Chapter 9

```
> with(plots):
Warning, the name changecoords has been redefined
```

```
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
```

```
> with(DEtools):
Warning, the name adjoint has been redefined
```

Questions1

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': pi1:='pi1': pi2:='pi2':
> Q:=q1+q2;
```

$$Q := q1 + q2$$

```
> p:=A-B*Q;
```

$$p := A - B\,(q1 + q2)$$

```
> pi1:=p*q1-a*q1;
```

$$\pi 1 := (A - B\,(q1 + q2))\,q1 - a\,q1$$

```
> pi2:=p*q2-a*q2;
```

$$\pi 2 := (A - B\,(q1 + q2))\,q2 - a\,q2$$

```
> solve(diff(pi1,q1)=0,q2);
```

$$-\frac{2\,B\,q1 - A + a}{B}$$

```
> solve(diff(pi2,q2)=0,q2);
```

$$-\frac{1}{2}\,\frac{a - A + B\,q1}{B}$$

```
> solq1:=simplify(solve(-(2*B*q1-A+a)/B=-1/2*(a-A+B*q1)/B,q1));
```

$$solq1 := -\frac{1}{3}\,\frac{-A + a}{B}$$

```
> solq2:=simplify(subs(q1=solq1,-(2*B*q1-A+a)/B));
```

$$solq2 := -\frac{1}{3}\,\frac{-A + a}{B}$$

Hence, equilibrium quantities are identical.

Question 2

This question is best done with paper and pencil. Summations over *j* not equal to *i* are not conveniently dealt with in software packages.

Question 3

(i)

Although the question asks for this to be done on a spreadsheet, we shall here use *Maple* and present a more formal derivation.

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': pi1:='pi1': pi2:='pi2':
> Q:=q1+q2;
```

$$Q := q1 + q2$$

```
> p:=A-B*Q;
```

$$p := A - B\,(q1 + q2)$$

```
> pi1:=p*q1-a1*q1;
```

$$\pi 1 := (A - B\,(q1 + q2))\,q1 - a1\,q1$$

```
> pi2:=p*q2-a2*q2;
```

$$\pi 2 := (A - B\,(q1 + q2))\,q2 - a2\,q2$$

```
> solve(diff(pi1,q1)=0,q2);
```

$$-\frac{2\,B\,q1 - A + a1}{B}$$

```
> solve(diff(pi2,q2)=0,q2);
```

$$-\frac{1}{2}\,\frac{a2 - A + B\,q1}{B}$$

```
> solq1:=simplify(solve(-(2*B*q1-A+a1)/B=-1/2*(a2-A+B*q1)/B,q1)
  );
```

$$solq1 := \frac{1}{3}\,\frac{A - 2\,a1 + a2}{B}$$

```
> solq2:=simplify(subs(q1=solq1,-(2*B*q1-A+a1)/B));
```

$$solq2 := -\frac{1}{3}\,\frac{-A - a1 + 2\,a2}{B}$$

(ii)

We already have the following as the reaction functions in the static model:

$$q1 = \frac{A - a1}{2\,B} - \frac{1}{2}\frac{q2}{} \quad \text{for firm 1}$$

$$q2 = \frac{A - a2}{2\,B} - \frac{1}{2}\frac{q1}{} \quad \text{for firm 2}$$

So the dynamic reaction fuctions are:

$$\mathrm{q1}(t) = \frac{A - a1}{2\,B} - \frac{1}{2}\,\mathrm{q2}(t-1)$$

$$\mathrm{q2}(t) = \frac{A - a2}{2\,B} - \frac{1}{2}\,\mathrm{q1}(t-1)$$

```
> rsolve({q1(t) = (A-a1)/(2*B)-1*q2(t-1)/2,q2(t) =
  (A-a2)/(2*B)-1*q1(t-1)/2,q1(0)=q10,q2(0)=q20},{q1(t),q2(t)
  });
```

$$\{q1(t)=\frac{1}{6}\Big(-3\left(\frac{1}{2}\right)^t a2+\left(\frac{-1}{2}\right)^t a2-2\left(\frac{-1}{2}\right)^t A+2\,a2+2\,A+3\left(\frac{1}{2}\right)^t a1+\left(\frac{-1}{2}\right)^t a1$$
$$-4\,a1-3\left(\frac{1}{2}\right)^t B\,q20+3\left(\frac{1}{2}\right)^t B\,q10+3\left(\frac{-1}{2}\right)^t B\,q20+3\left(\frac{-1}{2}\right)^t B\,q10\Big)/B,\ q2(t)=\frac{1}{6}\Big($$
$$3\left(\frac{1}{2}\right)^t a2+\left(\frac{-1}{2}\right)^t a2-2\left(\frac{-1}{2}\right)^t A-4\,a2+2\,A-3\left(\frac{1}{2}\right)^t a1+\left(\frac{-1}{2}\right)^t a1+2\,a1$$
$$+3\left(\frac{1}{2}\right)^t B\,q20-3\left(\frac{1}{2}\right)^t B\,q10+3\left(\frac{-1}{2}\right)^t B\,q20+3\left(\frac{-1}{2}\right)^t B\,q10\Big)/B\}$$

This could be simplified but we do not do this here. What matters is that the stability is governed purely by the coefficients of q2(t-1) and q1(t-1) in the reaction functions of firms 1 and 2 respectively. Since these are less than unity, then the system is asymptotically stable. In fact, the coefficients must each take the value of -1/2. Consequently the coefficients of q10 and q20 in the results involve only $\left(\frac{1}{2}\right)^t$ or $\left(-\frac{1}{2}\right)^t$ and so tend to zero in the limit regardless of the values of q10 and q20. (Note that the term *B* cancells.)

Question 4

(i)

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': pi1:='pi1': pi2:='pi2':
> p:=9-Q;
```

$$p := 9-Q$$

```
> Q:=q1+q2;
```

$$Q := q1+q2$$

```
> TC1:=a1*q1; TC2:=a2*q2;
```

$$TC1 := a1\,q1$$
$$TC2 := a2\,q2$$

```
> pi1:=p*q1-TC1;
```

$$\pi1 := (9-q1-q2)\,q1-a1\,q1$$

```
> pi2:=p*q2-TC2;
```

$$\pi2 := (9-q1-q2)\,q2-a2\,q2$$

```
> solve(diff(pi1,q1)=0,q2);
```

$$-2\,q1+9-a1$$

```
> solve(diff(pi2,q2)=0,q2);
```

$$\frac{9}{2}-\frac{1}{2}q1-\frac{1}{2}a2$$

```
> solq1:=solve(-2*q1+9-a1=9/2-1/2*q1-1/2*a2,q1);
```

$$solq1 := 3-\frac{2}{3}a1+\frac{1}{3}a2$$

```
> solq2:=subs(q1=solq1,-2*q1+9-a1);
```

$$solq2 := 3+\frac{1}{3}a1-\frac{2}{3}a2$$

```
> solq1-solq2;
```

$$-a1+a2$$

Hence, equilibrium q_1 is greater than equilibrium q_2 if $a_1 < a_2$

(ii) $a_1 = 3$ and $a_2 = 5$

The model is then

$$p = 9 - Q$$
$$Q = q1 + q2$$
$$TC1 = 3\,q1$$
$$TC2 = 5\,q2$$

```
> subs(a1=3,pi1);
```

$$(9-q1-q2)\,q1-3\,q1$$

```
> subs(a2=5,pi2);
```

$$(9-q1-q2)\,q2-5\,q2$$

```
> solve(diff((9-q1-q2)*q1-3*q1,q1)=0,q1);
```

$$3-\frac{1}{2}q2$$

```
> solve(diff((9-q1-q2)*q2-5*q2,q2)=0,q2);
```

$$2-\frac{1}{2}q1$$

Our reaction functions are therefore

$$\mathrm{q1}(t) = 3-\frac{1\,\mathrm{q2}(t-1)}{2}$$

$$\mathrm{q2}(t) = 2-\frac{1\,\mathrm{q1}(t-1)}{2}$$

(a) firm 1 monopolist

```
> rsolve({q1(t)=3-(1/2)*q2(t-1),q2(t)=2-(1/2)*q1(t-1),q1(
0)=3,q2(0)=0},{q1(t),q2(t)});
```

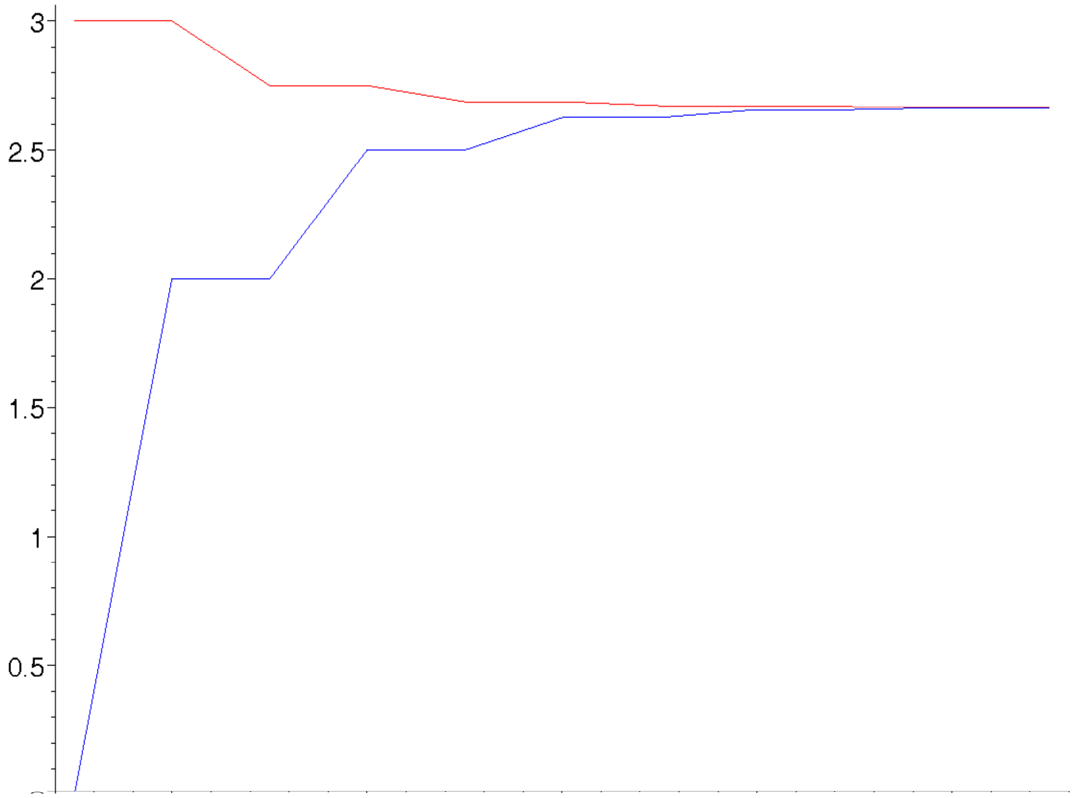
$$\left\{\mathrm{q1}(t)=\frac{1}{2}\left(\frac{1}{2}\right)^t-\frac{1}{6}\left(\frac{-1}{2}\right)^t+\frac{8}{3},\ \mathrm{q2}(t)=-\frac{1}{2}\left(\frac{1}{2}\right)^t-\frac{1}{6}\left(\frac{-1}{2}\right)^t+\frac{2}{3}\right\}$$

(b) firm 2 monopolist

