Appendix B: Exercises

B.1 Introduction to Digital Image Processing

1. Edge detection and edge enhancement.
   (a) Specify a differential operator (high-pass filter) to detect the horizontal edges in an image. Do the same for vertical edge detection.
   (b) How can edges of arbitrary direction be detected using the above two operators?
   (c) How can these operators be exploited for edge enhancement?

2. What is the effect of a convolution with the following 3×3 masks?

1	2	1
2	4	2
1	2	1

−1	0	1
−2	0	2
−1	0	1

0	−1	0
−1	4	−1
0	−1	0

0	−1	0
−1	5	−1
0	−1	0

   Are these operators used in clinical practice? Explain.

3. Calculate the convolution of the following 3×3 convolution mask and the image in the demarcated rectangle

3×3 mask

−1	0	1
−2	0	2
−1	0	1

image

0	0	0	0	1	1	1	1	1	1
0	0	0	0	1	1	1	1	1	1
0	0	0	0	1	1	1	1	1	1
0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1
0	0	0	0	0	1	1	1	1	1
0	0	0	0	1	1	1	1	1	1
0	0	0	0	1	1	1	1	1	1
0	0	0	0	1	1	1	1	1	1

4. What is the effect of the following convolution operators on an image?
   - The Laplacian of a Gaussian $\nabla^2 g\left(\vec{r}\right)$.
   - The difference of two Gaussians

     $$g_1\left(\vec{r}\right) - g_2\left(\vec{r}\right) \text{ with different } \sigma.$$

   - The 3×3 convolution mask

1	1	1
1	−8	1
1	1	1

5. Unsharp masking is defined as

   $$(1+\alpha)\, I\left(x,y\right) - \alpha\, g * I\left(x,y\right),$$

   with $I\left(x,y\right)$ the image, g a Gaussian, and α a parameter. The following convolution mask is an approximation of unsharp masking

−1/8	−2/8	−1/8
−2/8	?	−2/8
−1/8	-2/8	−1/8

   Calculate the missing central value.

B.2 Radiography

1. X-rays.
   (a) What is the physical difference between X-rays, γ-rays, light and radio waves? How do they interact with tissue (in the absence of a magnetic field)?
   (b) Draw the X-ray tube spectrum, i.e., the intensity distribution of X-rays as a function of the frequency of emitted X-ray photons (1) at the exit of the X-ray tube before any filtering takes place, and (2) after the filter but before the X-rays have reached the patient.
   (c) How does the tube voltage influence the wavelength of the X-rays?
   (d) Draw the linear attenuation coefficient (for an arbitrary tissue type) as a function of the energy.

2. What is the effect of the kV and mAs of an X-ray tube on

   (a) the patient dose, and
   (b) the image quality?

3. A digital radiograph is acquired but the CNR in the region of interest is insufficient. Calculate the relative CNR and relative dose of the following options

   (a) double the mAs;
   (b) take two radiographs and calculate the average image;
   (c) apply a window/level image operation;
   (d) change the kV.

4. A radiograph of a structure consisting of bone and soft tissue (see Figure B.1) is acquired by a screen-film detector.

   The exposure time is 1 ms. The radiographic film has a sensitometric curve $D = 2 \log E$. The film-screen system has an absorption efficiency of 25%. Assume that the X-rays are monochromatic and the linear attenuation coefficients of bone, soft tissue, and air are respectively 0.50 cm^{-1}, 0.20 cm^{-1}, 0.00 cm^{-1}.

   (a) Calculate the optical density D in positions A through E of the image.
   (b) Calculate the contrast, i.e., the difference in density, between positions B and C. How can this contrast be improved?

5. In mammography the breasts are compressed with a paddle. Explain why.

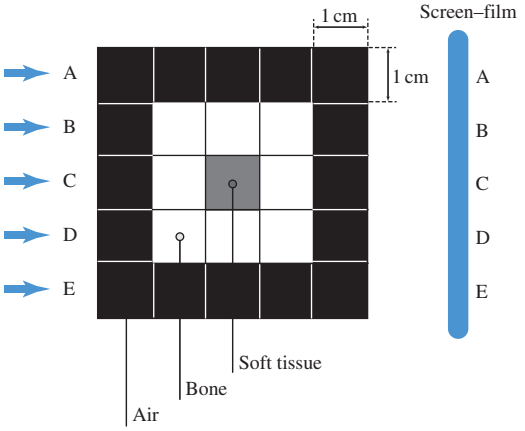


Figure B.1

B.3 X-ray Computed Tomography

1. Linear absorption coefficient.

   (a) Although the linear absorption coefficient μ depends on the energy, this dependence is not taken into account in filtered backprojection. Explain.
   (b) What is the effect of this approximation on the image quality?

2. Given an image consisting of two small bars, and projection $p(r, \theta)$ at angle θ (Figure B.2).

   An enlargement of this projection at angle θ looks as in Figure B.3 and can mathematically be written as

$$p(r) = \Pi\left(\frac{r}{3\Delta l}\right) - \Pi\left(\frac{r}{\Delta l}\right).$$

   (a) Calculate the Fourier transform of this projection.
   (b) The projection is now measured with an X-ray beam with width $\Delta s = 3\Delta l$. Assume a rectangular slice sensitivity profile (SSP).

   – Calculate the resulting Fourier transform of this measured projection.
   – Draw the measured projection (as a function of r).
   – What is the maximal size of Δs you would recommend?

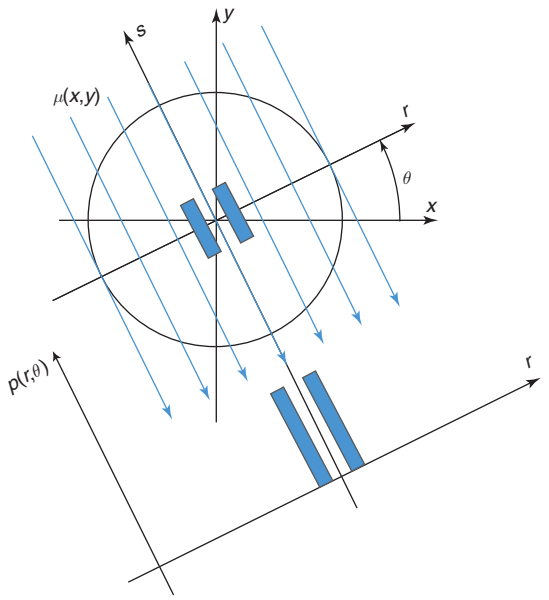


Figure B.2

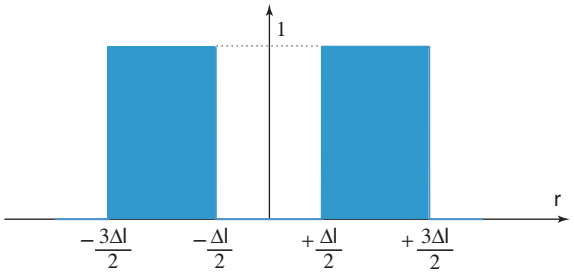


Figure B.3

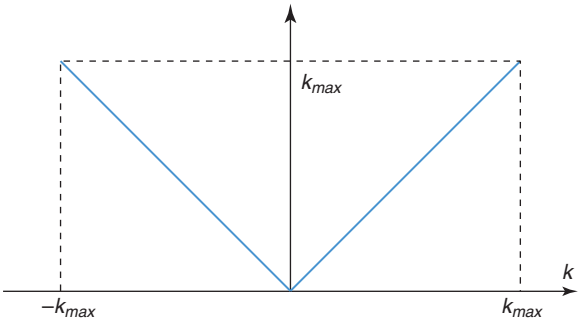


Figure B.4

– What is the maximal sampling distance Δr you would recommend?

3. (a) Filtered backprojection (FBP) uses the ramp filter $|k|$ cut off at k_{max} (Figure B.4)
   (b) Prove that the inverse Fourier transform of this function is

$$q(r) = \frac{k_{max}\sin(2\pi k_{max}r)}{\pi r} - \frac{1-\cos(2\pi k_{max}r)}{2\pi^2 r^2}$$

   knowing that this filter (i.e., the ramp filter $|k|$ cut off at k_{max}) can be written as the difference of a block and a triangle. Start by writing the exact mathematical expression of this block and triangle.

A sharp cut off at k_{max} is often avoided by suppressing the highest spatial frequencies. Assume the filter function as shown in Figure B.5 instead of the ramp filter.

Calculate the inverse Fourier transform of this function and compare it to $q(r)$.

4. Multi-slice helical CT with 64 detector rows, 180° interpolation, 4 rotations per second, and scan length 40 cm.

In Section 3.3.3.1 we have assumed a rectangular slice sensitivity profile (SSP) of width Δz.

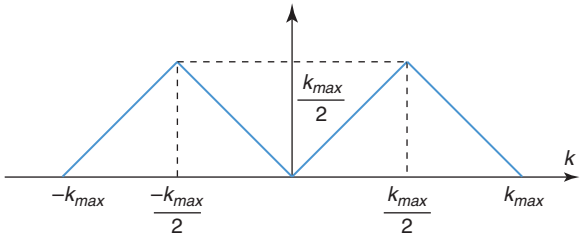


Figure B.5

In practice, however, the sensitivity will be higher in the center of the slice profile. Let us assume a triangular SSP $\Lambda(\frac{z}{\Delta z})$ with $\Delta z = 0.6$ mm in the center of the FOV.

(a) What is the effect on the resolution as compared to a rectangular SSP of width $\Delta z = 0.5$ mm. Explain.
(b) Calculate the maximal value of the table feed TF to avoid aliasing?
(c) Calculate the scan time.

Note: the Fourier transform of $\Lambda(\frac{z}{\Delta z})$ is

$$\frac{\Delta z}{2}\text{sinc}^2(\pi\frac{\Delta z}{2}k).$$

5. Given are two different tissues a and b. Two different detector sizes are used (Figure B.6).

In the first case the detector is twice as large as in the second case.

(a) Calculate the linear attenuation coefficients μ_a, μ_b, and μ_{a+b} from the input intensity I_i and the output intensities I_o, I_{oa}, and I_{ob}.
(b) Show that μ_{a+b} is always an underestimate of the mean linear attenuation $(\mu_a + \mu_b)/2$.
(c) What is the influence of this underestimate on a reconstructed CT image? Explain.

6. Assume multi-slice scanning without table motion. The detector width in the center of the FOV is 0.5 mm. Assume a block-shaped SSP in the axial direction (z-direction, see Figure B.7).

(a) Draw schematically the Fourier transform in the z-direction of the measured (sampled) projections.
(b) What is the maximum useful frequency of the sampled signal in the z-direction?
(c) What is the minimal distance δ in the z-direction between small details to be distinguishable? (Represent neighboring details by a sinusoidal function.)

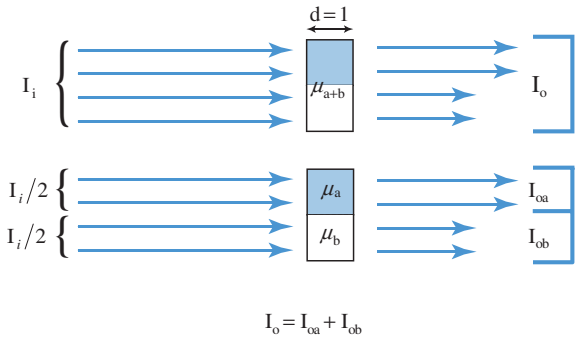


Figure B.6

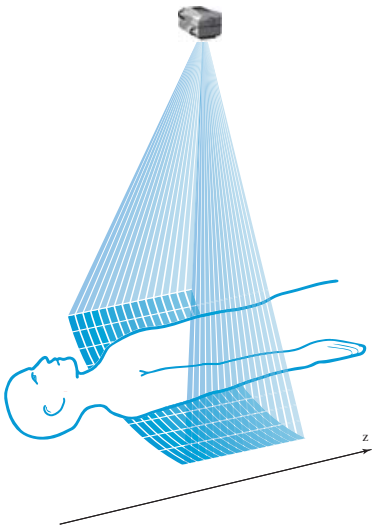


Figure B.7

7. Helical CT. Assume that β is the angular position of the X-ray tube and z its axial position.

   (a) Draw the data acquisition trajectory in the (β, z) space. What is the maximal table feed to avoid aliasing in case of 360° interpolation?
   (b) What is the maximal table feed to avoid aliasing in case of 180° interpolation? Show this in the (β, z) space.
   (c) What is the maximal table feed to avoid aliasing in case of dual-source CT in which the two X-ray tubes are positioned 90° apart (and operate simultaneously at equal kV). Draw the data acquisition trajectories in the (β,z) space for the two X-ray sources and show how the use of two sources influences the table feed.

8. Rewrite Eqs. 3.49–3.53 for maximum-likelihood reconstruction of CT images if the Poisson distribution is replaced by a Gaussian distribution.
9. Cardiac CT. The following conditions are given.

   – A CT scanner with 128 detector rows.
   – The detector width in the center of the FOV is 0.5 mm.
   – A full rotation (360Ł) of the X-ray tube takes 0.33 s.
   – A full dataset for reconstruction requires projection values for a range of 210°.
   – Maximum 1/4 of the heart cycle can be used for acquiring projection data.
   – The heart rhythm is 72 bpm.
   – The scan length is 20 cm.

   (a) Calculate the duration of 1/4 heart cycle (in seconds).
   (b) Calculate (in seconds) the time needed to obtain projection values for a range of 210°.
   (c) What can you conclude from (a) and (b)?
   (d) Assume that the table shift per heart beat equals the total width of the detector rows (i.e., the total z-collimation). Calculate the acquisition time.
   (e) The assumption under (d) is approximate. Explain why? How does this approximation influence the acquisition time?

10. Dual-energy CT. Given

    – Water and iodine are used as the basis materials.
    – A patient is injected with a iodine containing fluid. The peak concentration of injected iodine in the blood for this patient is 10 mg/ml.
    – The voxels consist of bone, soft tissue, water, and/or iodine.

    (a) Draw schematically the tissue-specific coefficients of the voxels (in mg/ml).
    (b) How can we obtain a precontrast (unenhanced) and a contrast scan?

11. Dual-energy CT.

    (a) Explain, using mathematical equations, how the images of Figure B.8 were obtained.
    (b) What are the advantages and applications of dual-energy CT?
    (c) What is photon counting and what is its relationship with dual-energy CT?

