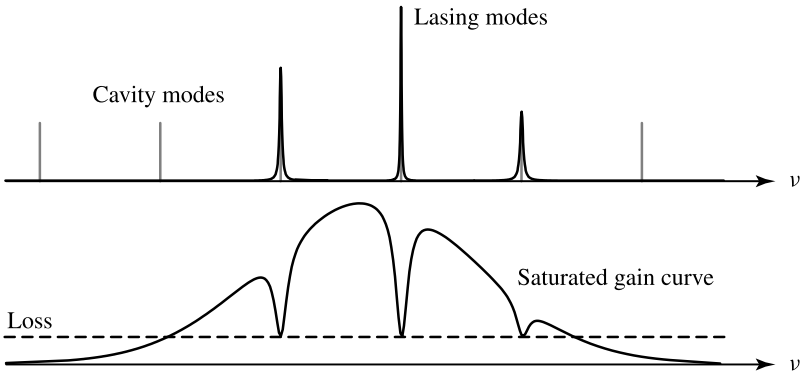

Figure 11.9 Spectral hole burning effect in the gain saturation of a laser in the case of inhomogeneous broadening. Multiple longitudinal modes oscillate simultaneously at a sufficiently high pumping level. The gain at each lasing frequency is saturated at the loss level. Mode pulling effect is ignored in this illustration.

medium if their longitudinal mode frequencies are sufficiently separated. Therefore, an inhomogeneously broadened laser can also oscillate in multiple transverse modes.

The linewidth of an oscillating laser mode is still described by (11.19). From this relation, we see that in practice the round-trip field gain factor G of a laser in steady-state oscillation cannot be exactly equal to unity because the laser linewidth cannot be zero, due to the existence of spontaneous emission. In reality, in steady-state oscillation the value of G is slightly less than unity, with the small difference made up by spontaneous emission. Clearly, the linewidth of an oscillating laser mode is determined by the amount of spontaneous emission that is channeled into the laser mode. Therefore, (11.19) is not very useful for calculating the linewidth of a laser mode in steady-state oscillation without knowing the exact value of G in the presence of spontaneous emission. Instead, a detailed analysis taking into account spontaneous emission yields the following *Schawlow–Townes relation* for the linewidth of a laser mode in terms of the laser parameters:

$$\Delta\nu_{\mathrm{ST}} = \frac{2\pi h\nu(\Delta\nu_{\mathrm{c}})^2}{P_{\mathrm{out}}} N_{\mathrm{sp}} = \frac{h\nu}{2\pi\tau_{\mathrm{c}}^2 P_{\mathrm{out}}} N_{\mathrm{sp}}, \tag{11.65}$$

where P_{out} is the output power of the laser mode being considered and N_{sp} is the spontaneous emission factor defined in (10.114). The effect of spontaneous emission on the linewidth of an oscillating laser mode enters the relation in (11.65) through the population densities of the upper and the lower laser levels in the form of the spontaneous emission factor. Because $N_{\mathrm{sp}} \geq 1$, the ultimate lower limit of the laser linewidth, which is known as the *Schawlow–Townes limit*, is that given in (11.65) with $N_{\mathrm{sp}} = 1$.

EXAMPLE 11.3 The Nd : YAG crystal used for the microchip laser described in Examples 11.1 and 11.2 has a spontaneous linewidth of 150 GHz at the laser wavelength