```
> rsolve({q1(t)=3-(1/2)*q2(t-1),q2(t)=2-(1/2)*q1(t-1),q1(
0)=0,q2(0)=2},{q1(t),q2(t)});
```

$$\left\{\mathrm{q1}(t)=-2\left(\frac{1}{2}\right)^t-\frac{2}{3}\left(\frac{-1}{2}\right)^t+\frac{8}{3},\ \mathrm{q2}(t)=2\left(\frac{1}{2}\right)^t-\frac{2}{3}\left(\frac{-1}{2}\right)^t+\frac{2}{3}\right\}$$

```
> q1firm1:=seq(1/2*(1/2)^t-1/6*(-1/2)^t+8/3,t=0..10):
> q1firm2:=seq(-2*(1/2)^t-2/3*(-1/2)^t+8/3,t=0..10):
> firm1:=listplot([q1firm1],colour=red):
> firm2:=listplot([q1firm2],colour=blue):
> display(firm1,firm2);
```



The equilibrium is reached sooner when firm 1 is the monopolist. This should not be surprising, since firm 1 has the lower unit costs.

Question 5

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': q3:='q3': pi1:='pi1':
  pi2:='pi2':pi3:='pi3':
```

(i)

```
> pi1:=(9-q1-q2-q3)*q1-5*q1;
  pi2:=(9-q1-q2-q3)*q2-5*q2;
  pi3:=(9-q1-q2-q3)*q3-5*q3;
```

$$\pi1 := (9 - q1 - q2 - q3)\, q1 - 5\, q1$$

$$\pi2 := (9 - q1 - q2 - q3)\, q2 - 5\, q2$$

$$\pi3 := (9 - q1 - q2 - q3)\, q3 - 5\, q3$$

```
> diff(pi1,q1);
```

$$-2\, q1 + 4 - q2 - q3$$

```
> diff(pi2,q2);
```

$$-2\, q2 + 4 - q1 - q3$$

```
> diff(pi3,q3);
```

$$-2\, q3 + 4 - q1 - q2$$

```
> solve({-2*q1+4-q2-q3=0,-2*q2+4-q1-q3=0,-2*q3+4-q1-q2=0},{q1,q2,q3});
```

$$\{q3 = 1, q1 = 1, q2 = 1\}$$

Hence the equilibrium is closer to the origin, i.e., the smaller the equilibrium output levels the higher the marginal costs.

(ii)

```
> solve(-2*q1+4-q2-q3=0,q1);
```

$$2 - \frac{1}{2}q2 - \frac{1}{2}q3$$

```
> solve(-2*q2+4-q1-q3,q2);
```

$$2 - \frac{1}{2}q1 - \frac{1}{2}q3$$

```
> solve(-2*q3+4-q1-q2,q3);
```

$$2 - \frac{1}{2}q1 - \frac{1}{2}q2$$

Hence, the reaction curves are:

$$q1(t) = 2 - \frac{1\ q2(t-1)}{2} - \frac{1\ q3(t-1)}{2}$$

$$q2(t) = 2 - \frac{1\ q1(t-1)}{2} - \frac{1\ q3(t-1)}{2}$$

$$q3(t) = 2 - \frac{1\ q1(t-1)}{2} - \frac{1\ q2(t-1)}{2}$$

(iii)

```
> rsolve({q1(t) = 2-1*q2(t-1)/2-1*q3(t-1)/2,q2(t) =
  2-1*q1(t-1)/2-1*q3(t-1)/2,q3(t) =
  2-1*q1(t-1)/2-1*q2(t-1)/2,q1(0)=q10,q2(0)=q20,q3(0)=q30},{
  q1(t),q2(t),q3(t)});
```

$$\{q2(t) = -(-1)^t + 1 + \frac{1}{3}(-1)^t q30 + \frac{1}{3}(-1)^t q10 + \frac{1}{3}(-1)^t q20 - \frac{1}{3}\left(\frac{1}{2}\right)^t q30 - \frac{1}{3}\left(\frac{1}{2}\right)^t q10 + \frac{2}{3}\left(\frac{1}{2}\right)^t q20, q1(t) = -(-1)^t + 1 + \frac{1}{3}(-1)^t q30 + \frac{1}{3}(-1)^t q10 + \frac{1}{3}(-1)^t q20 - \frac{1}{3}\left(\frac{1}{2}\right)^t q30 + \frac{2}{3}\left(\frac{1}{2}\right)^t q10 - \frac{1}{3}\left(\frac{1}{2}\right)^t q20, q3(t) = -(-1)^t + 1 + \frac{1}{3}(-1)^t q30 + \frac{1}{3}(-1)^t q10 + \frac{1}{3}(-1)^t q20 + \frac{2}{3}\left(\frac{1}{2}\right)^t q30 - \frac{1}{3}\left(\frac{1}{2}\right)^t q10 - \frac{1}{3}\left(\frac{1}{2}\right)^t q20\}$$

Because of the occurrence of the terms $(-1)^t$ then the system also oscillates, eventually oscillating with constant amplitude.

Question 6

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': q3:='q3':pi1:='pi1':
  pi2:='pi':pi3:='pi3':
>
```

(i)

```
> pi1:=(9-q1-q2-q3)*q1-3*q1;
  pi2:=(9-q1-q2-q3)*q2-2*q2;
  pi3:=(9-q1-q2-q3)*q3-q3;
```

$$\pi1 := (9 - q1 - q2 - q3)\, q1 - 3\, q1$$

$$\pi2 := (9 - q1 - q2 - q3)\, q2 - 2\, q2$$

$$\pi3 := (9 - q1 - q2 - q3)\, q3 - q3$$

```
> diff(pi1,q1);
```

$$-2\, q1 + 6 - q2 - q3$$

```
> diff(pi2,q2);
```

$$-2\, q2 + 7 - q1 - q3$$

```
> diff(pi3,q3);
```

$$-2\, q3 + 8 - q1 - q2$$

```
> solve({-2*q1+6-q2-q3=0,-2*q2+7-q1-q3=0,-2*q3+8-q1-q2=0},{q
  1,q2,q3});
```

$$\{q1 = \frac{3}{4}, q2 = \frac{7}{4}, q3 = \frac{11}{4}\}$$

(ii)

```
> solve(-2*q1+6-q2-q3,q1);
```

$$3 - \frac{1}{2}\, q2 - \frac{1}{2}\, q3$$

```
> solve(-2*q2+7-q1-q3,q2);
```

$$\frac{7}{2} - \frac{1}{2}\, q1 - \frac{1}{2}\, q3$$

```
> solve(-2*q3+8-q1-q2,q3);
```

$$4 - \frac{1}{2}\, q1 - \frac{1}{2}\, q2$$

Hence, the reaction curves are:

$$q1(t) = 3 - \frac{1\, q2(t-1)}{2} - \frac{1\, q3(t-1)}{2}$$

$$q2(t) = \frac{7}{2} - \frac{1\, q1(t-1)}{2} - \frac{1\, q3(t-1)}{2}$$

$$q3(t) = 4 - \frac{1\ q1(t-1)}{2} - \frac{1\ q2(t-1)}{2}$$

Monopoly points are (3,0,0) for firm 1, (0,7/2,0) for firm 2 and (0,0,4) for firm 3.

(a) firm 1 monopolist

```
> rsolve({q1(t) = 3-1*q2(t-1)/2-1*q3(t-1)/2,q2(t) =
  7/2-1*q1(t-1)/2-1*q3(t-1)/2,q3(t) =
  4-1*q1(t-1)/2-1*q2(t-1)/2,q1(0)=3,q2(0)=0,q3(0)=0},{q1(
  t),q2(t),q3(t)});
```

$$\{q2(t) = -\frac{3}{4}(-1)^t + \frac{7}{4} - \left(\frac{1}{2}\right)^t,\ q3(t) = -\frac{3}{4}(-1)^t - 2\left(\frac{1}{2}\right)^t + \frac{11}{4},$$
$$q1(t) = -\frac{3}{4}(-1)^t + 3\left(\frac{1}{2}\right)^t + \frac{3}{4}\}$$

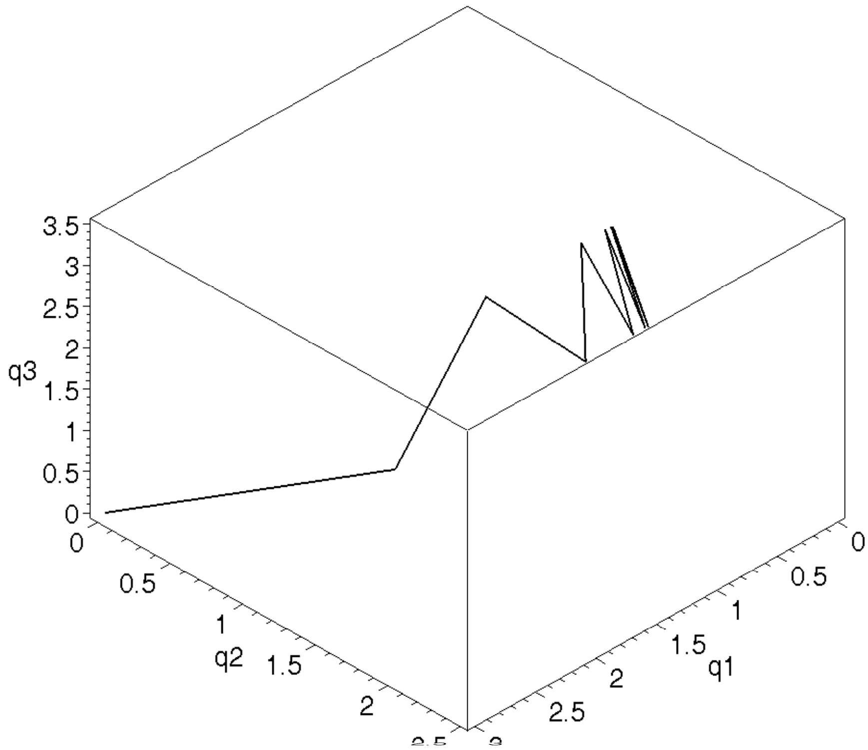
```
> q1:=t->-3/4*(-1)^t+3*(1/2)^t+3/4;
  q2:=t->-3/4*(-1)^t+7/4-(1/2)^t;
  q3:=t->-3/4*(-1)^t-2*(1/2)^t+11/4;
```

$$q1 := t \rightarrow -\frac{3}{4}(-1)^t + 3\left(\frac{1}{2}\right)^t + \frac{3}{4}$$

$$q2 := t \rightarrow -\frac{3}{4}(-1)^t + \frac{7}{4} - \left(\frac{1}{2}\right)^t$$

$$q3 := t \rightarrow -\frac{3}{4}(-1)^t - 2\left(\frac{1}{2}\right)^t + \frac{11}{4}$$