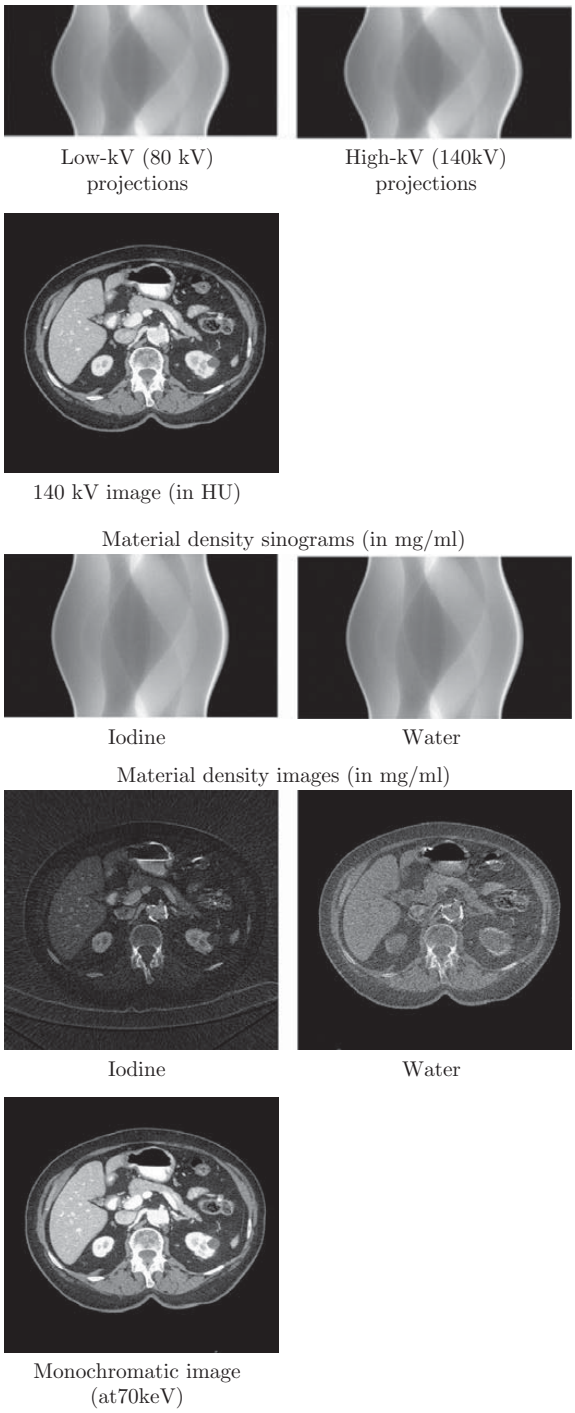


Figure B.8

12. CT of the lungs on a 64-row scanner, 120 kV, 90 mA s, pitch 1, 360Ł rotation time 0.33 s, detector width 0.60 mm, slice thickness 1 mm, scan length 38.4 cm, $CTDI_{vol}$ 10 mGy.
    (a) Calculate the scan time.
    (b) Calculate the estimated effective dose. Certain organs are only partially and/or indirectly (scatter) irradiated. The following table gives for each of the irradiated organs the percentage of irradiated tissue, and the tissue weighting factor w_T.

colon	0.5%	0.12
lungs	100%	0.12
breast	100%	0.12
stomach	50%	0.12
bone marrow	25%	0.12
thyroid gland	15%	0.04
liver	50%	0.04
esophagus	100%	0.04
bladder	1%	0.04
skin	25%	0.01
bone surface	30%	0.01
remainder	30%	0.12

13. CT of the lungs. The following table shows the irradiated organs, the irradiated portion of the organs (in %), and the tissue weighting factor w_T.

lungs	100%	0.12
breast	100%	0.12
stomach	65%	0.12
bone marrow	30%	0.12
liver	50%	0.04
esophagus	00%	0.04
remainder	30%	0.12

    The average regional conversion factor $k = 0.016$ mSv/(mGy.cm). Calculate the scan length.
14. How can the resolution of a CT image be improved without increasing the dose? How can the SNR of a CT image be improved without increasing the dose?
15. Micro-CT. A micro-CT scanner operates according to the same principles as a clinical CT scanner, but its size is adapted to small animals and small objects. The pixel size is in the order of micrometers. Assume that a patient undergoes a CT-scan and afterwards a small piece of soft tissue is

removed and further investigated using micro-CT. Compare for this case (i.e., clinical CT of human body versus micro-CT of small, in vitro, soft tissue)

(a) The current, time (mAs) en voltage (kV). Explain.
(b) The image quality factors assuming that the best image quality is desired. Explain.

16. Assume a dedicated maxillofacial cone-beam CT scanner operating at 100 kV and 40 mAs. The beam thickness in the center of the FOV is 60 mm. $CTDI_w$ = 90 mGy and the average regional conversion factor $k = 2.0 \times 10^{-3}$ mSv/(mGy cm).

(a) Calculate the effective dose. Show the details of your calculations.
(b) Draw the X-ray beam intensity as a function of the wavelength and show how the kV and the mA influence the intensity? How do the kV and the mA influence the absorbed dose?
(c) What is the influence of dental amalgam fillings (a mixture of metals) on the image quality? How can the quality be improved in this case? Explain.

B.4 Magnetic Resonance Imaging

1. Explain the artifacts in the images of Figure B.9.
2. Assume an MRI spin-echo (SE) sequence with B_0 = 0.5 T (see Figure B.10).
   The following conditions are given.

   – In all the images TR = 2000 ms. From (a) to (d) TE = 25 ms, TE = 50 ms, TE = 100 ms, and TE = 200 ms respectively.
   – T_1 (white brain matter) ~ 500 ms and T_1 (gray brain matter) ~ 650 ms.

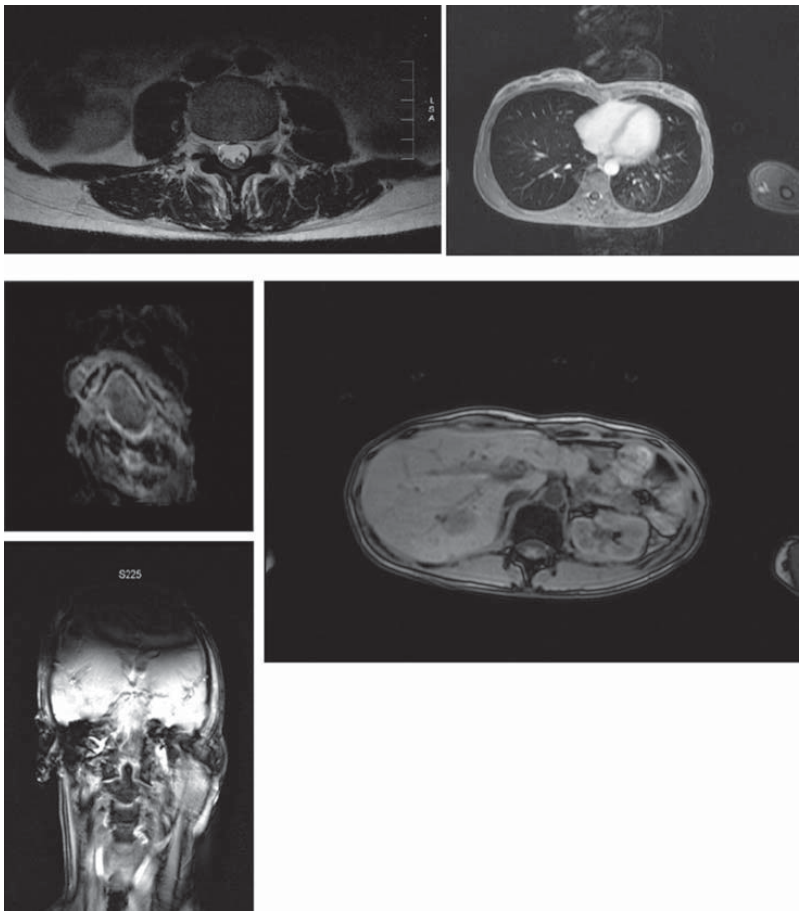
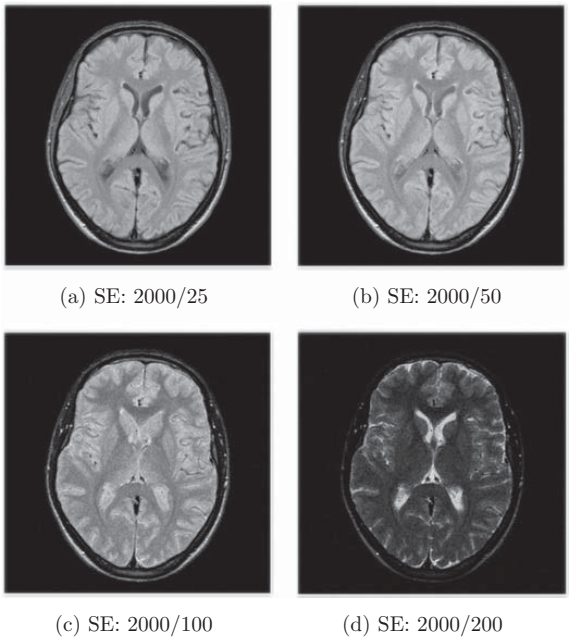


Figure B.9



(a) SE: 2000/25 (b) SE: 2000/50

(c) SE: 2000/100 (d) SE: 2000/200

Figure B.10

- T_2 (white brain matter) $\sim$ 90 ms and T_2 (gray brain matter) $\sim$ 100 ms.
- T_1 (CSF) $>$ 3000 ms and T_2 (CSF) $>$ 2000 ms.
- The proton density of gray matter is 14% higher than that of white matter.

The relative signal intensity can be expressed as

$$s(t) = \rho\, e^{-\frac{TE}{T_2}} \left[1 - e^{-\frac{TR}{T_1}}\right].$$

(a) First, draw (schematically) the longitudinal magnetization (M_z) as a function of time after a 90° pulse for white and gray matter and for CSF (cerebrospinal fluid).
(b) Next, draw (schematically) the transverse magnetization (M_{xy}) as a function of time after a 90° pulse for white and gray matter and for CSF (note that TR = 2000 ms).
(c) Explain now on this last diagram why the contrast between CSF and surrounding white brain and brain matter varies in Figure B.10(a–d).

3. T_1-weighted spin-echo sequences of one 2D slice of a phantom containing three different tissues are acquired. The repetition time (TR) is 400 ms. FOV 300 × 300 mm. Number of phase encoding steps 240. Slice thickness 8 mm.

   - Tissue A with T_1 = 80 ms and T_2 = 50 ms.
   - Tissue B with T_1 = 400 ms and T_2 = 100 ms.
   - Tissue C with T_1 = 2000 ms and T_2 = 1000 ms.

   Assume that the tissues all have the same proton density.

   (a) Draw the T_1-relaxation of each tissue as accurately as possible.
   (b) Draw the T_2-relaxation of each tissue as accurately as possible, taking the T_1-relaxation and the repetition time (TR = 400 ms) into account.
   (c) Assume an image with no T_2-weighting.
      - On a gray scale from 0 (dark) to 100 (bright), specify the gray value of each of the three tissues.
      - What is the acquisition time? Show the details of your calculations.
   (d) TE = 50 ms. Same questions as (c).

4. (a) See Figure B.11. If we assume that their proton densities are almost identical, draw the longitudinal and transverse relaxation curves for liver (star, T_1 550 ms, T_2 50 ms), spleen (circle, T_1 750 ms, T_2 80 ms), fat (square, T_1 200 ms, T_2 100 ms) and peritoneal water (triangle, T_1 3500 ms, T_2 2000 ms). Use an excitation RF pulse of 90° as start of the relaxation curves.
   (b) Draw for each image the spin-lattice and spin-spin relaxation curves and indicate TR and TE on these curves.

5. See Figure B.12.

   - Both images are obtained with a SE sequence (90° excitation pulse), TR = 1500 ms.
   - In the right image fat was suppressed with an inversion pulse (STIR = short TI inversion recovery).
   - Assume identical proton densities for all tissues.

   For both images

   (a) Draw the approximate longitudinal and transverse relaxation curves for fat-like behaving bone marrow (star, T_1 300 ms, T_2 100 ms), muscle (circle, T_1 1200 ms, T_2 120 ms), and water (triangle, T_1 3500 ms, T_2 2000 ms).
   (b) Indicate TE on the transverse relaxation curves.

6. (a) See Figure B.13(a). Assuming identical proton densities for muscle and water, draw the approximate longitudinal and transverse relaxation curves for water in the lungs (star, T_1 3500 ms, T_2 2000 ms) and muscle (arrow, T_1 1200 ms, T_2 120 ms). Indicate on these graphs the moment of the measurement. Explain why the healthy regions of the lungs appear dark in this image.
   (b) See Figure B.13(b). Same assumption and questions as in (a). This image was acquired after injection of a gadolinium-based contrast agent. Draw also the longitudinal and transverse relaxation curves of blood (with gadolinium) (circle) and explain why the signal of blood is bright.

7. The MR images in Figure B.14 were acquired with a SE sequence (90° pulse) at 1.5 T. For the lower right image a STIR (short TI inversion recovery) pulse sequence was used. T_1 (CSF) > 3000 ms and T_2 (CSF) > 2000 ms; T_1 (fat) = 200 ms and T_2 (fat) = 100 ms.
   (a) Draw the magnetization $|M_z|$ as a function of time for CSF and for fat for a SE sequence without and with an inversion pulse respectively.
   (b) Calculate the inversion time TI.
   (c) Draw the magnetization $|M_{xy}|$ as a function of time for CSF and for fat for each of the images (i.e., for TR = 3000 with and without saturation, and for TR = 300). Note that in practice the magnitude of the complex signal is calculated. Hence, negative signals are inverted.
   (d) Explain the contrast between CSF and fat in each of the four images.

