinear optical models, in Parts I and II of this book we have always adopted the plane-wave approximation, whereas now in Part III we study the transverse effects.

The interaction of light with linear inhomogeneous media can give rise to spatial structures of interesting and remarkable complexity. However, the field of optical pattern formation concerns mainly the interaction with nonlinear media, where the phenomena emerge spontaneously as a consequence of an instability; another name that is commonly used to designate optical pattern formation is "transverse nonlinear optics" or "optical morphogenesis". Historically, the interest in optical pattern formation emerged as a natural evolution of the previous development of the field of optical instabilities and chaos, when the main attention shifted gradually from purely temporal effects to spatio-temporal phenomena. The evolution was made possible also by the spectacular increase of the computational capabilities of available computers. For both of the fields of optical instabilities and optical pattern formation, continuous inspiration arose from the formulation of general disciplines such as Haken's synergetics [91, 92] and Prigogine's theory of dissipative structures [234, 235].

The existence of transverse effects has been well known since the earliest days of laser physics. In order to obtain the simple Gaussian transverse structure which is desired for most applications, one introduces apertures in the laser cavity. Otherwise, the system spontaneously generates more or less complex configurations. These phenomena were, however, mostly considered undesirable or difficult to control. A basic understanding, though, was achieved with the analysis of diffractive effects upon propagation of Gaussian beams with phenomena such as self-focussing and self-defocussing.

Despite the existence of an important early literature on this subject (see Refs. 8–22 in [306]), systematic investigations on optical pattern formation were initiated only in the

1980s. A great impulse was given by the attention devoted to the case of nonlinear materials contained in optical cavities in the low-transmission and single-longitudinal-mode limits, because such systems are described by sets of partial differential equations with two spatial variables plus time, exactly as in the case of two-dimensional patterns in hydrodynamics and nonlinear chemical reactions [307–309]. On the other hand, the study of systems with a single feedback mirror [310, 311] produced the best compromise between simplicity of theoretical treatment and accessibility to experimental realization.

The simplest models for studying optical pattern formation assume translational symmetry in the transverse plane. This implies that, if there are mirrors, they must be plane mirrors; and, if there is a driving field, it must have a plane-wave configuration. This is the most fundamental setting, because it allows one to study the spontaneous onset of a pattern from a homogeneous state with breaking of the translational symmetry, as a consequence of an instability arising from the interplay of nonlinearity and diffraction. In this case one can perform analytically both the calculation of the homogeneous stationary solution and its linear-stability analysis. When there is a cavity, the modes of the empty cavity correspond to the plane waves tilted with respect to the propagation direction, with a continuous frequency spectrum.

The assumption of translational symmetry implies that the system is infinitely extended in the transverse directions, and this is conveniently formalized by using periodic boundary conditions. Clearly, this kind of model is strongly idealized, because in practice one always has a beam confined to a certain region of the transverse plane. A confinement can be introduced in different ways, e.g. by including in the model the transverse shape of the medium or, if an input beam is present, by considering a Gaussian or a flat-top transverse profile. In this case, everything must be calculated numerically. When the transverse dimensions of the confinement region are much larger than the length which characterizes the spatial modulation of the pattern, one can recover the results of the idealized model qualitatively.

In the cavity case, one can obtain the transverse confinement by considering spherical instead of planar mirrors. In this case the system has only rotational symmetry; the modes of the empty cavity correspond to Gauss–Laguerre functions, with a discrete frequency spectrum. An important advantage of this configuration is that the number of modes in play can be controlled by the geometrical parameters of the cavity. For example, by reducing the Fresnel number one can cut off Gauss–Laguerre modes of high order. By varying the radius of curvature of the spherical mirrors and their distance, one controls the frequency spacing between adjacent transverse modes.

The overwhelming diverse phenomena one meets in optical pattern formation display several similarities with those, for example, in hydrodynamics and nonlinear chemical reactions, where diffusion and not diffraction couples to the nonlinearity to govern the arising patterns. A formal analogy between laser equations and hydrodynamics is described in Section 27.5. In definite domains of the parameter space, the exact dynamical equations can be approximated by Ginzburg–Landau or Swift–Hohenberg equations, or similar equations; pattern formation in a wide variety of fields has been described by these types of equations. For the sake of keeping our book to a reasonable size, we do not discuss such equations.

Undoubtedly, hydrodynamics and nonlinear chemical reactions have a much longer tradition than optics has in the study of pattern formation. However, radiation–matter interaction is fundamental in physics and chemistry, and this is already a strong motivation for studying pattern formation in optics. In addition, optics presents two special features that are interesting and stimulating. First, optical systems are very fast and have a large frequency bandwidth; hence they lend themselves naturally to applications, e.g. in telecommunications and information technology. The most relevant example of a useful application of optical structures is provided by solitonic transmission in optical fibers. The investigations of transverse nonlinear optics offer, in principle, the possibility of an approach to parallel optical information processing, by encoding information in the transverse structure of the electric field.

The second special feature is that optical systems are macroscopic or mesoscopic, yet they are capable of displaying interesting quantum effects, even at room temperature. This has led to investigations on the quantum aspects in transverse optical structures, which in turn have contributed in an important way to the birth of a new field called quantum imaging [67].

Chapter 26 studies the propagation of the electric field in vacuum, including transverse effects, and calculates the fundamental Gaussian beams and the higher-order Gauss–Hermite or Gauss–Laguerre beams. This classic piece of analysis is essential to describe the modes of cavities with spherical mirrors, both in the case of Fabry–Perot cavities and in the case of ring cavities.

The remaining chapters deal with optical pattern formation arising from the nonlinear interaction of radiation with matter, with the illustration of key concepts necessary to understand the nature of the investigations in the field of optical pattern formation and their developments. Chapters 27 and 28 treat the case of cavities with planar mirrors and Chapter 29 the configuration of cavities with spherical mirrors. Outstanding aspects relating to applications concerning the temporal version of cavity solitons are illustrated in Section 28.2.

The final chapter, Chapter 30, treats the aspects which are most interesting for the possibility of practical applications of the field of optical pattern formation. We describe the formation and manipulation of cavity solitons in cavities with planar mirrors. Especially interesting is the case of broad-area vertical-cavity surface-emitting lasers (VCSELs) driven by an external coherent field.

Reviews of the field of optical pattern formation can be found in [229,306,312,314–323]; a special issue can be found in [324]. Transverse effects in cold atomic systems are discussed in [325].