```
> points:=[seq([q1(t),q2(t),q3(t)],t=0..10)]:
> pointplot3d(points,
        axes=BOXED, connect=true, thickness=2,
        labels=["q1","q2","q3"],
        colour=black);
```




```
>
```

(b) firm 2 as monopolist

```
> q1:='q1':q2:='q2':q3:='q3':
> rsolve({q1(t) = 3-1*q2(t-1)/2-1*q3(t-1)/2,q2(t) =
  7/2-1*q1(t-1)/2-1*q3(t-1)/2,q3(t) =
  4-1*q1(t-1)/2-1*q2(t-1)/2,q1(0)=0,q2(0)=7/2,q3(0)=0},{q
  1(t),q2(t),q3(t)});
```

$$\{q1(t) = -\frac{7}{12}(-1)^t - \frac{1}{6}\left(\frac{1}{2}\right)^t + \frac{3}{4}, q3(t) = -\frac{7}{12}(-1)^t - \frac{13}{6}\left(\frac{1}{2}\right)^t + \frac{11}{4},$$

$$q2(t) = -\frac{7}{12}(-1)^t + \frac{7}{4} + \frac{7}{3}\left(\frac{1}{2}\right)^t\}$$

```
> q1:=t->-7/12*(-1)^t-1/6*(1/2)^t+3/4;
  q2:=t->-7/12*(-1)^t+7/4+7/3*(1/2)^t;
  q3:=t->-7/12*(-1)^t-13/6*(1/2)^t+11/4;
```

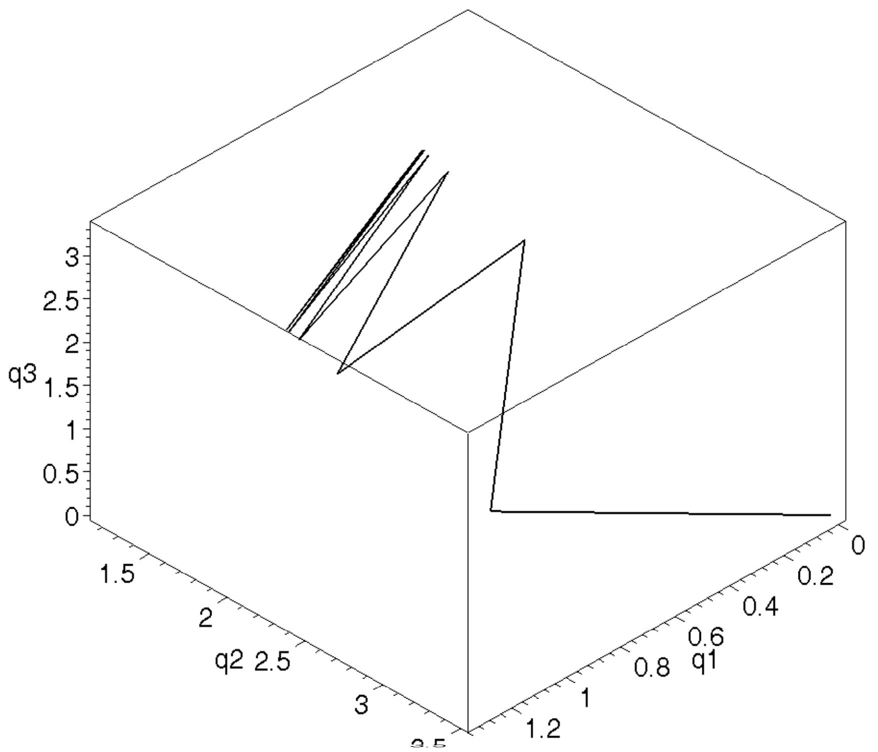
$$q1 := t \to -\frac{7}{12}(-1)^t - \frac{1}{6}\left(\frac{1}{2}\right)^t + \frac{3}{4}$$

$$q2 := t \to -\frac{7}{12}(-1)^t + \frac{7}{4} + \frac{7}{3}\left(\frac{1}{2}\right)^t$$

$$q3 := t \to -\frac{7}{12}(-1)^t - \frac{13}{6}\left(\frac{1}{2}\right)^t + \frac{11}{4}$$

```
> points:=[seq([q1(t),q2(t),q3(t)],t=0..10)]:
> pointplot3d(points,
        axes=BOXED, connect=true, thickness=2,
```

```
labels=["q1","q2","q3"],
colour=black);
```




```
>
```

(c) firm 3 monopolist

```
> q1:='q1':q2:='q2':q3:='q3':
> rsolve({q1(t) = 3-1*q2(t-1)/2-1*q3(t-1)/2,q2(t) =
  7/2-1*q1(t-1)/2-1*q3(t-1)/2,q3(t) =
  4-1*q1(t-1)/2-1*q2(t-1)/2,q1(0)=0,q2(0)=0,q3(0)=4},{q1(
  t),q2(t),q3(t)});
```

$$\{q1(t) = -\frac{5}{12}(-1)^t - \frac{1}{3}\left(\frac{1}{2}\right)^t + \frac{3}{4}, q2(t) = -\frac{5}{12}(-1)^t + \frac{7}{4} - \frac{4}{3}\left(\frac{1}{2}\right)^t,$$

$$q3(t) = -\frac{5}{12}(-1)^t + \frac{5}{3}\left(\frac{1}{2}\right)^t + \frac{11}{4}\}$$

```
> q1:=t->-5/12*(-1)^t-1/3*(1/2)^t+3/4;
  q2:=t->-5/12*(-1)^t+7/4-4/3*(1/2)^t;
  q3:=t->-5/12*(-1)^t+5/3*(1/2)^t+11/4;
```

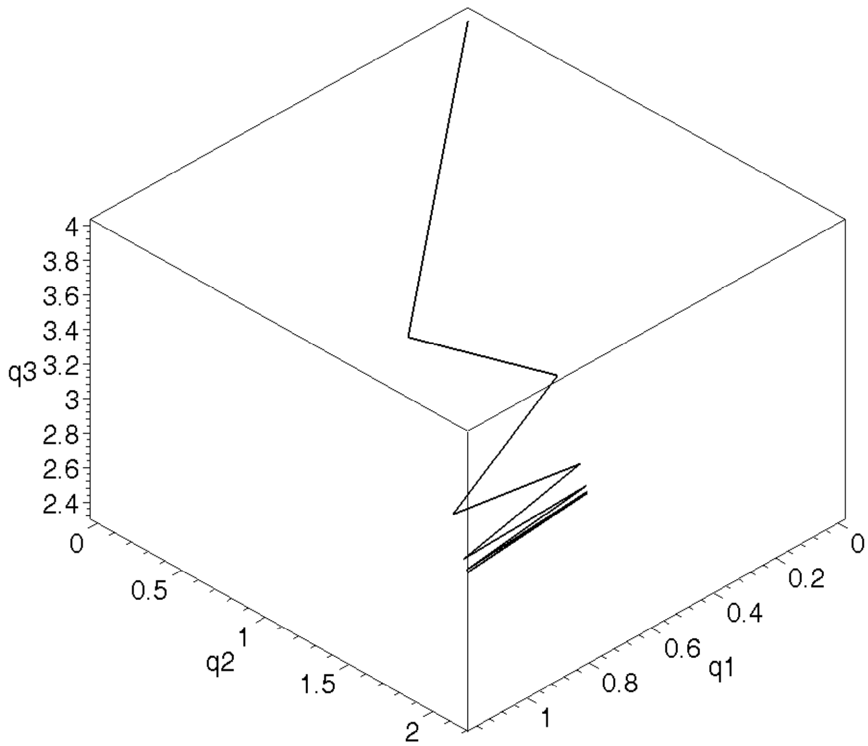
$$q1 := t \to -\frac{5}{12}(-1)^t - \frac{1}{3}\left(\frac{1}{2}\right)^t + \frac{3}{4}$$

$$q2 := t \to -\frac{5}{12}(-1)^t + \frac{7}{4} - \frac{4}{3}\left(\frac{1}{2}\right)^t$$

$$q3 := t \to -\frac{5}{12}(-1)^t + \frac{5}{3}\left(\frac{1}{2}\right)^t + \frac{11}{4}$$

```
> points:=[seq([q1(t),q2(t),q3(t)],t=0..10)]:
```

```
> pointplot3d(points,
        axes=BOXED, connect=true, thickness=2,
        labels=["q1","q2","q3"],
        colour=black);
```




```
>
```

(iii)

```
> q1:='q1':q2:='q2':q3:='q3':
```

```
> rsolve({q1(t) = 3-1*q2(t-1)/2-1*q3(t-1)/2,q2(t) =
  7/2-1*q1(t-1)/2-1*q3(t-1)/2,q3(t) =
  4-1*q1(t-1)/2-1*q2(t-1)/2,q1(0)=q10,q2(0)=q20,q3(0)=q30},{
  q1(t),q2(t),q3(t)});
```

$$\left\{ q2(t) = -\frac{7}{4}(-1)^t + \frac{7}{4} + \frac{1}{3}(-1)^t q10 + \frac{1}{3}(-1)^t q30 + \frac{1}{3}(-1)^t q20 - \frac{1}{3}\left(\frac{1}{2}\right)^t q10 - \frac{1}{3}\left(\frac{1}{2}\right)^t q30 + \frac{2}{3}\left(\frac{1}{2}\right)^t q20, \right.$$
$$q1(t) = -\frac{7}{4}(-1)^t + \left(\frac{1}{2}\right)^t + \frac{3}{4} + \frac{1}{3}(-1)^t q10 + \frac{1}{3}(-1)^t q30 + \frac{1}{3}(-1)^t q20 + \frac{2}{3}\left(\frac{1}{2}\right)^t q10 - \frac{1}{3}\left(\frac{1}{2}\right)^t q30 - \frac{1}{3}\left(\frac{1}{2}\right)^t q20,$$
$$\left. q3(t) = -\frac{7}{4}(-1)^t - \left(\frac{1}{2}\right)^t + \frac{11}{4} + \frac{1}{3}(-1)^t q10 + \frac{1}{3}(-1)^t q30 + \frac{1}{3}(-1)^t q20 - \frac{1}{3}\left(\frac{1}{2}\right)^t q10 + \frac{2}{3}\left(\frac{1}{2}\right)^t q30 - \frac{1}{3}\left(\frac{1}{2}\right)^t q20 \right\}$$

Question 7

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': q3:='q3': pi1:='pi1':
  pi2:='pi2': pi3:='pi3':
>
```