8. The following table shows typical values of T_1 and T_2 at 1.5 Tesla for a few substances

Tissue	T_1 (ms)	T_2 (ms)
fat	260	80
liver	500	40
muscle	870	45
aqueous	1500	1400

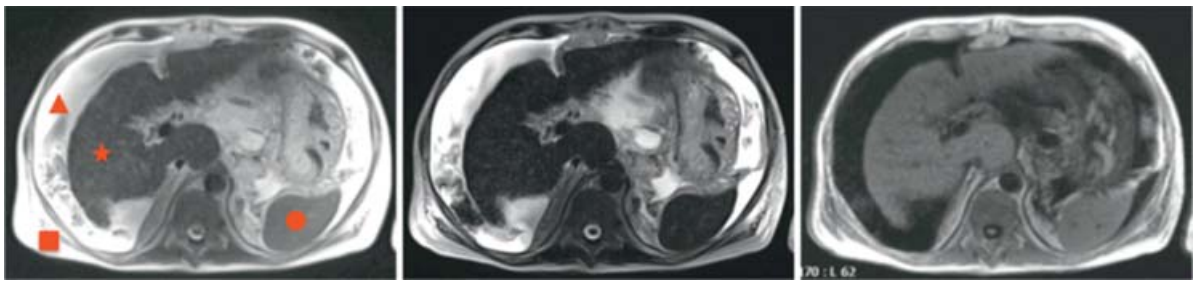


Figure B.11

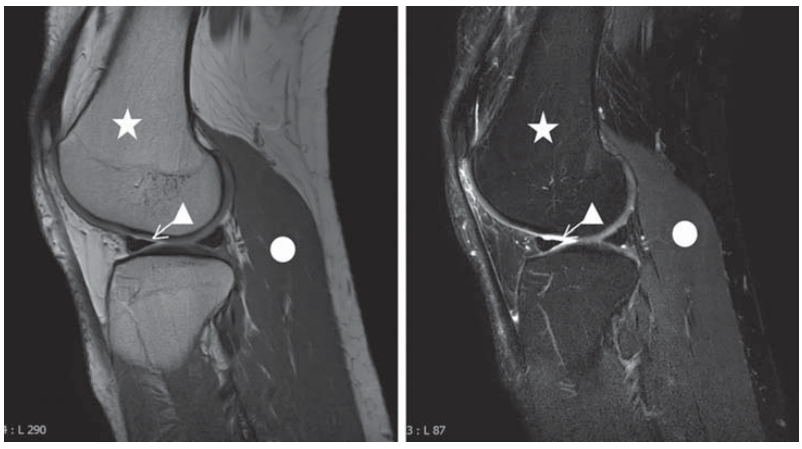
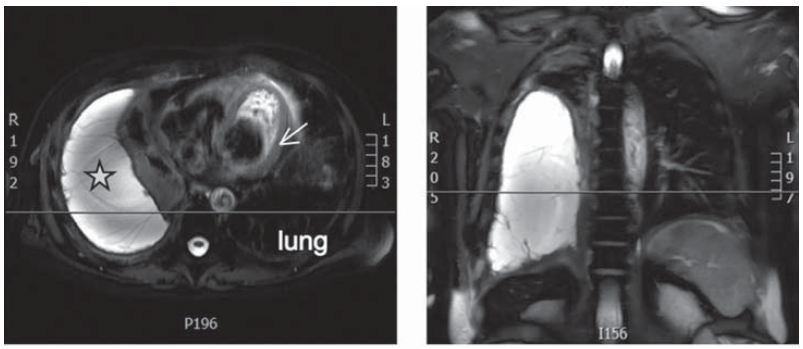
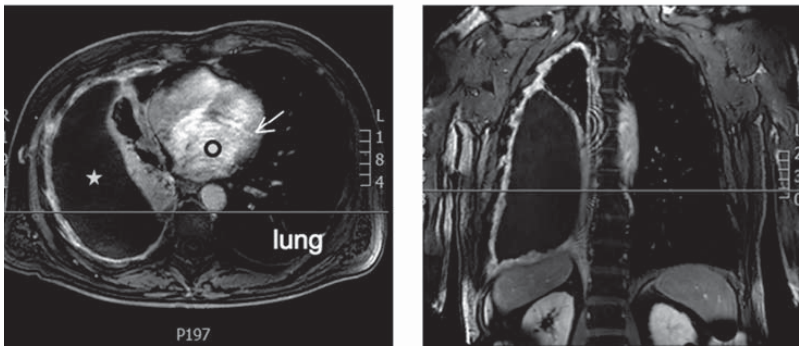


Figure B.12



(a) 2D SE with fat suppression.



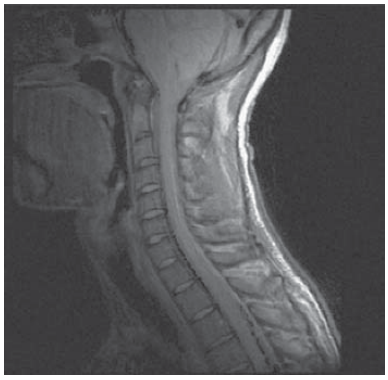
(b) 3D GE with fat suppression.

Figure B.13

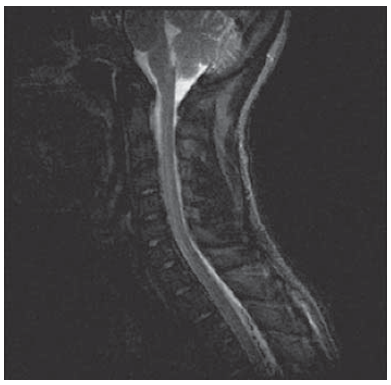
The simulated images in Figure B.15 show an identical abdominal slice, obtained with inversion-recovery (IR) sequences at 1.5 Tesla. The window/level parameters are the same for all these images. The parameters of the sequence (TR, TE, TI), however, are different.

(a) Calculate the ideal inversion time TI for fat suppression.
(b) Draw, for each sequence (with parameters TR, TE, TI), the longitudinal magnetization $|M_z|$ of the four given substances as a function of time. Mark the moment of the measurement in each diagram.
(c) Localize the four given substances based on your answer to the previous question.
(d) What would change if the images are acquired at 3 Tesla instead of 1.5 Tesla?

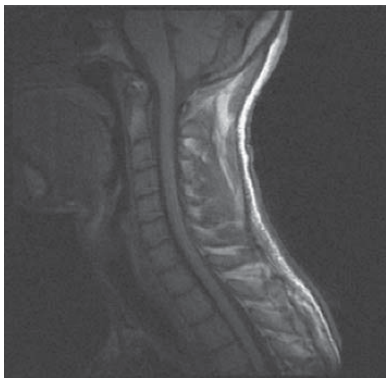
9. Suggest one or more categories for the lesion in the images of Figure B.16.
10. Assume that the magnetic field of an MRI magnet lies along the z-axis, i.e., $\vec{B} = (0, 0, B_0)$. The MRI system has three orthogonal gradient systems. We know that for protons $\gamma/2\pi = 42.57$ MHz/T.
    (a) What is the precession frequency of protons in a main magnetic field $B_0 = 1.5$ T? Give the result in Hz, not in rad/s.
    (b) The strengths of the gradients at a certain moment are G_x, G_y, G_z. What is the *magnitude* and the *direction* of the total external magnetic field in an arbitrary point (x,y,z) inside the imaging volume?
11. A 2-mm slice perpendicular to the z-axis at position $z = 0.1$ m is excited with a radiofrequency pulse at frequency f. Assume $B_0 = 1.5$ T and $G_z = 10$ mT/m. What is the frequency f (in Hz) of the RF pulse to excite this slice? And what is the bandwidth (in Hz)?
12. In 2D MRI a rectangular SSP requires that the RF pulse is a sinc function.
    (a) Explain. Draw this sinc function for a rectangular SSP with a width of 1 mm. In practice, however, this is impossible because a sinc function has an infinite extent. Therefore the sinc function is truncated.
    (b) What is the effect on the resulting SSP?
    (c) What is the maximum distance (in mm) between two slices to minimize aliasing in the



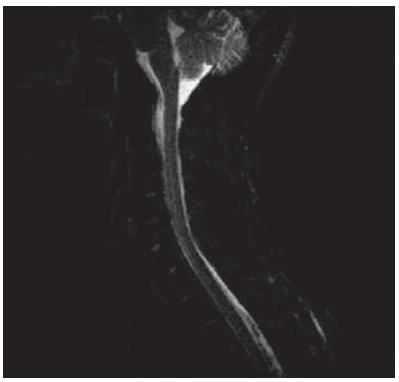
TR = 3000, TE = 30



TR = 3000, TE = 150



TR = 300, TE = 30



TR = 3000, TE = 150 TI = ?

Figure B.14

z-direction, i.e., perpendicular to the slices? Explain.

13. What is the corresponding $\vec{k}$-space of the phantom images in Figure B.17(a), (b), and (c)? Make your choice from images (1), (2), and (3). Explain your choice.
14. Chemical shift.

    (a) Draw schematically a pulse sequence for 2D chemical shift imaging.
    (b) What is the chemical shift artifact?
    (c) Calculate the acquisition time of this pulse sequence. (Choose acceptable values for the different parameters.)

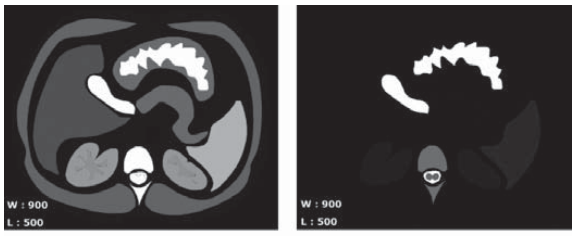
15. In order to reduce the acquisition time, multiple lines in the $\vec{k}$-space can be measured per excitation.

    (a) Assume that four lines per excitation are measured. Draw these lines in the $\vec{k}$-space.
    (b) What is the effect on the image quality (as compared to measuring only one line per excitation)

        – If the lowest frequencies are measured first?
        – If the highest frequencies are measured first?

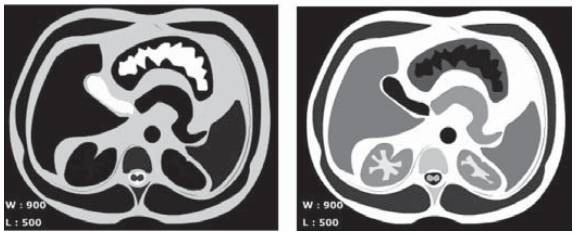
16. For imaging of tumoral invasion in fat-like bone marrow a T_1-weighted SE sequence is often used with the following parameters: field of view 200 × 150 mm, acquisition matrix 384 × 245, slice thickness 5 mm, TR 522 ms, TE 13 ms.

    (a) What is the resulting voxel size (x, y, and z) and the acquisition time for this sequence?
    (b) Considering that tumoral invasion behaves like water, and bone marrow behaves like fat, which signal intensity will both tissues give in this sequence? Show this by drawing the longitudinal and transversal relaxation curves



TR = 2415 ms, TE = 68 ms
TI = 160 ms

TR = 2415 ms, TE = 152 ms
TI = 160 ms



TR = 2415 ms, TE = 68 ms
TI = 399 ms

TR = 2415 ms, TE = 68 ms
TI = 916 ms

Figure B.15

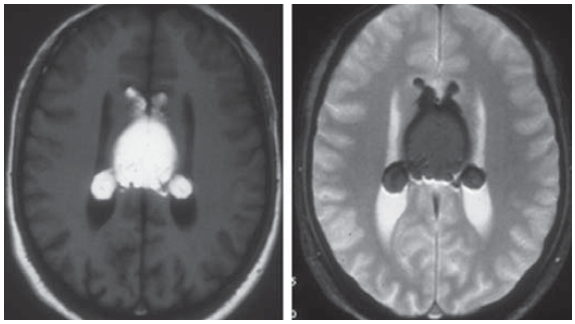


Figure B.16

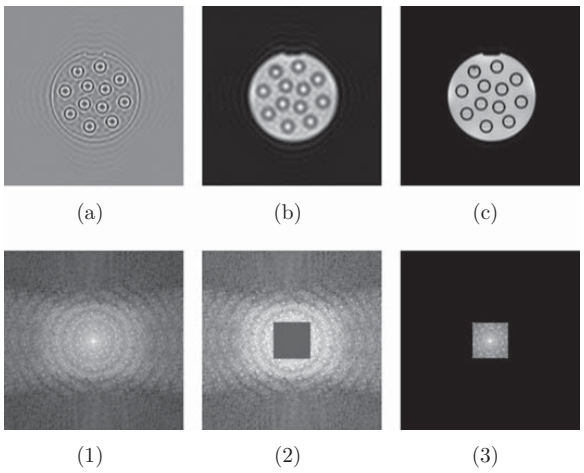


Figure B.17

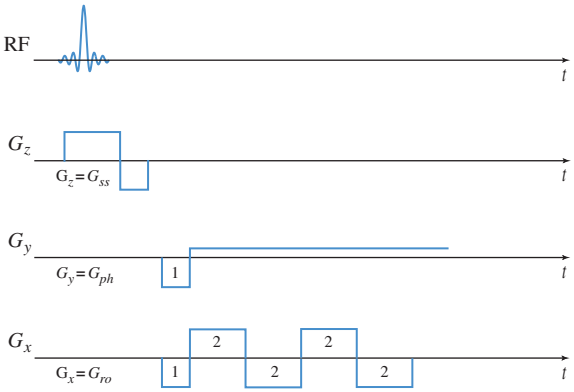


Figure B.18

for water and fat, and indicate the moment of the measurement.

(c) Knowing that the measurement of one line in $\vec{k}$-space requires less than 20 ms, it is possible to acquire multiple *slices* during the same acquisition time as calculated in (a). Explain how this can be obtained.

17. Assume that a single slice of a T_2-weighted turboSE sequence (30 echoes) is acquired with the following parameters: FOV 200 × 200 mm, matrix 128 (phase) × 256 (frequency), slice thickness 5 mm, TR 2500 ms, TE 80 ms.

    (a) Draw (schematically) the pulse sequence.
    (b) What is the pixel size and the acquisition time for this slice?
    (c) What is the influence of using multiple echoes on the image quality (resolution and CNR).
    (d) In order to improve the image quality images are often acquired multiple times and the resulting image is calculated as a pixel per pixel average of these acquisitions. What is the acquisition time if, instead of measuring the image only once, it is measured four times and averaged? What is the effect on the CNR?

18. Consider the pulse sequence in Figure B.18 (surface 2 equals two times surface 1). Draw the trajectory of $\vec{k}$ in the $\vec{k}$-space.

19. (a) Draw the $\vec{k}$-space trajectory for the following pulse sequence diagram (Figure B.19).

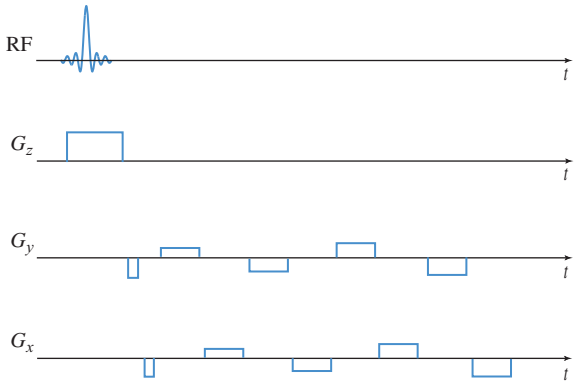


Figure B.19

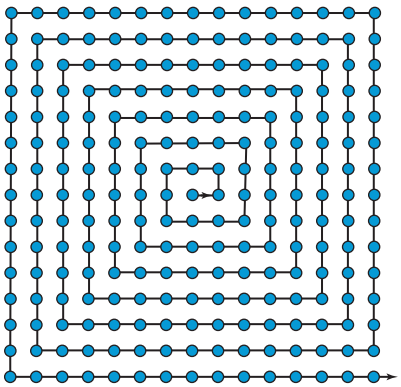


Figure B.20

(b) What is the effect on the image quality if this pulse sequence would be stopped prematurely (e.g., if only 30% of the $\vec{k}$-space is filled in)?

20. Draw the pulse scheme (i.e., RF pulses and magnetic gradient pulses) for the $\vec{k}$-space sampling shown in Figure B.20.

21. (a) Draw the $\vec{k}$-space trajectory for the pulse sequence diagram shown in Figure B.21.
    (b) If this pulse sequence would be stopped prematurely (e.g., if only 75% of the $\vec{k}$-space is covered), which parts of the $\vec{k}$-space would have been sampled and which would not? Describe the effect this will this have on the image quality.

22. MRI
    Given the single-shot pulse sequence in Figure B.22. It only requires one 90° RF pulse to measure the entire $\vec{k}$-space.

    (a) Draw the $\vec{k}$-space trajectory.

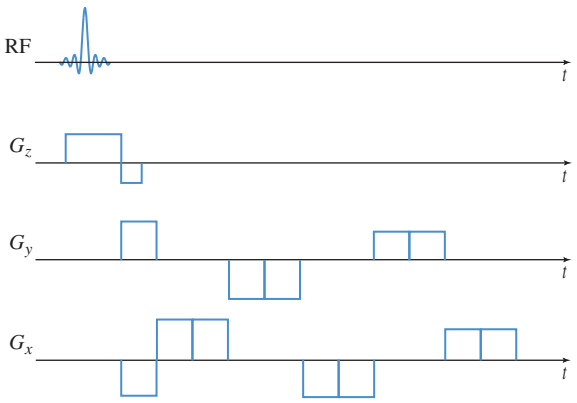


Figure B.21

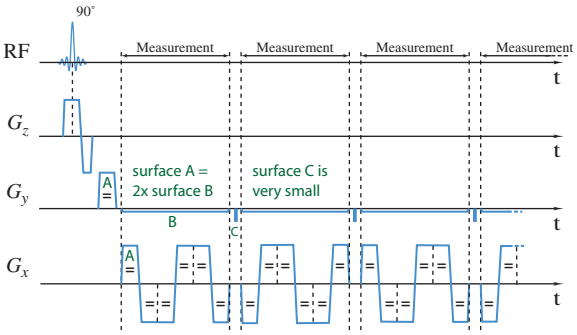


Figure B.22

(b) Give an example where this sequence could be used.
(c) Discuss the resolution and contrast of this sequence as compared to a 2D SE sequence with only one echo per excitation and the same number of phase encoding steps.
(d) The number of phase encoding steps in this example is very low. What will be the effect on the image quality?

23. (a) Draw the pulse scheme (i.e., the RF pulses and magnetic gradient pulses) for the trajectory in the $\vec{k}$-space of Figure B.23.
    (b) What is the influence on the image if the scanning is stopped prematurely?
    (c) What is the influence on the image if the scan direction is inverted (i.e., inwards)?
    (d) What is the influence on the image if Δt is reduced and all the other parameters remain unchanged?

24. Given

$$k_x(t) = \frac{\gamma}{2\pi} at\cos(bt)$$
$$k_y(t) = \frac{\gamma}{2\pi} at\sin(bt) \qquad a, b > 0.$$

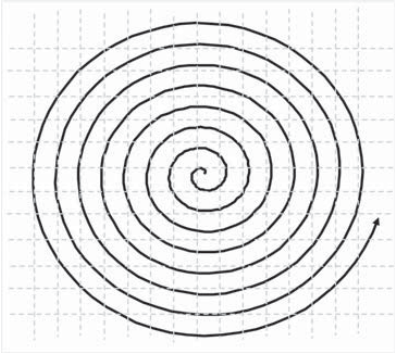


Figure B.23

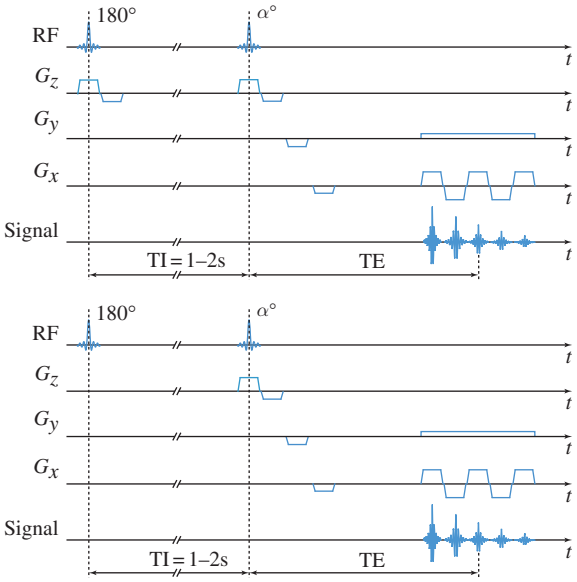


Figure B.24

(a) Draw the trajectory in the $\vec{k}$-space.
(b) Calculate the necessary gradients $G_x(t)$ and $G_y(t)$.
(c) Draw the corresponding magnetic gradient pulse sequence.

25. Given the two pulse sequences (Figure B.24). The difference between both sequences is the presence/absence of a gradient G_z at the 180°-RF-pulse.

(a) Draw the trajectory traversed in $\vec{k}$-space.
(b) What is the influence of the 180°-pulse on the hydrogen nuclei of intravascular arterial water?
(c) What does the subtraction image show (i.e., the subtraction of the images obtained with the two pulse sequences)?

26. In MRI moving blood yields additional dephasing and a corresponding signal loss. Assume a constant blood velocity.

(a) Draw a 2D gradient-echo sequence that removes the velocity-induced phase shift at t = TE. Explain this by means of mathematical equations.
(b) Draw a 2D SE sequence with the same effect.
(c) Draw a 2D gradient-echo sequence to calculate the velocity in the x-direction. Assume that the 2D slice is perpendicular to the z-axis. Explain this by means of mathematical equations.

27. Explain the DWI and ADC images (Figure B.25) in the region of the liver and the spleen.
28. In molecular imaging research, gene expressions in vivo can be visualized by means of the marker ferritin, which has the property of capturing iron. Which imaging technique is used to visualize this process? Explain.
29. Explain the images in Figure B.26.
30. Assume that the x-direction is the phase-encoding direction in Figure B.27.

(a) In Figure B.27(a) the FOV in the x-direction is chosen too small. What is the effect on the image quality?
(b) In Figure B.27(b) the FOV in the x-direction is chosen twice as large by oversampling, i.e., by doubling the number of pixels in the x-direction (while keeping the same pixel size and spatial resolution). Which acquisition parameter(s) have to be modified to achieve this? Explain.
(c) Does the acquisition time of the sequence for (b) change as compared to the sequence for (a)? Explain.

What can be said about the acquisition time if the number of pixels in the y-direction (frequency-encoding direction) were doubled?

31. A patient with thickness L is scanned using a coil with bandwidth BW (in Hz). Note that the different frequencies that are received by this coil are defined by the range of precession frequencies of the spins.

(a) What are the conditions necessary to avoid aliasing artifacts in the read-out direction?

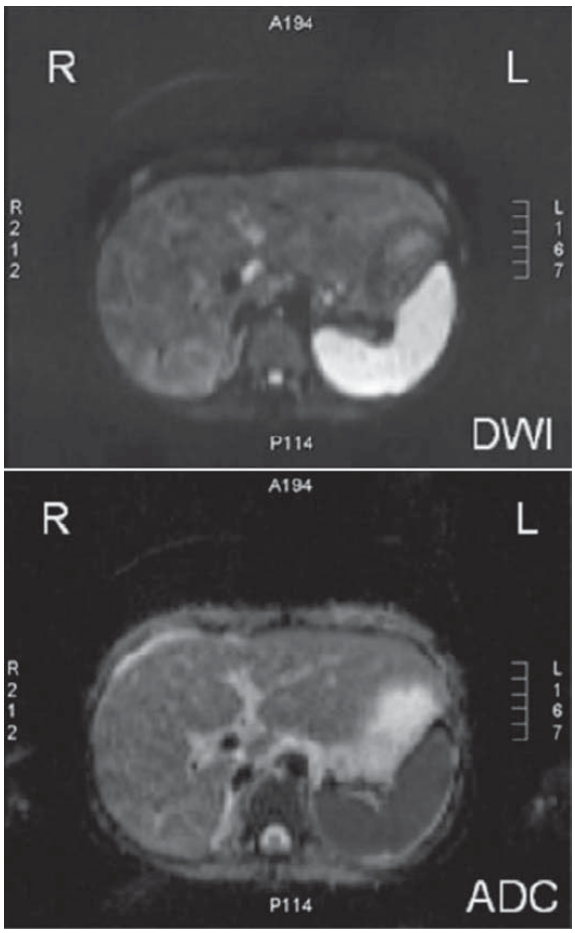


Figure B.25

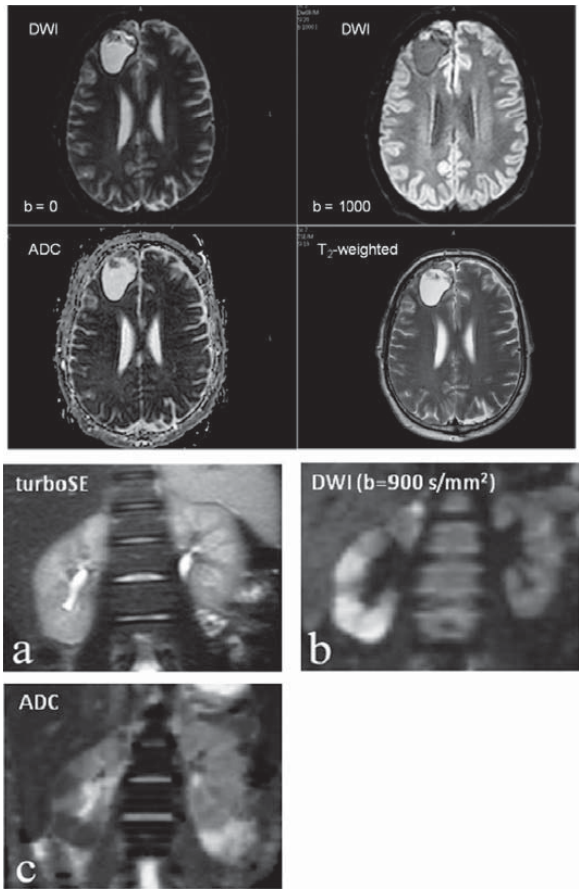


Figure B.26

(b) What is the maximal gradient amplitude as a function of BW and L that is necessary to avoid aliasing?
(c) What is the relationship between BW and the sampling distance Δt?

32. An image is acquired with FOV = 8 cm and 256 phase encoding gradient steps. The phase encoding gradient equals 10 mT/m. The radiologist prefers an image with the highest resolution and without artifacts. Calculate the pulse duration of the phase encoding gradient. You may assume that the pulse has a rectangular shape.
33. The MR images in Figure B.28 were acquired with a SE sequence (90° pulse) at 1.5 T. Explain the origin of the artifact in the right image.
34. Which type of artifact do you recognize in the images of Figure B.29? Describe how this artifact is generated. How can this artifact be avoided without reducing the image resolution? Does this have an effect on the acquisition time?
35. Given are a turboSE sequence with 10 echoes, TR = 500 ms, TE (first echo) = 20 ms; image size 240×160 pixels (hence, N_{ph} = 160); slice thickness 5 mm. The heart rate of the patient is 60 bpm.

(a) What is the acquisition time for one slice of the liver?
(b) What is the acquisition time for one slice of the heart? The measurements are synchronized with the ECG.

B.5 Nuclear Medicine Imaging

1. Radioactivity.

(a) How can the half-life of a radioactive isotope be calculated?
(b) Give a realistic value of the half-life for some radioactive tracers.

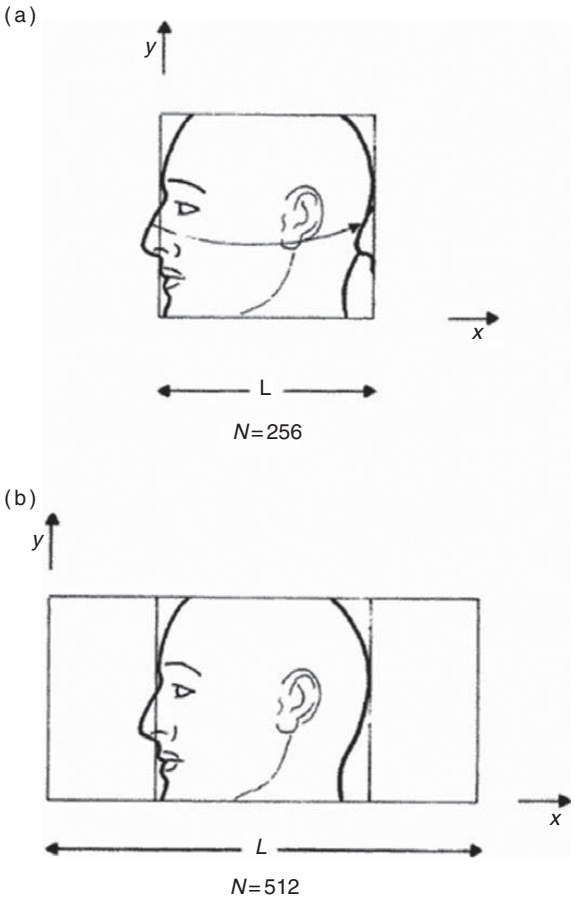
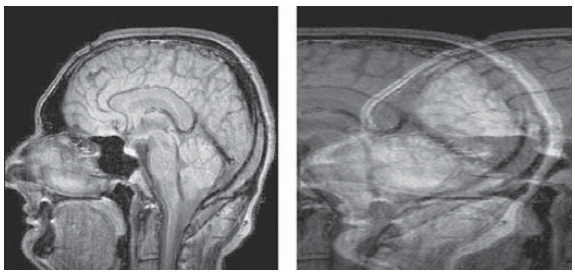


Figure B.27



(a) TR = 3000, TE = 10 without artifact (b) TR = 3000, TE = 10 with artifact

Figure B.28

(c) Which recommendations would you give to the patient and his/her environment?