(i)

```
> pi1:=(15-2*q1-2*q2-2*q3)*q1-5*q1;
  pi2:=(15-2*q1-2*q2-2*q3)*q2-3*q2;
  pi3:=(15-2*q1-2*q2-2*q3)*q3-2*q3;
```

$$\pi1 := (15 - 2\,q1 - 2\,q2 - 2\,q3)\,q1 - 5\,q1$$

$$\pi2 := (15 - 2\,q1 - 2\,q2 - 2\,q3)\,q2 - 3\,q2$$

$$\pi3 := (15 - 2\,q1 - 2\,q2 - 2\,q3)\,q3 - 2\,q3$$

```
> diff(pi1,q1);
```

$$-4\,q1 + 10 - 2\,q2 - 2\,q3$$

```
> diff(pi2,q2);
```

$$-4\,q2 + 12 - 2\,q1 - 2\,q3$$

```
> diff(pi3,q3);
```

$$-4\,q3 + 13 - 2\,q1 - 2\,q2$$

```
> solve({-4*q1+10-2*q2-2*q3,-4*q2+12-2*q1-2*q3,-4*q3+13-2*q1
  -2*q2},{q1,q2,q3});
```

$$\{q1 = \frac{5}{8}, q2 = \frac{13}{8}, q3 = \frac{17}{8}\}$$

(ii)

```
> solve(-4*q1+10-2*q2-2*q3,q1);
```

$$\frac{5}{2} - \frac{1}{2}\,q2 - \frac{1}{2}\,q3$$

```
> solve(-4*q2+12-2*q1-2*q3,q2);
```

$$3 - \frac{1}{2}\,q1 - \frac{1}{2}\,q3$$

```
> solve(-4*q3+13-2*q1-2*q2,q3);
```

$$\frac{13}{4} - \frac{1}{2}\,q1 - \frac{1}{2}\,q2$$

Hence, the reaction curves are:

$$q1(t) = \frac{5}{2} - \frac{1\ q2(t-1)}{2} - \frac{1\ q3(t-1)}{2}$$

$$q2(t) = 3 - \frac{1\ q1(t-1)}{2} - \frac{1\ q3(t-1)}{2}$$

$$q3(t) = \frac{13}{4} - \frac{1\ q1(t-1)}{2} - \frac{1\ q2(t-1)}{2}$$

Monopoly points are (5/2,0,0) for firm 1, (0,3,0) for firm 2 and (0,0,13/4) for firm 3.

(a) firm 1 monopolist

```
> rsolve({q1(t) = 5/2-1*q2(t-1)/2-1*q3(t-1)/2,q2(t) =
  3-1*q1(t-1)/2-1*q3(t-1)/2,q3(t) =
  13/4-1*q1(t-1)/2-1*q2(t-1)/2,q1(0)=5/2,q2(0)=0,q3(0)=0}
  ,{q1(t),q2(t),q3(t)});
```

$$\{q2(t) = -\frac{5}{8}(-1)^t - \left(\frac{1}{2}\right)^t + \frac{13}{8},\ q3(t) = -\frac{5}{8}(-1)^t - \frac{3}{2}\left(\frac{1}{2}\right)^t + \frac{17}{8},$$

$$q1(t) = -\frac{5}{8}(-1)^t + \frac{5}{2}\left(\frac{1}{2}\right)^t + \frac{5}{8}\}$$

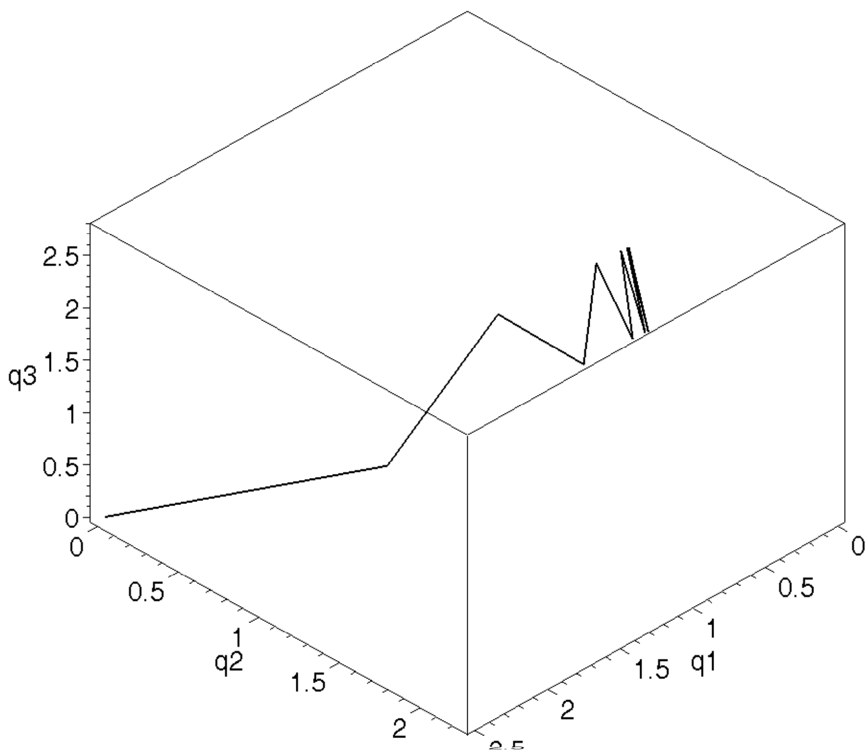
```
> q1:=t->-5/8*(-1)^t+5/2*(1/2)^t+5/8;
  q2:=t->-5/8*(-1)^t-(1/2)^t+13/8;
  q3:=t->-5/8*(-1)^t-3/2*(1/2)^t+17/8;
```

$$q1 := t \to -\frac{5}{8}(-1)^t + \frac{5}{2}\left(\frac{1}{2}\right)^t + \frac{5}{8}$$

$$q2 := t \to -\frac{5}{8}(-1)^t - \left(\frac{1}{2}\right)^t + \frac{13}{8}$$

$$q3 := t \to -\frac{5}{8}(-1)^t - \frac{3}{2}\left(\frac{1}{2}\right)^t + \frac{17}{8}$$

```
> points:=[seq([q1(t),q2(t),q3(t)],t=0..10)]:
> pointplot3d(points,
        axes=BOXED, connect=true, thickness=2,
        labels=["q1","q2","q3"],
        colour=black);
```



```
>
```

(b) firm 2 as monopolist

```
> q1:='q1':q2:='q2':q3:='q3':
> rsolve({q1(t) = 5/2-1*q2(t-1)/2-1*q3(t-1)/2,q2(t) =
  3-1*q1(t-1)/2-1*q3(t-1)/2,q3(t) =
  13/4-1*q1(t-1)/2-1*q2(t-1)/2,q1(0)=0,q2(0)=3,q3(0)=0},{
  q1(t),q2(t),q3(t)});
```

$$\{q2(t) = -\frac{11}{24}(-1)^t + \frac{11}{6}\left(\frac{1}{2}\right)^t + \frac{13}{8},\ q3(t) = -\frac{11}{24}(-1)^t - \frac{5}{3}\left(\frac{1}{2}\right)^t + \frac{17}{8},$$

$$q1(t) = -\frac{11}{24}(-1)^t - \frac{1}{6}\left(\frac{1}{2}\right)^t + \frac{5}{8}\}$$

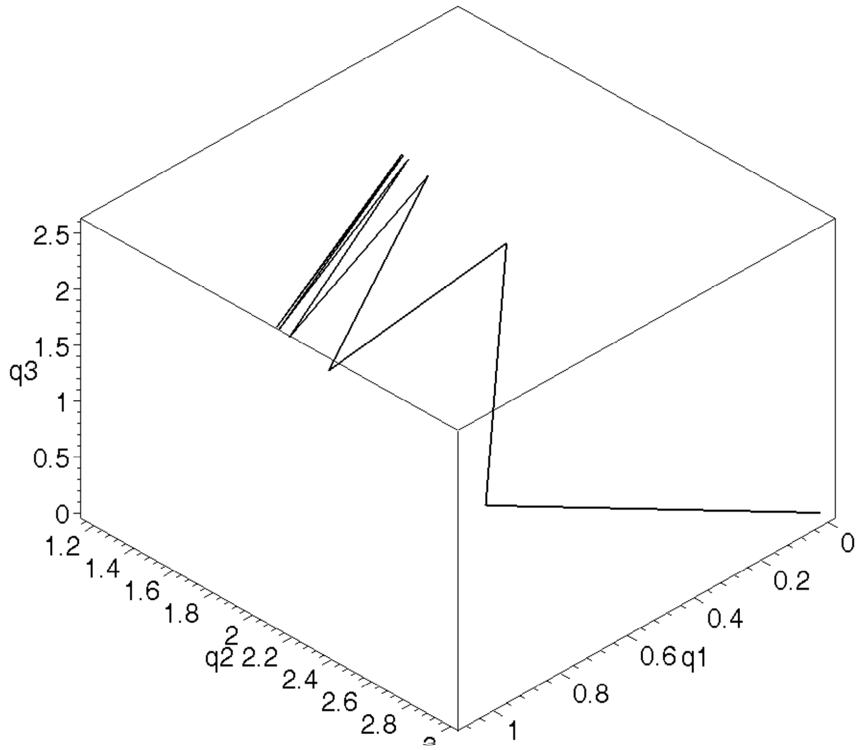
```
> q1:=t->-11/24*(-1)^t-1/6*(1/2)^t+5/8;
  q2:=t->-11/24*(-1)^t+11/6*(1/2)^t+13/8;
  q3:=t->-11/24*(-1)^t-5/3*(1/2)^t+17/8;
```

$$q1 := t \to -\frac{11}{24}(-1)^t - \frac{1}{6}\left(\frac{1}{2}\right)^t + \frac{5}{8}$$

$$q2 := t \to -\frac{11}{24}(-1)^t + \frac{11}{6}\left(\frac{1}{2}\right)^t + \frac{13}{8}$$

$$q3 := t \to -\frac{11}{24}(-1)^t - \frac{5}{3}\left(\frac{1}{2}\right)^t + \frac{17}{8}$$

```
> points:=[seq([q1(t),q2(t),q3(t)],t=0..10)]:
> pointplot3d(points,
      axes=BOXED, connect=true, thickness=2,
      labels=["q1","q2","q3"],
      colour=black);
```




```
>
```

(c) *firm 3 monopolist*

```
> q1:='q1':q2:='q2':q3:='q3':
> rsolve({q1(t) = 5/2-1*q2(t-1)/2-1*q3(t-1)/2,q2(t) =
  3-1*q1(t-1)/2-1*q3(t-1)/2,q3(t) =
  13/4-1*q1(t-1)/2-1*q2(t-1)/2,q1(0)=0,q2(0)=0,q3(0)=13/4
  },{q1(t),q2(t),q3(t)});
```

$$\{q1(t) = -\frac{3}{8}(-1)^t - \frac{1}{4}\left(\frac{1}{2}\right)^t + \frac{5}{8}, q3(t) = -\frac{3}{8}(-1)^t + \frac{3}{2}\left(\frac{1}{2}\right)^t + \frac{17}{8},$$

$$q2(t) = -\frac{3}{8}(-1)^t - \frac{5}{4}\left(\frac{1}{2}\right)^t + \frac{13}{8}\}$$

```
> q1:=t->-3/8*(-1)^t-1/4*(1/2)^t+5/8;
  q2:=t->-3/8*(-1)^t-5/4*(1/2)^t+13/8;
  q3:=t->-3/8*(-1)^t+3/2*(1/2)^t+17/8;
```