2. What is the problem when using filtered backprojection in nuclear medicine imaging?
3. Explain how the two images in Figure B.30 were acquired. What is the difference between them and why?

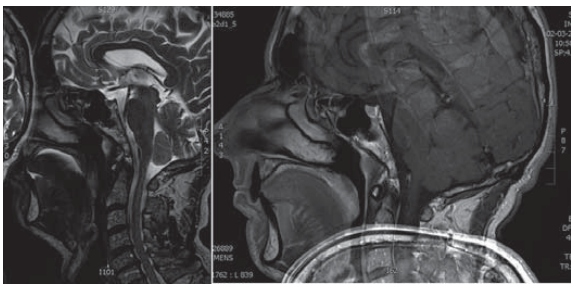


Figure B.29

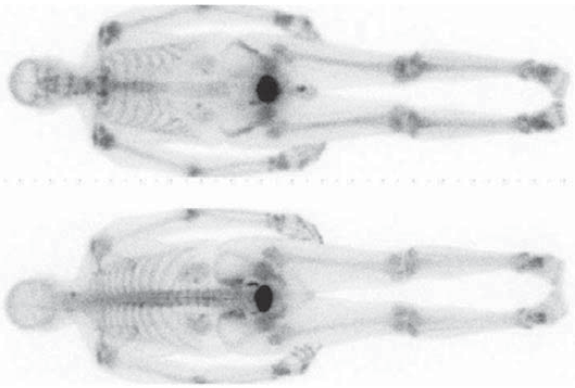


Figure B.30

4. A colleague in a PET center would like to know whether they should put on a lead apron to protect themselves against the irradiation from the positron emitters. We know that the mass density of lead is 11.35 g/cm^3 and that its linear attenuation coefficient for this kind of γ-rays is 1.75 cm^{-1}.
   (a) An apron that absorbs $^3/_4$ of the irradiation would provide satisfactory protection. What is the thickness of lead (in cm) required to obtain a transmission of 25% (i.e., $^3/_4$ is absorbed)? Assume a perpendicular incidence of the radiation with the apron.
   (b) What is the weight of this lead apron with a transmission of 25% if about 1.5 m^2 (flexible, but lead containing) material is needed? Neglect the other material components in the apron.
   (c) What is your advice with respect to the question of putting on a lead apron? Assume that 10 kg is the maximum bearable weight for an apron.
5. Given is a positron emitting point source at position $x = x^*$ in a homogeneously attenuating

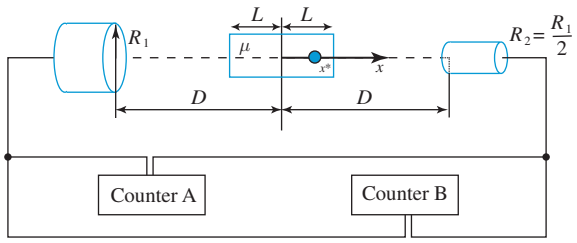


Figure B.31

medium (center $x = 0$, $-L \leq x \leq L$) with attenuation coefficient μ (Figure B.31). Detector 1 has radius R_1 and detector 2 has radius $R_2 = \frac{R_1}{2}$. The detectors count all the incoming photons (i.e., the absorption efficiency is 100%). Counter A counts all photons independent of the detector, while counter B counts only the coincidences. Because $D >> L$, $D + L \approx D$. If $\mu = 0$, detector 1 would count N photons per time unit.

(a) Calculate the average number of photons per time unit measured by counter A as a function of μ, x, and N. Calculate the standard deviation for repeated measurements.
(b) Repeat these calculations for counter B.

6. How does a gamma camera react on a simultaneous (i.e., within a time window ΔT) hit of two photons of 140 keV each if the energy window is [260 keV, 300 keV]?

   What is the probability of a simultaneous (i.e., within a time window ΔT) hit of two photons as a function of the activity A (i.e., average number of photons per time unit) and the time resolution ΔT?

7. (a) A positron emitting point source is positioned in the center of a homogeneous attenuating cylinder with radius r and attenuation coefficient μ (Figure B.32). Electronic coincidence circuits connect the opposing detectors. They measure y_1 and y_2 photon pairs respectively. The sensitivity of the detectors is 50%. All the detectors have the same size and distance to the point source. Calculate the activity in the center of the cylinder from the measurements y_1 and y_2 using maximum-likelihood reconstruction.
   (b) A single photon emitting point source is positioned in the center of a homogeneously attenuating cylinder with radius $r = 1$ cm and attenuation coefficient $\mu = \ln(4)\ \text{cm}^{-1}$ (Figure B.32). All the detectors have the same size and distance to the point source. The efficiency of the four detectors is 1%. It takes the absorption efficiency and the influence of the geometry (only limited photons travel in the direction of a detector) into account. During a short measurement each detector absorbs exactly 1 photon. What is the maximum likelihood of the total number of photons that were emitted during the measurement?

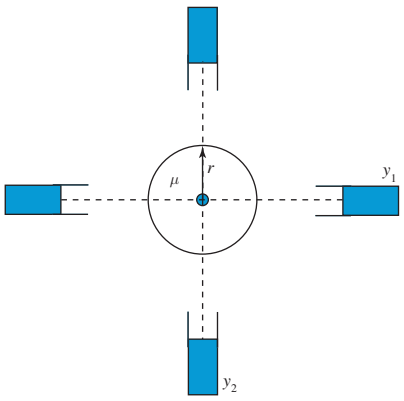


Figure B.32

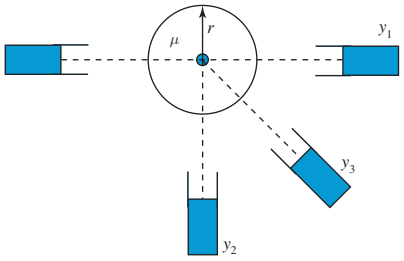


Figure B.33

8. A positron emitting point source is positioned in the center of a homogeneously attenuating cylinder with radius r and attenuation coefficient μ (Figure B.33). Two opposing detectors, connected by an electronic coincidence circuit, measure y_1 photon pairs and two other single-photon detectors measure y_2 and y_3 photons respectively. The thickness of the detectors is sufficiently large to detect all the incoming photons (i.e., the absorption efficiency is 100%). All the detectors have the same size and distance to the point source.
   Calculate the activity in the center of the cylinder from the measurements y_1, y_2, and y_3 using maximum-likelihood reconstruction.
9. How does Compton scatter influence the spatial resolution in SPECT and PET respectively?

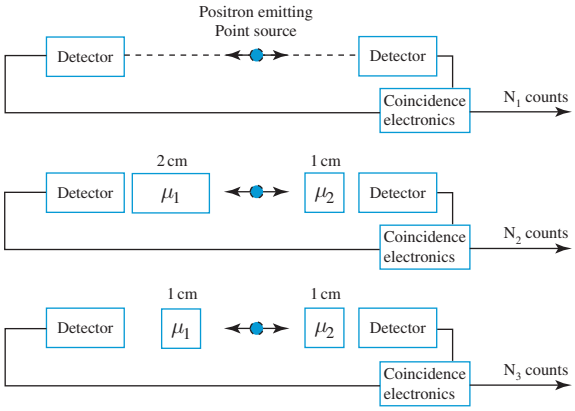


Figure B.34

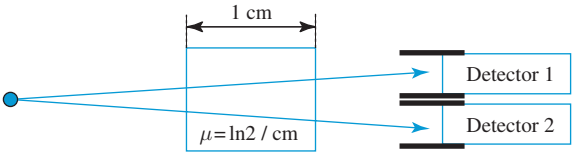


Figure B.35

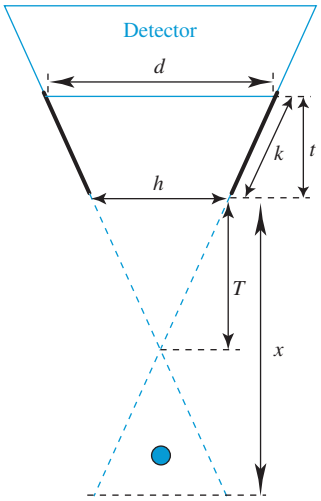


Figure B.36

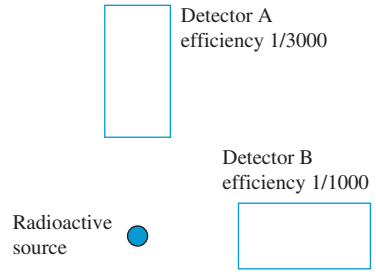


Figure B.37

10. Two opposing detectors, connected by an electronic coincidence circuit, perform three subsequent measurements (Figure B.34). The only difference between the measurements is that the attenuation is modified by adding homogeneous blocks between the positron emitting point source and the detectors. The measurements N_1, N_2, and N_3 and the attenuation depths are given. Calculate the linear absorption coefficients μ_1 and μ_2.
11. A radioactive point source is positioned in front of two detectors (Figure B.35). After a measurement time of one hour, two photons per second have been captured by each of the detectors.

    Next, an attenuating block with attenuation depth 1 cm and an attenuation coefficient of $\ln 2\ \text{cm}^{-1}$ is added and a new measurement is performed, this time during only one second.

    What is the probability that during this measurement of one second exactly one photon is captured by detector 1 and four photons by detector 2?
12. Given are a square detector with collimator with known geometry, and a point source at distance x (Figure B.36).

    (a) Calculate the sensitivity of the point source at distance x.

    (b) Calculate the FWHM of the point spread function at distance x.

    Perform the calculations for both $x \leq T$ and $x \geq T$.
13. Given a point source and two detectors (Figure B.37). The efficiency of both detectors is known and takes both the absorption efficiency and the influence of the geometry (only limited photons travel in the direction of a detector) into account. During a short measurement exactly one photon is absorbed by each detector. What is the maximum likelihood of the total number of photons that were emitted during this measurement?
14. Given a phantom with homogeneous attenuation coefficient μ and homogeneous positron activity λ per length unit (Figure B.38), a measurement is performed with a single-photon detector and another with a pair of opposing detectors connected by an electronic coincidence circuit. The efficiency of all the detectors is constant (S).

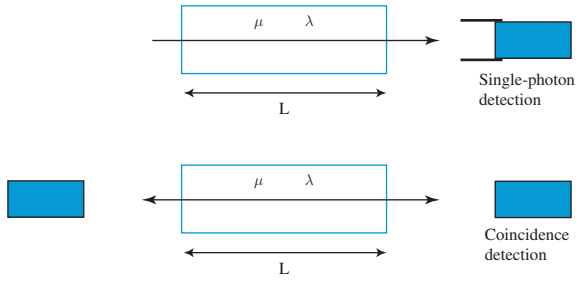


Figure B.38

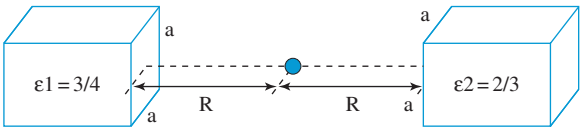


Figure B.39

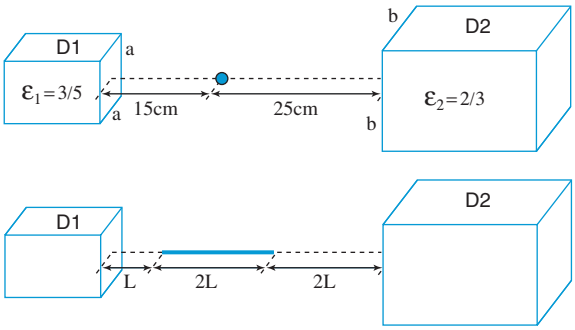


Figure B.40

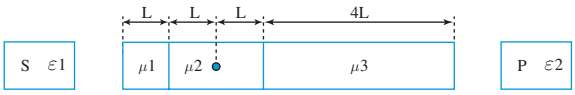


Figure B.41

Calculate the expected number of measured photons for both cases.

15. Given a positron emitting point source in the center of a detector pair connected by an electronic coincidence circuit (Figure B.39). The distance R from the point source to the detectors is much larger than the detector size a. The detectors consist of different materials. The absorption efficiency of detector 1 is 3/4 and that of detector 2 is 2/3.

    (a) Calculate the fraction of emitted photon pairs that yields a true coincidence event.
    (b) Calculate the fraction of emitted photon pairs that yields a single event.

16. Given a source positioned on the central axis through two detectors (see Figure B.40). Detector 1 has a surface of $a \times a$ ($a = 1$ cm) and an absorption efficiency of 3/5. Detector 2 has a surface of $b \times b$ ($b = 2$ cm) and an absorption efficiency of 2/3.

    (a) Given a single photon emitting point source, calculate for each detector the fraction of emitted photons that will be detected.
    (b) Given a positron emitting point source, the detectors are connected by an electronic coincidence circuit. Calculate the fraction of emitted photon pairs that yields a true coincidence event.
    (c) Same setup as in (b). Assume the point source is replaced by a positron emitting line source with a homogeneous attenuation coefficient μ and homogeneous activity λ per length unit. The detectors are connected by an electronic coincidence circuit. Calculate the number of true coincidence events detected by the detector pair. (Take the variation in geometrical efficiency into account.)

17. Given two photon detectors (S and P), a positron emitting point source that emits A photon pairs, and three blocks with attenuation coefficients $\mu_1 = \frac{1}{L}$, $\mu_2 = \frac{1}{2L}$, and $\mu_3 = \frac{1}{L}$ respectively (Figure B.41). The efficiency of the detectors is known (ϵ_1 and ϵ_2 respectively) and takes both the absorption efficiency and the influence of the geometry (only limited photons travel in the direction of the detector) into account.

    (a) Calculate the expected number of detected photons in S.
    (b) Calculate the expected number of detected photon pairs if S and P are connected by an electronic coincidence circuit.

18. A point source that emits gamma photons with an energy of 140 keV is located on the line that connects the center of the detector surface of two detectors (Figure B.42). Assume that the distance from the point source to each detectors is much larger than the width a of the detectors. The detectors are made of a different material. The absorption efficiency of detector 1 is 3/4 and of detector 2 is 1/2.

    (a) Calculate the fraction of emitted photons detected by each detector, assuming no attenuating substance between the point source and the detectors (see Figure B.42(a)).