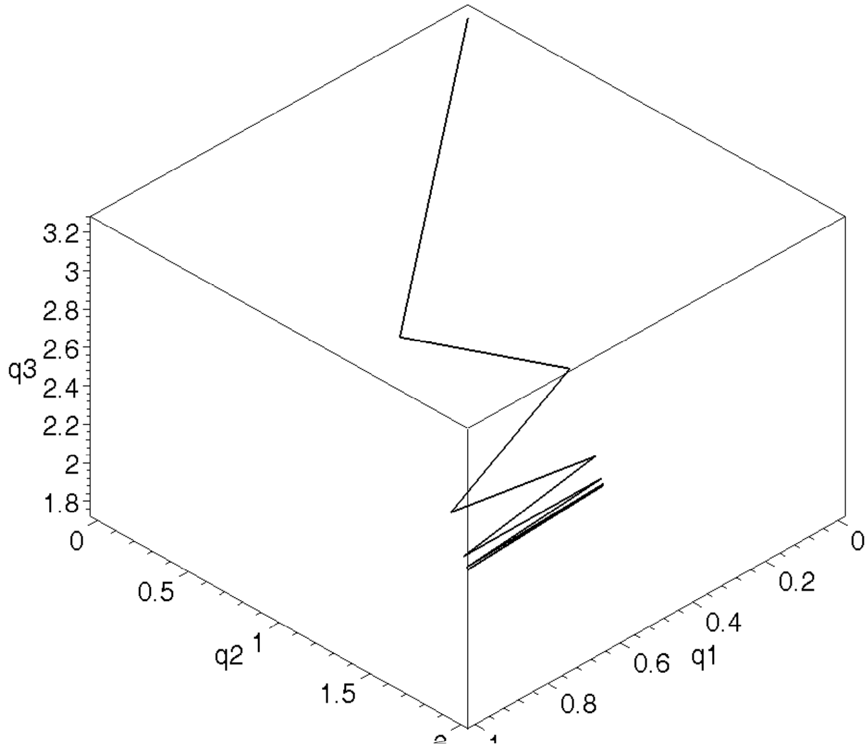
$$q1 := t \to -\frac{3}{8}(-1)^t - \frac{1}{4}\left(\frac{1}{2}\right)^t + \frac{5}{8}$$

$$q2 := t \to -\frac{3}{8}(-1)^t - \frac{5}{4}\left(\frac{1}{2}\right)^t + \frac{13}{8}$$

$$q3 := t \to -\frac{3}{8}(-1)^t + \frac{3}{2}\left(\frac{1}{2}\right)^t + \frac{17}{8}$$

```
> points:=[seq([q1(t),q2(t),q3(t)],t=0..10)]:
> pointplot3d(points,
      axes=BOXED, connect=true, thickness=2,
```

```
                labels=["q1","q2","q3"],
                colour=black);
```




```
>
```

(iii)

```
> q1:='q1':q2:='q2':q3:='q3':
```

```
> rsolve({q1(t) = 5/2-1*q2(t-1)/2-1*q3(t-1)/2,q2(t) =
  3-1*q1(t-1)/2-1*q3(t-1)/2,q3(t) =
  13/4-1*q1(t-1)/2-1*q2(t-1)/2,q1(0)=q10,q2(0)=q20,q3(0)=q30
  },{q1(t),q2(t),q3(t)});
```

$$\{q1(t) = -\frac{35}{24}(-1)^t + \frac{5}{6}\left(\frac{1}{2}\right)^t + \frac{5}{8} + \frac{1}{3}(-1)^t q20 + \frac{1}{3}(-1)^t q10 + \frac{1}{3}(-1)^t q30 - \frac{1}{3}\left(\frac{1}{2}\right)^t q20 + \frac{2}{3}\left(\frac{1}{2}\right)^t q10 - \frac{1}{3}\left(\frac{1}{2}\right)^t q30,\ q3(t) = -\frac{35}{24}(-1)^t - \frac{2}{3}\left(\frac{1}{2}\right)^t + \frac{17}{8} + \frac{1}{3}(-1)^t q20 + \frac{1}{3}(-1)^t q10 + \frac{1}{3}(-1)^t q30 - \frac{1}{3}\left(\frac{1}{2}\right)^t q20 - \frac{1}{3}\left(\frac{1}{2}\right)^t q10 + \frac{2}{3}\left(\frac{1}{2}\right)^t q30,\ q2(t) = -\frac{35}{24}(-1)^t - \frac{1}{6}\left(\frac{1}{2}\right)^t + \frac{13}{8} + \frac{1}{3}(-1)^t q20 + \frac{1}{3}(-1)^t q10 + \frac{1}{3}(-1)^t q30 + \frac{2}{3}\left(\frac{1}{2}\right)^t q20 - \frac{1}{3}\left(\frac{1}{2}\right)^t q10 - \frac{1}{3}\left(\frac{1}{2}\right)^t q30\}$$

```
>
```

(iv)

No. Because of the presence of $(-1)^t$, the system will eventually oscillate.

Question 8

```
>
```

(i)

(a)

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': pi1:='pi1':
  pi2:='pi2':
> Q:=q1+q2;
```

$$Q := q1 + q2$$

```
> p:=20-3*Q;
```

$$p := 20 - 3\,q1 - 3\,q2$$

```
> pi1:=p*q1-4*q1;
  pi2:=p*q2-4*q2;
```

$$\pi1 := (20 - 3\,q1 - 3\,q2)\,q1 - 4\,q1$$

$$\pi2 := (20 - 3\,q1 - 3\,q2)\,q2 - 4\,q2$$

```
> diff(pi1,q1);
```

$$-6\,q1 + 16 - 3\,q2$$

```
> diff(pi2,q2);
```

$$-6\,q2 + 16 - 3\,q1$$

```
> solve({-6*q1+16-3*q2=0,-6*q2+16-3*q1=0},{q1,q2});
```

$$\left\{q2 = \frac{16}{9},\, q1 = \frac{16}{9}\right\}$$

```
>
```

(b)

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': q3:='q3':
  pi1:='pi1': pi2:='pi2': pi3:='pi3':
> Q:=q1+q2+q3;
```

$$Q := q1 + q2 + q3$$

```
> p:=20-3*Q;
```

$$p := 20 - 3\,q1 - 3\,q2 - 3\,q3$$

```
> pi1:=p*q1-4*q1;
  pi2:=p*q2-4*q2;
  pi3:=p*q3-4*q3;
```

$$\pi1 := (20 - 3\,q1 - 3\,q2 - 3\,q3)\,q1 - 4\,q1$$

$$\pi2 := (20 - 3\,q1 - 3\,q2 - 3\,q3)\,q2 - 4\,q2$$

$$\pi3 := (20 - 3\,q1 - 3\,q2 - 3\,q3)\,q3 - 4\,q3$$

```
> diff(pi1,q1);
```

```
diff(pi2,q2);
diff(pi3,q3);
```

$$-6\ q1 + 16 - 3\ q2 - 3\ q3$$

$$-6\ q2 + 16 - 3\ q1 - 3\ q3$$

$$-6\ q3 + 16 - 3\ q1 - 3\ q2$$

```
> solve({-6*q1+16-3*q2-3*q3,-6*q2+16-3*q1-3*q3,-6*q3+16-3
  *q1-3*q2},{q1,q2,q3});
```

$$\{q3 = \frac{4}{3},\ q1 = \frac{4}{3},\ q2 = \frac{4}{3}\}$$

```
>
```

(c)

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': pi1:='pi1':
  pi2:='pi2':
> Q:=q1+q2;
```

$$Q := q1 + q2$$

```
> p:=20-3*Q;
```

$$p := 20 - 3\ q1 - 3\ q2$$

```
> pi1:=p*q1-4*q1^2;
  pi2:=p*q2-4*q2^2;
```

$$\pi 1 := (20 - 3\ q1 - 3\ q2)\ q1 - 4\ q1^2$$

$$\pi 2 := (20 - 3\ q1 - 3\ q2)\ q2 - 4\ q2^2$$

```
> diff(pi1,q1);
```

$$-14\ q1 + 20 - 3\ q2$$

```
> diff(pi2,q2);
```

$$-14\ q2 + 20 - 3\ q1$$

```
> solve({-14*q1+20-3*q2=0,-14*q2+20-3*q1},{q1,q2});
```

$$\{q1 = \frac{20}{17},\ q2 = \frac{20}{17}\}$$

```
>
```

(d)

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': q3:='q3':
  pi1:='pi1': pi2:='pi2': pi3:='pi3':
> Q:=q1+q2+q3;
```

$$Q := q1 + q2 + q3$$

```
> p:=20-3*Q;
```

$$p := 20 - 3\ q1 - 3\ q2 - 3\ q3$$

```
> pi1:=p*q1-4*q1^2;
  pi2:=p*q2-4*q2^2;
  pi3:=p*q3-4*q3^2;
```

$$\pi 1 := (20 - 3\ q1 - 3\ q2 - 3\ q3)\ q1 - 4\ q1^2$$

$$\pi 2 := (20 - 3\ q1 - 3\ q2 - 3\ q3)\ q2 - 4\ q2^2$$

$$\pi 3 := (20 - 3\,q1 - 3\,q2 - 3\,q3)\,q3 - 4\,q3^2$$

```
> diff(pi1,q1);
  diff(pi2,q2);
  diff(pi3,q3);
```

$$-14\,q1 + 20 - 3\,q2 - 3\,q3$$

$$-14\,q2 + 20 - 3\,q1 - 3\,q3$$

$$-14\,q3 + 20 - 3\,q1 - 3\,q2$$

```
> solve({-14*q1+20-3*q2-3*q3=0,-14*q2+20-3*q1-3*q3=0,-14*
  q3+20-3*q1-3*q2=0},{q1,q2,q3});
```

$$\{q1 = 1, q2 = 1, q3 = 1\}$$

```
>
```

(ii)

Here we are comparing models (a) and (b).