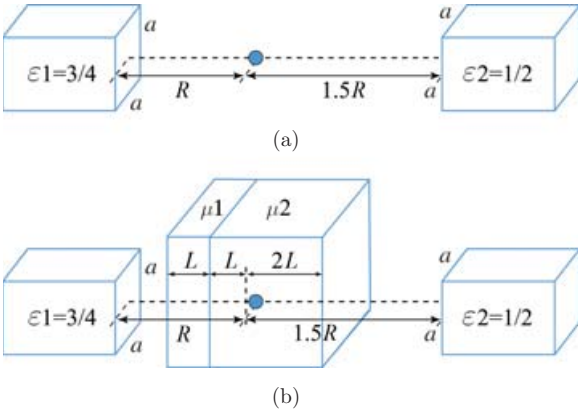


Figure B.42

(b) Calculate the fraction of emitted photons detected by each detector if the point source is located in the center (see Figure B.42(b)) of attenuating substances consisting of two materials with attenuation coefficients $\mu_1 = 1/(2L)$ and $\mu_2 = 1/L$.

(c) Same configuration of Figure B.42(b). Assume this time that the point source emits pairs of 511 keV gamma photons detected by a coincidence circuit. Assume further the same attenuation coefficients and absorption efficiencies as in question (b).

- Calculate the fraction of emitted photon pairs that are detected by the coincidence circuit.
- The assumption that the attenuation coefficients and absorption efficiencies at 140 keV and 511 keV are identical is not valid. Why not and how would this influence the result of the solution?

19. A positron emitting (^{18}F) point source is positioned in front of a detector. 3600 photons are counted during a first measurement of one hour. Next, a second measurement is performed, this time of only one second.

(a) Calculate the probability that exactly zero photons are detected.

(b) Calculate the probability that exactly two photons are detected.

20. Given a point source with activity $P = 1$ mCi at a distance $L = 30$ cm from a cube with size $h = 5$ cm and attenuation coefficient $\mu = 0.1$ cm^{-1} (Figure B.43). The density of the cube is 1 kg/l, and the half-life of the tracer is $T_{\frac{1}{2}} = 2$ h.

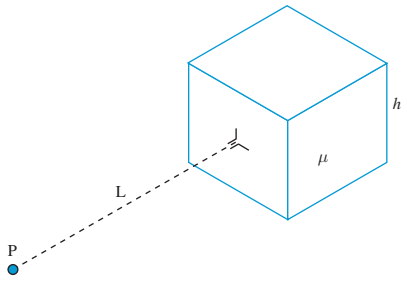


Figure B.43

Calculate the absorbed dose (in mGy) of the cube after several days. Note that 1 eV = 1.602×10^{-19} J and 1 mCi = 3.7×10^7 Bq.

B.6 Ultrasound Imaging

1. Ultrasonic waves.

(a) What are reflection, refraction, scatter, and absorption? What is their effect on an ultrasound image?

(b) What is the effect of the acoustic impedance on the reflection?

(c) What is the physical reason to avoid an air gap between the transducer and the patient? How can it be avoided?

(d) What is constructive interference? How is this used to focus the ultrasonic beam? And how is it used to sweep the ultrasonic beam?

2. Calculate the reflection coefficient at air/tissue transitions.
3. Which methods do you know are used to measure the velocity of blood?
4. Given an ultrasound scanner with the following characteristics

- 5 MHz phased array transducer.
- 16-bit, 20 MHz AD converter.
- 256 Mb image memory (RAM).
- Operating mode: B-mode acquisition; 3000 transmitted ultrasonic pulses per second; image depth 10 cm; number of scan lines 60; sector angle 60°.

The ultrasound velocity is 1530 m/s.

(a) What is the image frequency (frame rate)?

(b) How long does it take to fill the complete image memory with data?

(c) How many images can maximally be stored in memory?

5. In Continuous Wave (CW) Doppler the signal is subdivided into time segments. Explain the consequence of shorter time segments on the velocity resolution and on the temporal resolution.
6. Doppler imaging. Given

   – Velocity of blood: 0.3 m/s.
   – Pulse repetition period: 0.1 ms.
   – Transmitted pulse frequency: 5 MHz.
   – Velocity of ultrasound: 1500 m/s.

   (a) What is the maximal depth that can be measured with the given pulse repetition period?
   (b) What would be the maximal pulse repetition period to avoid aliasing?
   (c) Calculate the measured phase shift (in degrees) between subsequent pulses reflected by blood.

7. Using PW Doppler, samples s_j, $j = 1, 2, \ldots$ are taken of the following signal of a blood vessel

$$s_j = 18 \sin(\tfrac{2}{5}\pi j + 0.35) \text{ mV}.$$

   The pulse repetition frequency is 12 kHz. The frequency of the transmitted pulse is 2.5 MHz. The velocity of the ultrasonic signal in soft tissue is 1530 m/s.

   (a) What is the velocity of blood (in the direction of the transducer)?
   (b) What is the maximal velocity $v_{\max}$ that can be measured without artifacts?
   (c) What is the maximal distance from the transducer required to measure this maximal velocity $v_{\max}$ without artifacts?
   (d) What is the measured velocity of blood if its real velocity equals $v_{\max}$ + 1 m/s?

8. A 3D ultrasound system acquires an image of the heart with the following parameters

   – transmission pulse frequency: 6000 Hz.
   – ultrasound frequency: 2.5 MHz.
   – ultrasound speed: 1500 m/s.
   – cone angle: 60° in lateral and in elevation direction.
   – line density (lateral and elevation): 1 line per degree.
   – number of scan lines detected in parallel: 12.
   – heart rate: 60 bpm.

   (a) What is the maximal depth of the tissue that can be visualized?
   (b) Calculate the frame rate.
   (c) Does Color Flow (CF) Doppler imaging influence the frame rate? Give an example.
   (d) How can signals reflected by blood in the heart chambers be distinguished from those reflected by the heart muscle (Doppler tissue imaging)?

9. A radiologist would like to distinguish small details in the vessel wall. Assume that the distance between these small details is 0.5 mm, and that the blood vessel runs parallel to the surface of the tissue at a depth of 5 cm. The attenuation of the ultrasonic beam is $\frac{1 \text{ dB}}{\text{MHz}\cdot\text{cm}}$. The maximum attenuation to guarantee that the image is practically useful is 100 dB. The ultrasonic pulse duration is 2 periods. Assume that the ultrasound velocity in tissue is 1530 m/s.

   (a) What is the minimal frequency required to distinguish the small details along the vessel wall?
   (b) Given the maximum attenuation of 100 dB, what is the maximum frequency that can be used?
   (c) Which ultrasonic frequency can be recommended to obtain the best image quality (i.e., high resolution and high SNR)?

10. Assume that the point spread function (PSF) in the lateral direction is the following sinc^2 function

$$s(y) = \text{sinc}^2(\frac{\pi}{d}y)$$

    (a) What is the physical principle behind this pattern?
    (b) Calculate the modulation transfer function (MTF). Draw the PSF and the MTF.
    (c) What is the (approximate) value of the FWHM?
    (d) What is the maximal distance Δy required between two neighboring scan lines to avoid aliasing?
    (e) What is the highest available spatial frequency in the signal?
    (f) What is the minimal distance in the lateral direction between two distinguishable small calcifications? Represent neighboring calcifications as a sinusoidal function of y.

11. (a) Calculate the Fourier transform of the triangular function $\Lambda(\frac{x}{2L})$ based on the Fourier

transform of a rectangular function and on the convolution theorem. Draw the graphs of these functions in the spatial domain and in the Fourier domain.

(b) Assume that $\Lambda(\frac{x}{2L})$ is the lateral pressure field of an ultrasound transducer. How would you express the resolution in the lateral direction? What is the minimal lateral distance between two small details to be able to distinguish them, i.e., to perceive them as two distinct details separated from each other?

B.7 Medical Image Computing

1. Using the principle of dynamic programming, search the best edge from left to right in the following gradient image

1	1	1	1	0	0	0	1	3	3
3	3	5	4	3	2	3	3	5	4
4	5	4	4	5	4	5	5	2	2
3	3	2	2	5	5	3	3	0	0
1	2	1	3	1	2	4	1	4	4

The cost $C(i)$ of connecting pixel (x, y_1) with pixel $(x+1, y_2)$ is defined as follows

$$C(i) = (5 - \text{grad}(x+1, y_2) + |y_2 - y_1|)$$

with grad() the gradient in the y-direction (these values are shown in the image matrix above), x the column number, y the row number, and y_1 and y_2 arbitrary y values.

2. From images I_A and I_B two regions of interests (ROI) A and B are shown in Figure B.44. I_A and I_B are geometrically aligned.

   (a) Calculate the sum of squared differences (SSD) and the mutual information (MI) of the regions of interest.

   (b) Do the same when image I_B is translated one pixel to the right. Use matrix B_t instead of B this time. What can you conclude?

$$A = \begin{bmatrix} 0 & 0 & 8 & 7 & 8 & 5 & 0 & 0 \\ 0 & 0 & 2 & 9 & 5 & 7 & 0 & 0 \\ 0 & 0 & 4 & 6 & 4 & 3 & 0 & 0 \\ 0 & 0 & 4 & 6 & 2 & 4 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 9 & 9 & 1 & 5 & 6 & 3 & 9 & 9 \\ 9 & 9 & 1 & 1 & 3 & 4 & 9 & 9 \\ 9 & 9 & 2 & 3 & 2 & 8 & 9 & 9 \\ 9 & 9 & 2 & 3 & 1 & 2 & 9 & 9 \end{bmatrix}$$

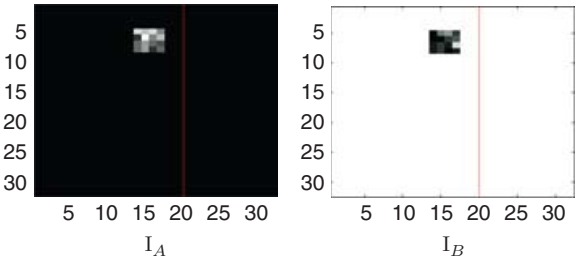


Figure B.44

$$B_t = \begin{bmatrix} 9 & 9 & 9 & 1 & 5 & 6 & 3 & 9 \\ 9 & 9 & 9 & 1 & 1 & 3 & 4 & 9 \\ 9 & 9 & 9 & 2 & 3 & 2 & 8 & 9 \\ 9 & 9 & 9 & 2 & 3 & 1 & 2 & 9 \end{bmatrix}$$

3. Calculate the mutual information (MI) of the following arrays

9	0	4	9
9	7	4	0
1	8	0	9
1	0	3	8

0	8	3	0
0	5	2	8
9	1	8	0
7	8	7	2

4. To perform a pixel classification of a T_1-weighted brain scan, a digital atlas can be used as prior knowledge. The atlas consists of a T_2-weighted brain image and an image in which each voxel value expresses the probability that this voxel belongs to white brain matter, gray brain matter, or CSF. Explain the segmentation method.
5. Statistical pixel classification.
   - Assume that we have a SPECT image of a patients brain taken at two different dates. We assume that the imaging conditions of both scans are identical with the exception of the position of the patient.
     (a) What do the gray values represent?
     (b) How can we make a subtraction image based on both scans?
     (c) Assume that the gray values in the subtraction image are normally distributed. Draw the histogram.
     (d) Assume that the gray value in each pixel of the subtraction image equals zero (i.e., the null hypothesis). The null hypothesis is rejected in a voxel if the probability to find any voxel with a higher gray value is less than 0.05. How can we detect these voxels?
   - Same questions (a–d) for the case of a PD-weighted and T_2-weighted MR image of the

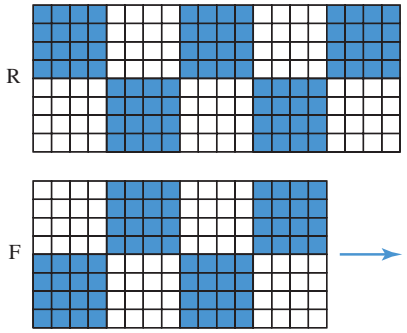


Figure B.45

brain (PD = proton density). Note that we have two subtraction images now instead of one.

6. (a) Which features can be used for white and gray brain matter classification?
   (b) Which features have we used in this textbook for face classification ?
   (c) And for the accurate delineation of hand bones in 2D radiographic images? Why has data classification for this application not been used as the problem-solving strategy?
7. Consider the binary 20 × 8 image R and 16 × 8 image F in Figure B.45. Calculate their joint histogram and MI given that the upper left corner of the images coincides. Repeat these calculations when image F shifts to the right about 1, 2, 3, and 4 pixels respectively.
8. Given the gray scale image with corresponding numerical values in Figure B.46. The trajectory with minimal energy $E = \int_0^1 -|\nabla I(v)|^2 ds$ has to be found with

$$\nabla = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix}$$

$$\frac{\partial}{\partial x} \approx \begin{matrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{matrix} \qquad \frac{\partial}{\partial y} \approx \begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$$

   (a) $-|\nabla I(v)|^2$ is first calculated. The resulting gray scale image is represented in Figure B.46. Calculate the corresponding numerical values.
   (b) Next, a search algorithm is applied. Which? Explain its mechanism. Indicate the trajectory with minimal energy.

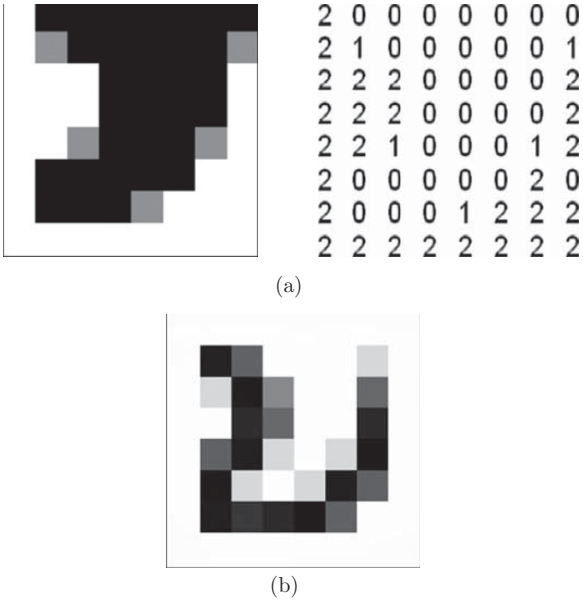


Figure B.46

9. Given a statistical shape model learned from a training set. Using PCA the eigenvalues $\lambda_1 = 16, \lambda_2 = 9, \lambda_3 = 4, \lambda_4 = 1$, ... are obtained. Assume that only the three most important modes of variation are employed for the model and that the parameters of the shape model are normally distributed.
   (a) Calculate the maximal value of the internal energy for the large majority of the training set. Specify the value you choose for "the large majority" (which has to be less than 100%).
   (b) What is the internal energy of the most likely shape?
   (c) Assume that we search in the image for a shape that is only partially visible due to insufficient CNR. Give a mathematical solution to localize the complete shape based on the trained shape model.
10. Given a statistical face model learned from a training set of 256 × 256 pictures (Figure B.47). By means of PCA, eigenfaces are calculated and the 20 most important modes of variation are further used. The parameters of the shape model are normally distributed.
   (a) What is the maximal value of the energy for the large majority of the training set? Specify a value for "the large majority" (which has to be less than 100%).