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': pi1:='pi1':
  pi2:='pi2':
> solve(-6*q1+16-3*q2,q1);
```

$$\frac{8}{3} - \frac{1}{2}\,q2$$

```
> solve(-6*q2+16-3*q1,q2);
```

$$\frac{8}{3} - \frac{1}{2}\,q1$$

The two reaction functions are therefore:

$$\mathrm{q1}(t) = \frac{8}{3} - \frac{1\,\mathrm{q2}(t-1)}{2}$$

$$\mathrm{q2}(t) = \frac{8}{3} - \frac{1\,\mathrm{q1}(t-1)}{2}$$

```
> rsolve({q1(t) =
  8/3-1*q2(t-1)/2,q2(t)=8/3-1*q1(t-1)/2,q1(0)=8/3,q2(0)=0},{
  q1(t),q2(t)});
```

$$\left\{\mathrm{q2}(t) = -\frac{4}{9}\left(\frac{-1}{2}\right)^t + \frac{16}{9} - \frac{4}{3}\left(\frac{1}{2}\right)^t, \mathrm{q1}(t) = -\frac{4}{9}\left(\frac{-1}{2}\right)^t + \frac{16}{9} + \frac{4}{3}\left(\frac{1}{2}\right)^t\right\}$$

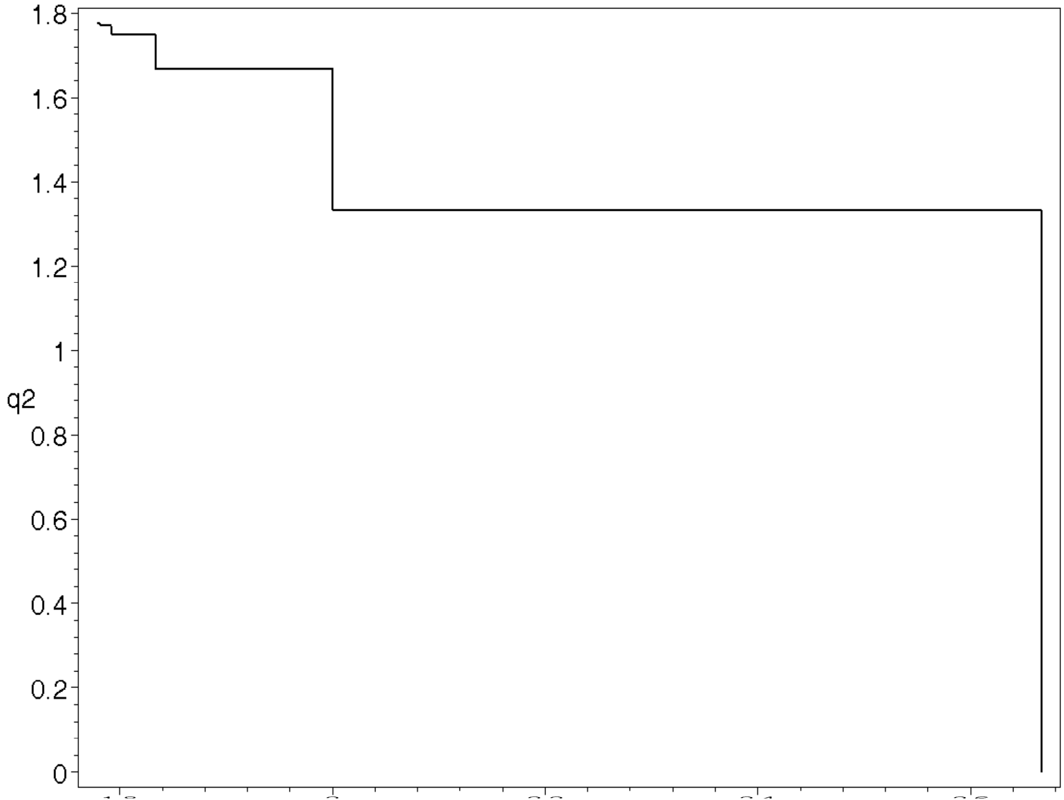
```
> q1:=t->-4/9*(-1/2)^t+16/9+4/3*(1/2)^t;
  q2:=t->-4/9*(-1/2)^t+16/9-4/3*(1/2)^t;
```

$$q1 := t \rightarrow -\frac{4}{9}\left(\frac{-1}{2}\right)^t + \frac{16}{9} + \frac{4}{3}\left(\frac{1}{2}\right)^t$$

$$q2 := t \rightarrow -\frac{4}{9}\left(\frac{-1}{2}\right)^t + \frac{16}{9} - \frac{4}{3}\left(\frac{1}{2}\right)^t$$

```
> points:=[seq([q1(t),q2(t)],t=0..10)]:
> pointplot(points,
```

```
axes=BOXED, connect=true, thickness=2,
labels=["q1","q2"],
colour=black);
```




```
> p:='p': Q:='Q': q1:='q1': q2:='q2': q3:='q3': pi1:='pi1':
  pi2:='pi2': pi3:='pi3':
> solve(-6*q1+16-3*q2-3*q3,q1);
```

$$\frac{8}{3}-\frac{1}{2}q2-\frac{1}{2}q3$$

```
> solve(-6*q2+16-3*q1-3*q3,q2);
```

$$\frac{8}{3}-\frac{1}{2}q1-\frac{1}{2}q3$$

```
> solve(-6*q3+16-3*q1-3*q2,q3);
```

$$\frac{8}{3}-\frac{1}{2}q1-\frac{1}{2}q2$$

The three reaction functions are therefore:

$$\mathrm{q1}(t)=\frac{8}{3}-\frac{1\ \mathrm{q2}(t)}{2}-\frac{1\ \mathrm{q3}(t)}{2}$$

$$\mathrm{q2}(t)=\frac{8}{3}-\frac{1\ \mathrm{q1}(t)}{2}-\frac{1\ \mathrm{q3}(t)}{2}$$

$$\mathrm{q3}(t)=\frac{8}{3}-\frac{1\ \mathrm{q1}(t)}{2}-\frac{1\ \mathrm{q2}(t)}{2}$$

If firm 1 is the monopolist then the initial point is (8/3,0,0).

```
> rsolve({q1(t) = 8/3-1*q2(t-1)/2-1*q3(t-1)/2,q2(t) =
```

```
8/3-1*q1(t-1)/2-1*q3(t-1)/2,q3(t) =
8/3-1*q1(t-1)/2-1*q2(t-1)/2,q1(0)=8/3,q2(0)=0,q3(0)=0},{q1
(t),q2(t),q3(t)});
```

$$\{\mathrm{q2}(t) = -\frac{4}{9}(-1)^t + \frac{4}{3} - \frac{8}{9}\left(\frac{1}{2}\right)^t, \mathrm{q1}(t) = -\frac{4}{9}(-1)^t + \frac{4}{3} + \frac{16}{9}\left(\frac{1}{2}\right)^t,$$

$$\mathrm{q3}(t) = -\frac{4}{9}(-1)^t + \frac{4}{3} - \frac{8}{9}\left(\frac{1}{2}\right)^t\}$$

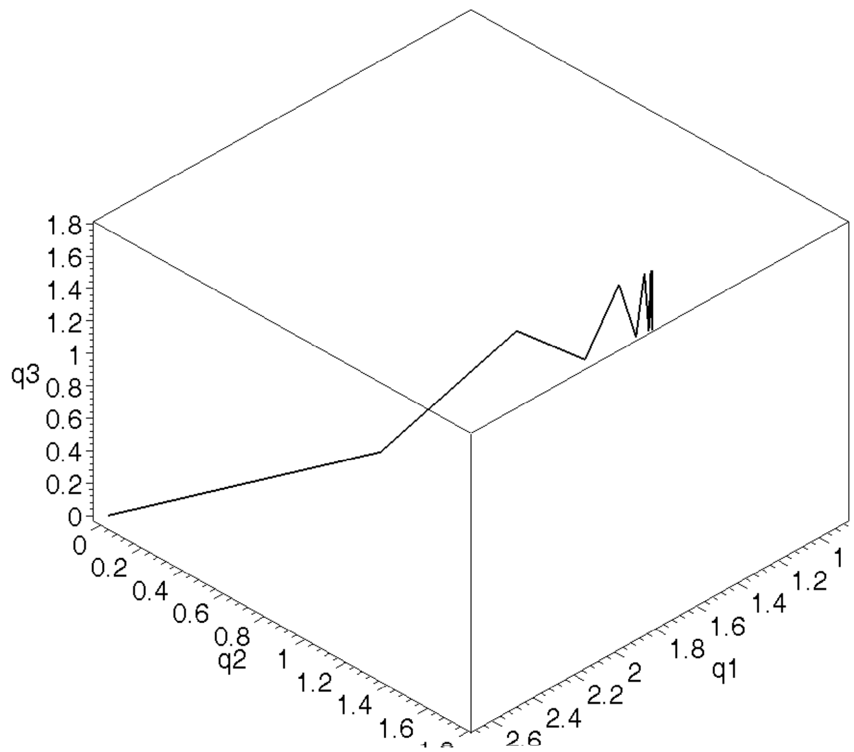
```
> q1:=t->-4/9*(-1)^t+4/3+16/9*(1/2)^t;
  q2:=t->-4/9*(-1)^t+4/3-8/9*(1/2)^t;
  q3:=t->-4/9*(-1)^t+4/3-8/9*(1/2)^t;
```

$$q1 := t \to -\frac{4}{9}(-1)^t + \frac{4}{3} + \frac{16}{9}\left(\frac{1}{2}\right)^t$$

$$q2 := t \to -\frac{4}{9}(-1)^t + \frac{4}{3} - \frac{8}{9}\left(\frac{1}{2}\right)^t$$

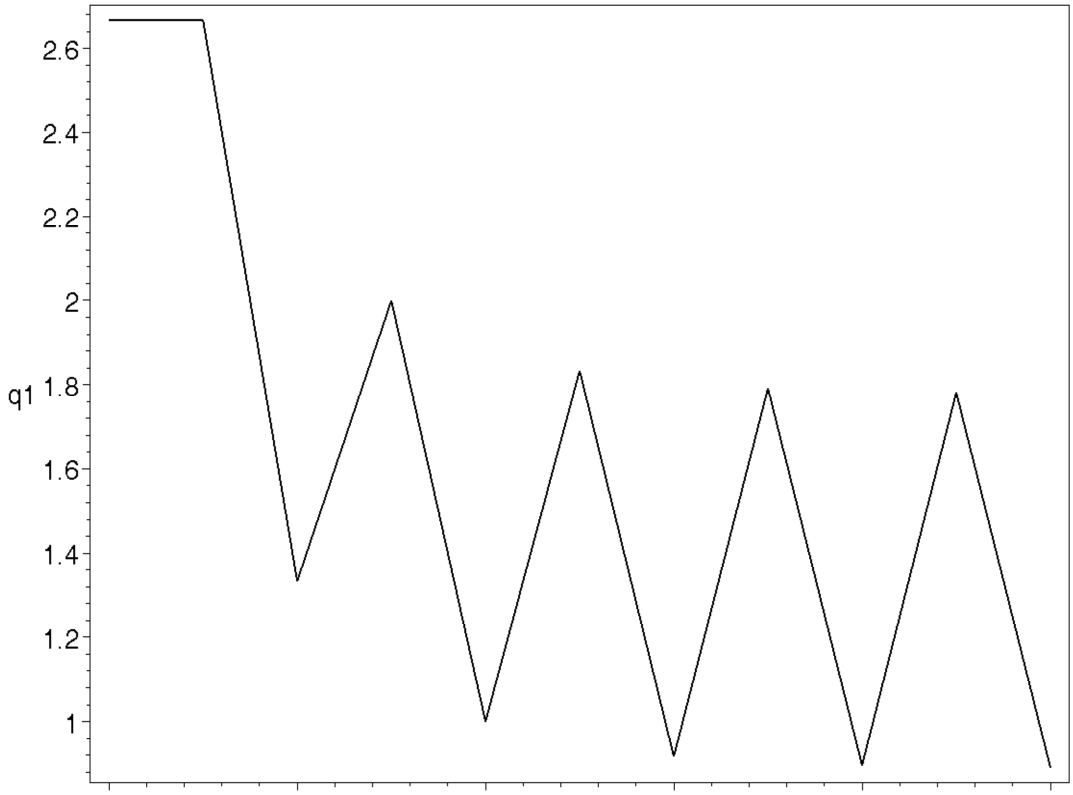
$$q3 := t \to -\frac{4}{9}(-1)^t + \frac{4}{3} - \frac{8}{9}\left(\frac{1}{2}\right)^t$$

```
> points:=[seq([q1(t),q2(t),q3(t)],t=0..10)]:
> pointplot3d(points,
       axes=BOXED, connect=true, thickness=2,
       labels=["q1","q2","q3"],
       colour=black);
```




```
> points1:=[seq([t,q1(t)],t=0..10)]:
> pointplot(points1,
```

```
axes=BOXED, connect=true, thickness=2,
labels=["t","q1"],
colour=black);
```



When $n = 2$ the system was stable. However, for $n = 3$ the system soon begins to oscilate. Here we show it only for q1 but it applies equally to q2 and q3, which is apparent from the presence of $(-1)^t$ in the solutions.

(iii)

Part (ii) already compares (a) and (b) and so here we consider just (c) and (d).

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': pi1:='pi1':
  pi2:='pi2':
> solve(-14*q1+20-3*q2,q1);
```

$$\frac{10}{7} - \frac{3}{14} q2$$

```
> solve(-14*q2+20-3*q1,q2);
```

$$\frac{10}{7} - \frac{3}{14} q1$$

Hence the reaction curves are:

$$q1(t) = \frac{10}{7} - \frac{3\, q2(t-1)}{14}$$

$$q2(t) = \frac{10}{7} - \frac{3\, q1(t-1)}{14}$$

If firm 1 is a monopoly then the intial point is (10/7,0).

```
> rsolve({q1(t) = 10/7-3*q2(t-1)/14,q2(t) =
```

```
10/7-3*q1(t-1)/14,q1(0)=10/7,q2(0)=0},{q1(t),q2(t)});
```

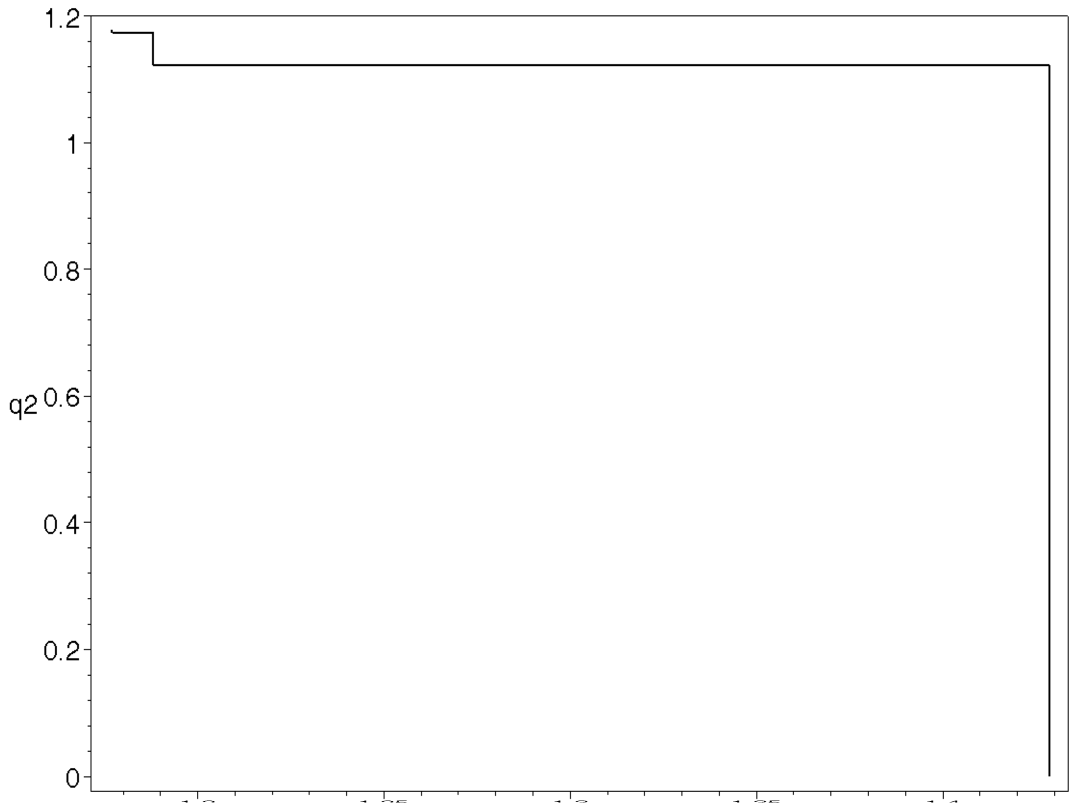
$$\{q1(t) = -\frac{55}{119}\left(\frac{-3}{14}\right)^t + \frac{20}{17} + \frac{5}{7}\left(\frac{3}{14}\right)^t, q2(t) = -\frac{55}{119}\left(\frac{-3}{14}\right)^t + \frac{20}{17} - \frac{5}{7}\left(\frac{3}{14}\right)^t\}$$

```
> q1:=t->-55/119*(-3/14)^t+20/17+5/7*(3/14)^t;
q2:=t->-55/119*(-3/14)^t+20/17-5/7*(3/14)^t;
```

$$q1 := t \to -\frac{55}{119}\left(\frac{-3}{14}\right)^t + \frac{20}{17} + \frac{5}{7}\left(\frac{3}{14}\right)^t$$

$$q2 := t \to -\frac{55}{119}\left(\frac{-3}{14}\right)^t + \frac{20}{17} - \frac{5}{7}\left(\frac{3}{14}\right)^t$$

```
> points:=[seq([q1(t),q2(t)],t=0..10)]:
> pointplot(points,
      axes=BOXED, connect=true, thickness=2,
      labels=["q1","q2"],
      colour=black);
```




```
> p:='p': Q:='Q': q1:='q1': q2:='q2': q3:='q3': pi1:='pi1':
pi2:='pi2': pi3:='pi3':
> solve(-14*q1+20-3*q2-3*q3,q1);
```

$$\frac{10}{7} - \frac{3}{14}q2 - \frac{3}{14}q3$$

```
> solve(-14*q2+20-3*q1-3*q3,q2);
```

$$\frac{10}{7} - \frac{3}{14}q1 - \frac{3}{14}q3$$

```
> solve(-14*q3+20-3*q1-3*q2,q3);
```

$$\frac{10}{7} - \frac{3}{14} q1 - \frac{3}{14} q2$$

Hence the reaction curves are:

$$q1(t) = \frac{10}{7} - \frac{3\ q2(t-1)}{14} - \frac{3\ q3(t-1)}{14}$$

$$q2(t) = \frac{10}{7} - \frac{3\ q1(t-1)}{14} - \frac{3\ q3(t-1)}{14}$$

$$q3(t) = \frac{10}{7} - \frac{3\ q1(t-1)}{14} - \frac{3\ q2(t-1)}{14}$$

With firm 1 the monopolist, then the initial point is (10/7,0,0).

```
> rsolve({q1(t) = 10/7-3*q2(t-1)/14-3*q3(t-1)/14,q2(t) =
  10/7-3*q1(t-1)/14-3*q3(t-1)/14,q3(t) =
  10/7-3*q1(t-1)/14-3*q2(t-1)/14,q1(0)=10/7,q2(0)=0,q3(0)=0}
  ,{q1(t),q2(t),q3(t)});
```

$$\left\{ q3(t) = -\frac{11}{21}\left(\frac{-3}{7}\right)^t + 1 - \frac{10}{21}\left(\frac{3}{14}\right)^t,\ q1(t) = -\frac{11}{21}\left(\frac{-3}{7}\right)^t + 1 + \frac{20}{21}\left(\frac{3}{14}\right)^t,\right.$$
$$\left. q2(t) = -\frac{11}{21}\left(\frac{-3}{7}\right)^t + 1 - \frac{10}{21}\left(\frac{3}{14}\right)^t \right\}$$

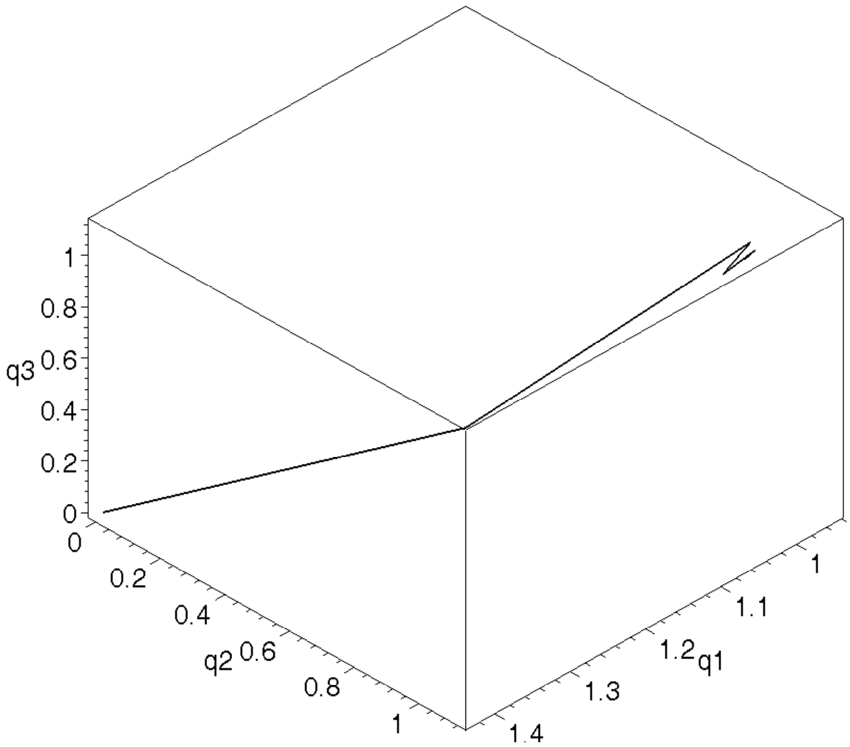
```
> q1:=t->-11/21*(-3/7)^t+1+20/21*(3/14)^t;
  q2:=t->-11/21*(-3/7)^t+1-10/21*(3/14)^t;
  q3:=t->-11/21*(-3/7)^t+1-10/21*(3/14)^t;
```

$$q1 := t \to -\frac{11}{21}\left(\frac{-3}{7}\right)^t + 1 + \frac{20}{21}\left(\frac{3}{14}\right)^t$$