Figure B.47

(b) What is the energy of the most likely face?
(c) Specify by means of a mathematical equation how an arbitrary face can be written as a function of the eigenfaces.
(d) Give a clinical example for which this method can be applied.

11. (a) Describe a 2D shape delineation method in which the shape is learned from a training set of radiographic images.
(b) Can you modify this method to perform a delineation in a 3D image?

12. A CT and a T_1-weighted MR image are acquired from a patient with a brain lesion. Both images need to be spatially aligned.

(a) Which similarity measure would you use? Formalize this similarity.
(b) In practice, this similarity measure should be a continuous function of the geometric transformation. How would you evaluate this similarity measure if the voxels of the CT and MR image do not exactly coincide?
(c) Besides a T_1-weighted MR image, a T_2-weighted MR image has also been acquired. Both MR images are assumed to be perfectly aligned. How would you extend the previously used similarity measure to align the CT image with the multispectral (i.e., T_1 + T_2) MR image?
(d) If the number of acquired MR sequences increases, straightforward implementation of this similarity measure may suffer from the curse of dimensionality. Explain why. How would you solve this problem without modifying the similarity measure?

13. T_1-weighted and T_2-weighted MRI scans of a patient's brain need to be spatially aligned.

(a) Which similarity measure would you use and why?
(b) Figure B.48 shows the joint histograms of the intensities for four geometric transformations. The intensity at position (x, y) in the joint histogram represents the number of voxels with intensity x in the T_1-image and intensity y in the T_2-image. Order the histograms from best to worst, i.e., start with the histogram that corresponds to the best transformation and ends with the histogram that corresponds to the worst transformation. Explain your answer.

14. Two CT images of the same patient can be registered by minimizing their sum of squared differences (SSD).

(a) Formalize this similarity measure, including its dependency on the geometric transformation parameters.
(b) Assume that the subtraction image of both images has a normal distribution. Proof that maximizing the likelihood of observing one image given the other is identical to minimizing the sum of squared differences.

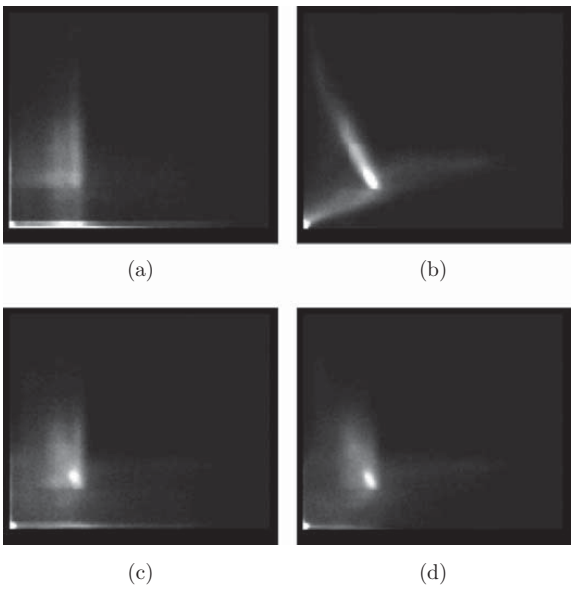


Figure B.48

(c) Assume that both images are acquired before and after the insertion of a metallic implant. Is a straightforward use of SSD in this particular case recommended? Explain. If not, which similarity measure would be used instead and why? Can the SSD-criterion be modified to improve its expected performance in the case of a metallic implant?

15. The Kullback-Leibler divergence is given by

$$\sum_x p(x) \log_2 p(x) - \sum_x p(x) \log_2 q(x)$$

From an information theoretical point of view it encodes the expected number of additional bits required to transmit a message sampled from probability density function $p(x)$ with a code optimized for messages generated from probability density function $q(x)$.

(a) How can this measure be used as a similarity measure for medical image registration?
(b) Which image registration tasks can be performed using this measure and why?

16. Scoliosis is a medical condition characterized by a deformity of the spine. It is typically diagnosed using a single 2D radiographic image and quantified by measuring 2D angles between characteristic lines of certain vertebrae. As opposed to this traditional 2D analysis, our goal is to measure the 3D relative pose of the vertebrae. This 3D information will be calculated from a single radiographic image using a 3D statistical model of each vertebra. For the modeling a set of exemplar lumbar CT scans is available. Assume that in each of these CT scans, the vertebrae have been labeled (C_{1-7}, T_{1-12}, L_{1-5}, S_{1-5}). The contours have not been outlined and are not explicitly used during the analysis.

(a) For each vertebra, a 3D statistical model is constructed to capture its appearance variability. Principal component analysis (PCA) is used.
- How would you construct this 3D appearance model?
- What is the meaning of the eigenvectors and the eigenvalues?
- What are the parameters in the model? What is their meaning?

(b) Next, these statistical models are used to calculate the 3D pose of the vertebrae from a single 2D radiographic image. Assume that the radiographic system is internally calibrated.
- Which geometric transformation would you use to match the 3D model with the 2D radiographic image? How many parameters are required?
- Which similarity measure (internal and external energy function) would you use?

17. A chest radiograph of a patient is compared to an older one of the same patient. To simplify this comparison, both images are registered using flexible geometric model matching.

(a) Which metric would you recommend to express the external energy? Explain your choice.
(b) How would you model the deformation between both images? Explain.
(c) The match between both images will not be perfect. Why not?

18. Echocardiography uses ultrasound waves to acquire 2D or 3D images of the heart. Explain a method for flexible registration of subsequent ultrasound images in a heart cycle.

B.8 Visualization for Diagnosis and Therapy

1. 3D dynamic information of a knee implant can be calculated from a 2D fluoroscopic image sequence (Figure B.49(a) shows a frame of such a sequence) if a 3D model of the knee implant (Figure B.49(b)) is available. A projection of the 3D model onto the fluoroscopic image is shown in Figure B.49(c). Assume that the fluoroscopy system has been (internally) calibrated. You are asked to develop an algorithm to register both metallic knee implant components (each defined in its own coordinate frame) to the fluoroscopic image sequence.

(a) Which geometric transformation is used and how many parameters are to be defined?
(b) Which similarity measure would you use?

The accuracy of the 3D motion of both parts of the knee implant is validated on data of human cadavers. Radio-opaque markers are drilled into

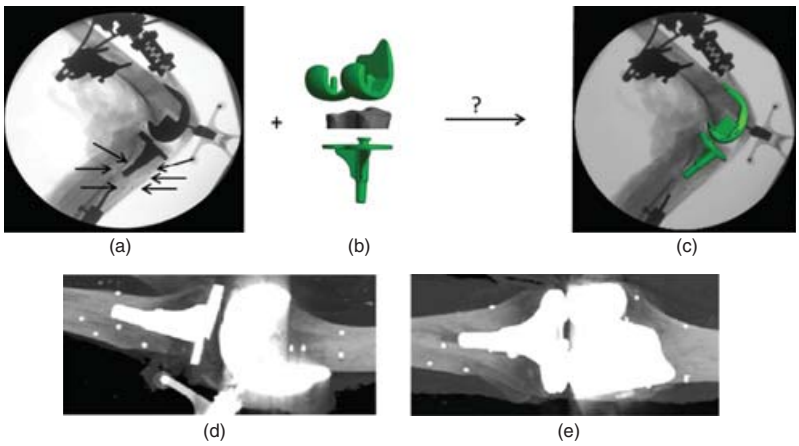


Figure B.49

the upper and lower leg of the cadaver. Next, a (static) CT scan and the (dynamic) fluoroscopic sequence are acquired. Figures B.49(d-e) show the maximum intensity projections (MIP) of a CT scan.

How would you calculate the 3D motion of each implant component based on the fluoroscopic sequence and the CT scan using the markers? How many markers are minimally required?

2. 3D edge enhancement can be obtained by

$$I'(\mathbf{r}) = I(\mathbf{r}) + \gamma \, \|\nabla(\mathbf{r})\|. \qquad \text{(B.1)}$$

How would you calculate $\nabla(\mathbf{r})$?

3. How can the measurements that are obtained with an intraoperative navigation system be matched with an intraoperative video image?
4. Augmented reality. A head-mounted display (HMD) integrates video images and ultrasound images in real time to assist a biopsy (Figure B.50). Which method would you recommend to perform the matching of both imaging modalities (video and ultrasound)? Assume that the internal calibration of each of the two imaging systems is known.
5. Given are preoperative CT images of a vertebra and planning data, both in the image space I. In the surgery space S the coordinates of a set of points along the surface of this vertebra and the position of the surgical instruments are measured with an optical navigation system.

   (a) Which algorithm can be used to match the surfaces of the vertebra in the 3D image space and the 3D surgery space (i.e., match I and S)? Explain this algorithm.

   (b) A 2D radiograph in the intraoperative image space X is taken during surgery. The goal is to project the 3D CT data, i.e., the surface of the outlined vertebra and planning data, onto this radiograph (i.e., match I and X). In order to do this, the radiographic system must be calibrated. Explain by means of equations how this is done.

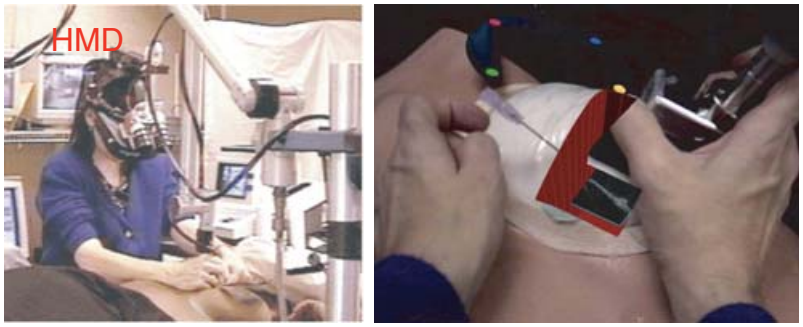
   (c) Next, the 3D CT space I is matched with the 2D intraoperative image space X. Explain how this can be done by point matching, taking advantage of the fact that I and S have already been matched.
   Note: it is not feasible to find corresponding (anatomical) points in I and X.

6. Augmented reality. A preoperative 3D CT or MR image has to be registered with 2D endoscopic video images. Assume that the endoscopic camera was calibrated.
   How can the video coordinates (u, v) of a point (x, y, z) in the preoperative images be calculated

   (a) in the simple case of a static endoscope, and
   (b) when the endoscope is in motion? Assume that the endoscope is a rigid instrument.

7. Image guided surgery. To perform a biopsy two on-site radiographs of the lesion are taken from two different directions. The positions of both the X-ray tube and the detector are unknown. A navigation system is used to localize the biopsy needle geometrically in real time. A number of markers, visible in both radiographs, are attached to the skin and their 3D coordinates (x, y, z) can be measured by the navigation system.

   (a) Calculate the 3D coordinates (x_l, y_l, z_l) of the lesion based on its projection in



(a) (b)

Figure B.50

both radiographs. Note that these coordinates cannot simply be measured with the navigation system like the 3D marker coordinates (x, y, z).

(b) How many markers are minimally needed?

8. Consider an orthopedic intervention where screws are inserted in the femur. The intervention is planned on a preoperative CT image. The surgeon wants to perform the intervention in an image-guided way. Specifically, (s)he wants to visualize the location of the surgical instruments in the planning CT. The operation room is equipped with an optical navigation system and a biplane (= 2) fluoroscopy system. Assume that no markers can be used preoperatively.

   (a) How can the intraoperative navigation space be matched with the intraoperative fluoroscopic images? How many markers are needed?
   (b) How can the preoperative CT image be matched with the biplane intraoperative fluoroscopic images? Can the surgical tools in the fluoroscopic images impede the registration? Explain. If this is the case, how then can this problem mathematically be solved?

9. Preoperative maxillofacial CT images of a patient were acquired together with one or more 2D photographs taken with a digital camera. By projecting the 2D photographs onto the 3D skin surface, derived from the CT images, a textured 3D surface of the head can be obtained.

   (a) How can a 3D surface of the face be obtained from CT images?
   (b) How can the texture of a 2D photograph be projected onto this 3D surface?

B.9 Miscellaneous

1. Image quality. Explain why the following statements are true or false.

   (a) The detection threshold of small details is defined by the resolution and the CNR.
   (b) The available number of pixels influences the resolution and the perceptibility (visibility) of details.
   (c) The use of multiple spin echoes per excitation in MRI has a negative effect on the in-plane resolution.
   (d) A larger external magnetic field in MRI increases the CNR.
   (e) The resolution in PET is limited by physical constraints.

2. The Laplacian of a Gaussian (LoG) is often approximated as a difference of Gaussians (DoG) with different standard deviations, i.e.,

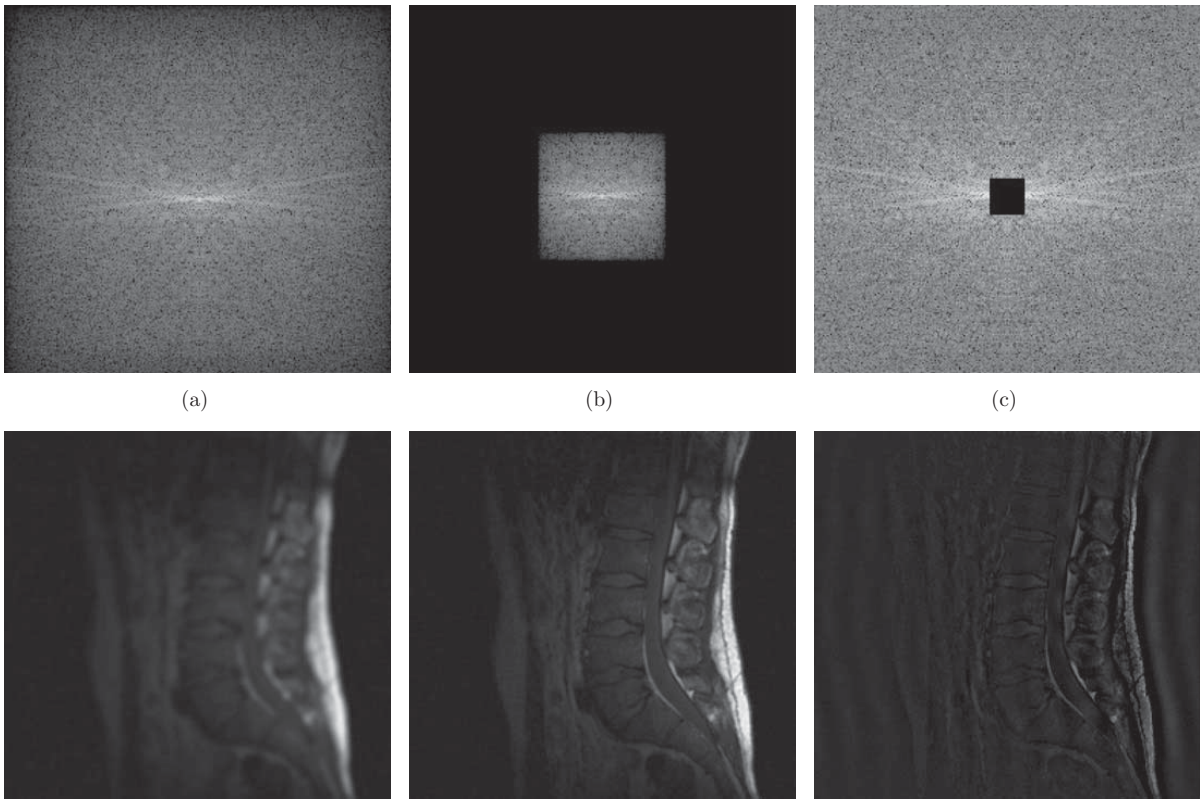
$$\nabla^2 g(\vec{r}) \approx \frac{1}{(k-1)\sigma}(g(\vec{r}, k\sigma) - g(\vec{r}, \sigma))$$

   for small-scale factors k.

   (a) Draw this function.
   (b) Is it useful for image enhancement? Explain.
   (c) Is it useful for image segmentation? Explain.
   (d) Is it useful for 3D visualization? Explain.

3. Figure B.51 shows three MR images of the lumbar spine and their $\vec{k}$-space in arbitrary order.

   (a) Which $\vec{k}$-space accompanies each of the three MR images? Explain.
   (b) The bottom left image and the bottom right image were combined using unsharp masking to obtain the image shown in Figure B.52. Explain.



(a) (b) (c)

(d) (e) (f)

Figure B.51

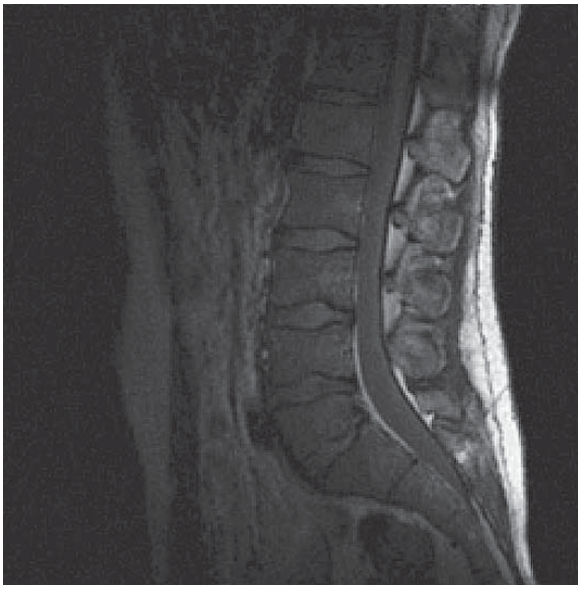


Figure B.52

4. CT is based on the projection theorem stating that the one-dimensional Fourier transform of the projections equals the two-dimensional Fourier transform of the image along a line in the 2D Fourier space, i.e.,

$$P(k,\theta) = F_1\{p_\theta(r)\} \leftrightarrow F(k_x,k_y) = F_2\{f(x,y)\}.$$

Hence, the image $f(x,y)$ can be reconstructed by calculating the inverse 2D Fourier transform.

   (a) In MRI it is possible to sample along radial lines in the $\vec{k}$-space (see Figure B.53(a)). Draw a suitable pulse sequence in the diagram of Figure B.53(b) to acquire samples from radial lines.
   (b) Can filtered backprojection be employed for MRI reconstruction as well?

5. Image reconstruction in CT and in MRI is based on Fourier theory. In both cases assumptions are made in order to apply this theory. In CT the X-ray beam is assumed to be monochromatic. In

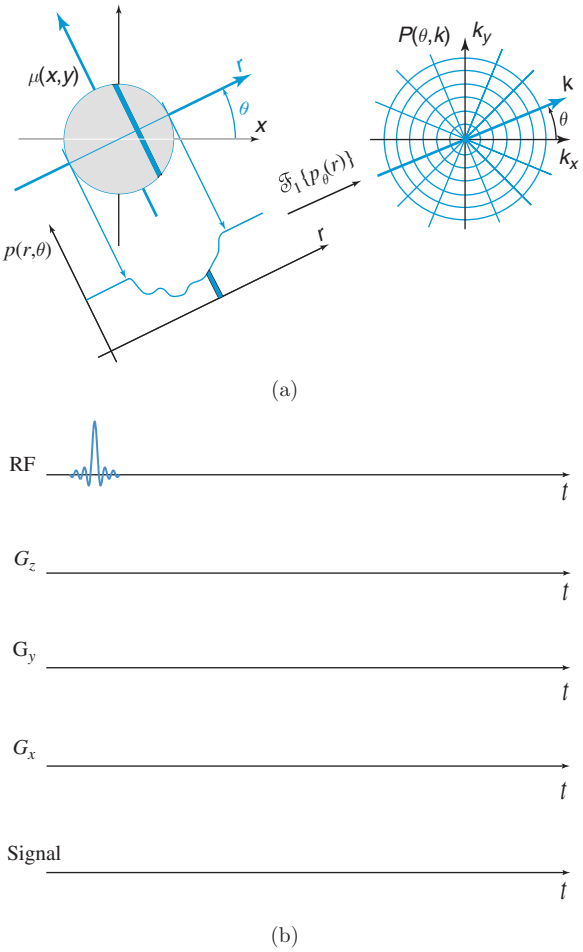


Figure B.53

MRI the relaxation effect during the short reading interval is neglected in the case of multiple echoes per excitation. What is the influence of these assumptions on the image quality in CT and MRI respectively?

6. Sagittal CT and MR images of the cervical spine are acquired (*C*1 through *C*7; length 20 cm). Calculate the scan time for both modalities. The following data are given

   (a) Helical CT: 128 detector rows; detector thickness = 0.6 mm in the center of the FOV; pitch = 0.8; rotation time = 1.0 s.
   (b) 2D turboSE MRI: TR = 2000 ms; TE = 90 ms; number of slices N_{sl} = 15; slice thickness = 3 mm; number of lines in the phase encoding direction N_{ph} = 192; number of samples per line N_r = 256; 15 lines are measured per excitation.

7. (a) Which types of electromagnetic radiation are used in CT, SPECT, MRI, and a microwave oven? What is their biological effect on the human body?
   (b) Which electromagnetic radiation can be employed in a body scanner in the airport?
8. How can we obtain a perfusion image using

   (a) Dual-energy CT?
   (b) MRI?
   (c) Nuclear medicine imaging?

   Explain your answer.
9. Aliasing in CT.

   (a) At which frequency k_{max} is the ramp filter cut off as a function of the X-ray beam width Δs? Explain.
   (b) How can aliasing be further reduced in the *xy*-plane?
   (c) What is the maximal pitch in helical CT to avoid aliasing? Explain.

   Aliasing in MRI. Assume that a sagittal slice is selected. Phase-encoding is done in the direction of the body axis.

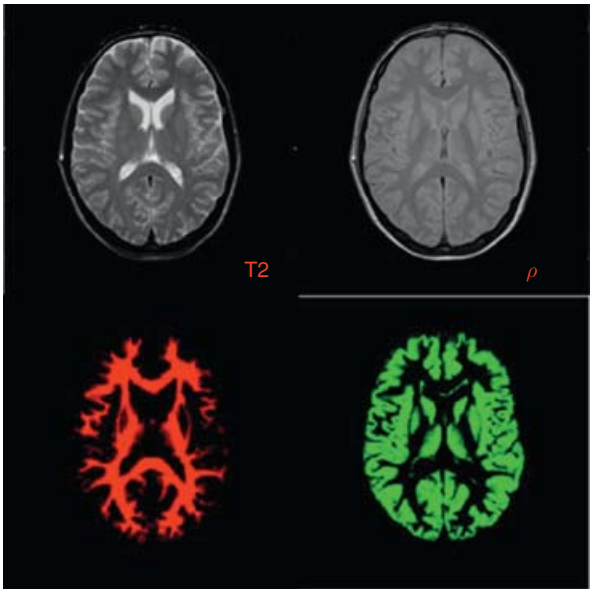
   (a) How is the effect of aliasing visible in the image?
   (b) Which constraint must be satisfied to avoid aliasing? Looking at this constraint, can aliasing be avoided?

   Aliasing in Doppler imaging.

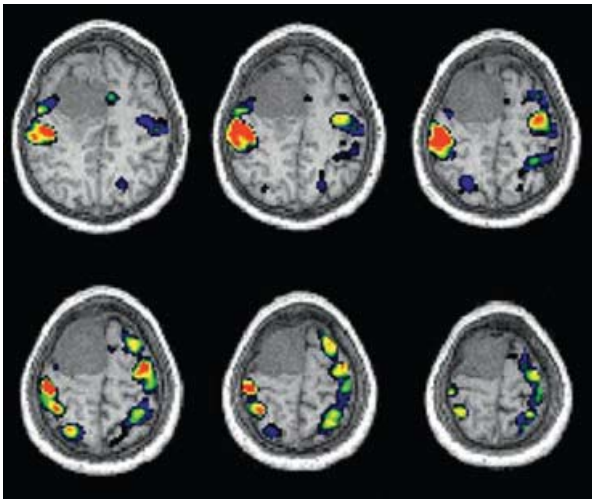
   (a) What is the maximal depth that can be measured without aliasing when the transmitted frequency f_T = 5 MHz and the blood velocity in the direction of the US waves v_a = 0.6 m/s?

10. Which methods do you know to obtain images of (a) blood vessels and (b) flow? Explain.
11. How have the images in Figure B.54 been obtained? Explain both the image formation and the image analysis method.
12. Dual-energy CT and image analysis.

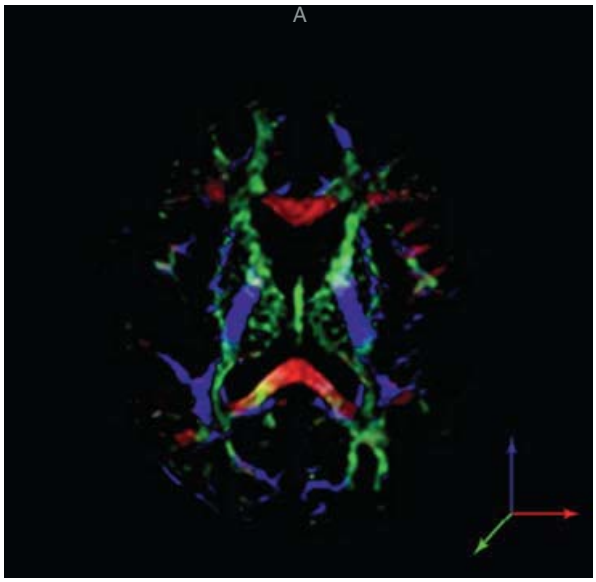
    (a) Explain the meaning of Figure B.55.
    (b) What is the (linear) partial volume effect? Can you notice it in this figure?
    (c) Given the data in this figure, how can we calculate a monochromatic image?

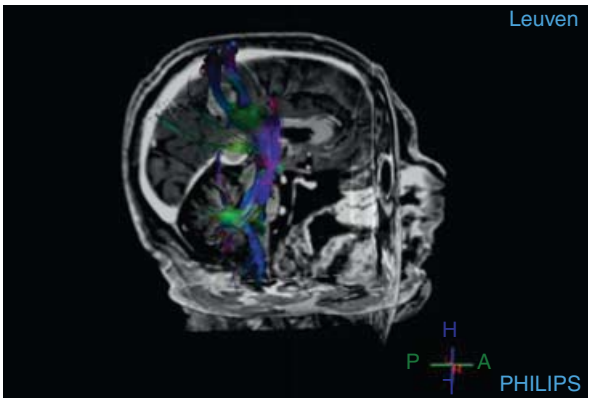


(a)



(b)





(c)

Figure B.54

(d) How can we obtain an image of the bony structures using statistical modeling?

13. MRI and image analysis. A bias field is a possible artifact in MRI.

(a) What is a bias field?
(b) Which technical imperfection is the cause of this artifact?
(c) What is the effect of this artifact on MRI-CT registration of the brain using mutual information?
(d) What is the effect of this artifact on white brain matter segmentation by means of pixel classification? Explain.
(e) How can this artifact be removed by means of image processing?

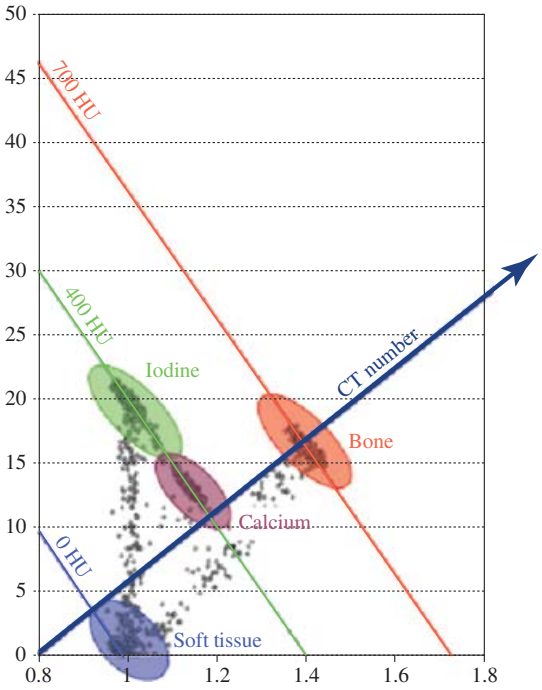


Figure B.55

14. X-ray and ultrasound intensity.

    (a) What is the definition and unit of *intensity* (I) of an X-ray beam and of an ultrasonic beam?

    (b) Draw and explain the X-ray tube spectrum (of emitted X-ray photons) as a function of the wavelength λ. The X-rays are attenuated by the irradiated body. Draw and explain the X-ray spectrum of the outgoing beam. (Draw the two curves in the same diagram.)

    (c) Calculate how the acoustic intensity changes if the sound level L increases from 100 dB to 110 dB.