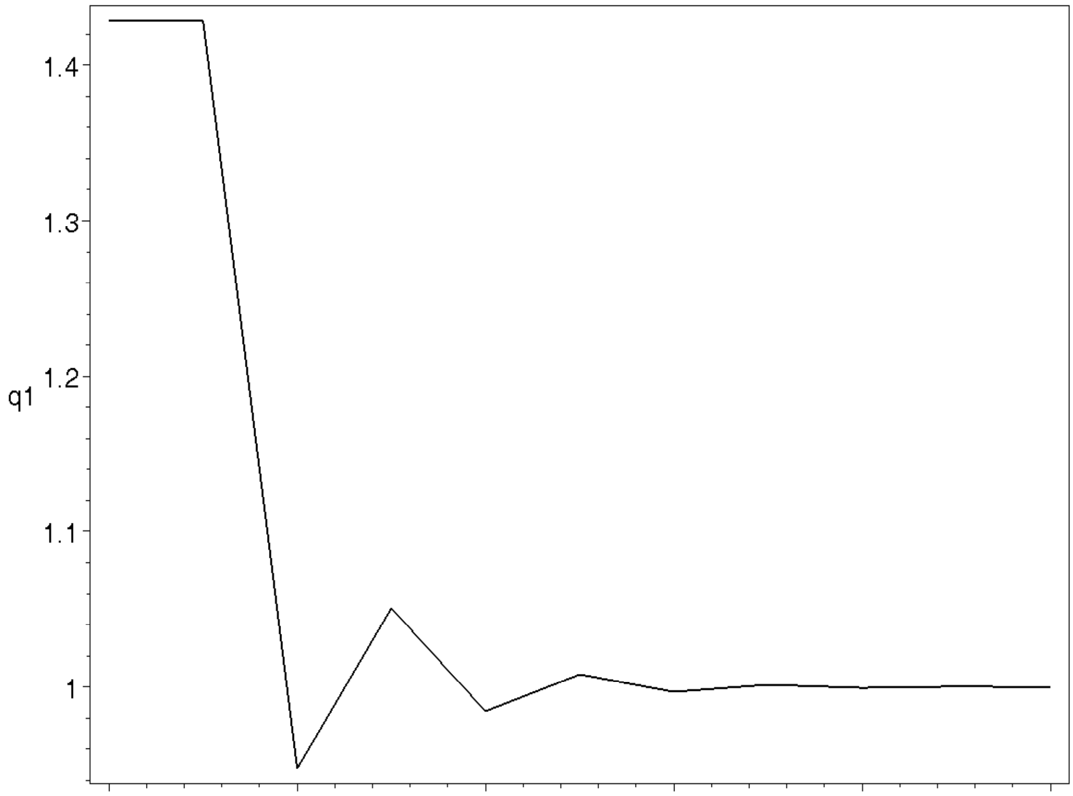
$$q2 := t \to -\frac{11}{21}\left(\frac{-3}{7}\right)^t + 1 - \frac{10}{21}\left(\frac{3}{14}\right)^t$$

$$q3 := t \to -\frac{11}{21}\left(\frac{-3}{7}\right)^t + 1 - \frac{10}{21}\left(\frac{3}{14}\right)^t$$

```
> points:=[seq([q1(t),q2(t),q3(t)],t=0..10)]:
> pointplot3d(points,
       axes=BOXED, connect=true, thickness=2,
       labels=["q1","q2","q3"],
       colour=black);
```




```
> points1:=[seq([t,q1(t)],t=0..10)]:
> pointplot(points1,
        axes=BOXED, connect=true, thickness=2,
        labels=["t","q1"],
        colour=black);
```



When marginal costs are not constant then the system is more stable and equilibrium is reached sooner. The oscillatory pattern observed for $n = 3$ under constant marginal costs now disappears and the system is stable.

>

Question 9

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': pi1:='pi1': pi2:='pi2':
```

(i)

The Cournot solution is the same as for model 8(i)(a), namely (16/9,16/9).

(ii)

In question 8(ii) we derived the reaction functions, which in their continuous dynamic form are:

$$x1(t) = \frac{8}{3} - \frac{1\ q2(t)}{2}$$

$$x2(t) = \frac{8}{3} - \frac{1\ q1(t)}{2}$$

Hence

$$\frac{\partial}{\partial t} q1(t) = .2\left(\frac{8}{3} - \frac{1\ q2(t)}{2} - q1(t)\right)$$

$$\frac{\partial}{\partial t} q2(t) = .2\left(\frac{8}{3} - \frac{1\ q1(t)}{2} - q2(t)\right)$$

```
> .2*(8/3-1*q2(t)/2-q1(t));
```

$$.5333333333 - .1000000000\ q2(t) - .2\ q1(t)$$

```
> .2*(8/3-1*q1(t)/2-q2(t));
```

$$.5333333333 - .1000000000\ q1(t) - .2\ q2(t)$$

The matrix of the system is therefore

```
> mA:=matrix([[-0.2,-0.1],[-0.1,-0.2]]);
```

$$mA := \begin{bmatrix} -.2 & -.1 \\ -.1 & -.2 \end{bmatrix}$$

```
> trace(mA);
```

$$-.4$$

```
> det(mA);
```

$$.03$$

```
> trace(mA)^2-4*det(mA);
```

$$.04$$

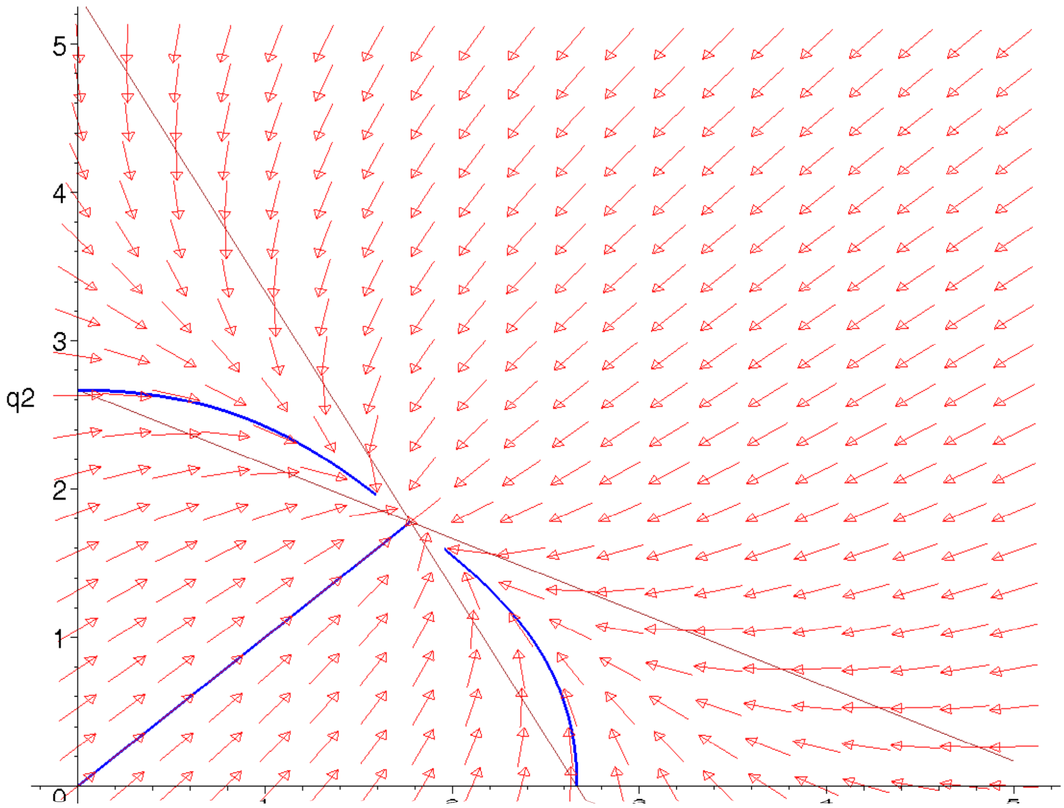
Hence the equilibrium is dynamically stable.

(iii)

The initial points are (8/3,0) for firm 1 a monopolist and (0,8/3) for firm 2 being a monopolist and (0,0) indicating both firms are to enter the industry.

```
> curves:=DEplot({diff(q1(t),t) =
```

```
.2*(8/3-1*q2(t)/2-q1(t)),diff(q2(t),t) =
.2*(8/3-1*q1(t)/2-q2(t))},[q1(t),q2(t)],t=0..20,q1=0..5,q2
=0..5,[[q1(0)=8/3,q2(0)=0],[q1(0)=0,q2(0)=8/3],[q1(0)=0,q2
(0)=0]],stepsize=.2,arrows=medium, linecolour=blue):
> lines:=plot({16/3-2*q1,8/3-(1/2)*q1},q1=0..5,q2=0..5,colou
r=brown,thickness=1):
> display(curves,lines);
```



The diagram reveals the stability of the Cournot solution.

Question 10

```
> p:='p': Q:='Q': q1:='q1': q2:='q2': pi1:='pi1': pi2:='pi2':
```

(i)

The Cournot solution is the same as for model 8(i)(c), namely (20/17,20/17).

(ii)

In question 8(iii) we derived the reaction functions, which in their continuous dynamic form are:

$$x1(t) = \frac{10}{7} - \frac{3\, q2(t)}{14}$$

$$x2(t) = \frac{10}{7} - \frac{3\, q1(t)}{14}$$

Hence

$$\frac{\partial}{\partial t} q1(t) = .2 \left(\frac{10}{7} - \frac{3\, q2(t)}{14} - q1(t) \right)$$

$$\frac{\partial}{\partial t} q2(t) = .2 \left(\frac{10}{7} - \frac{3\, q1(t)}{14} - q2(t) \right)$$

```
> .2*(10/7-3*q2(t)/14-q1(t));
```

$$.2857142857 - .04285714286\, q2(t) - .2\, q1(t)$$

```
> .2*(10/7-3*q1(t)/14-q2(t));
```

$$.2857142857 - .04285714286\, q1(t) - .2\, q2(t)$$

The matrix of the system is therefore

```
> mA:=matrix([[-0.2,-0.0429],[-0.0429,-0.2]]);
```

$$mA := \begin{bmatrix} -.2 & -.0429 \\ -.0429 & -.2 \end{bmatrix}$$

```
> trace(mA);
```

$$-.4$$

```
> det(mA);
```

$$.03815959$$

```
> trace(mA)^2-4*det(mA);
```

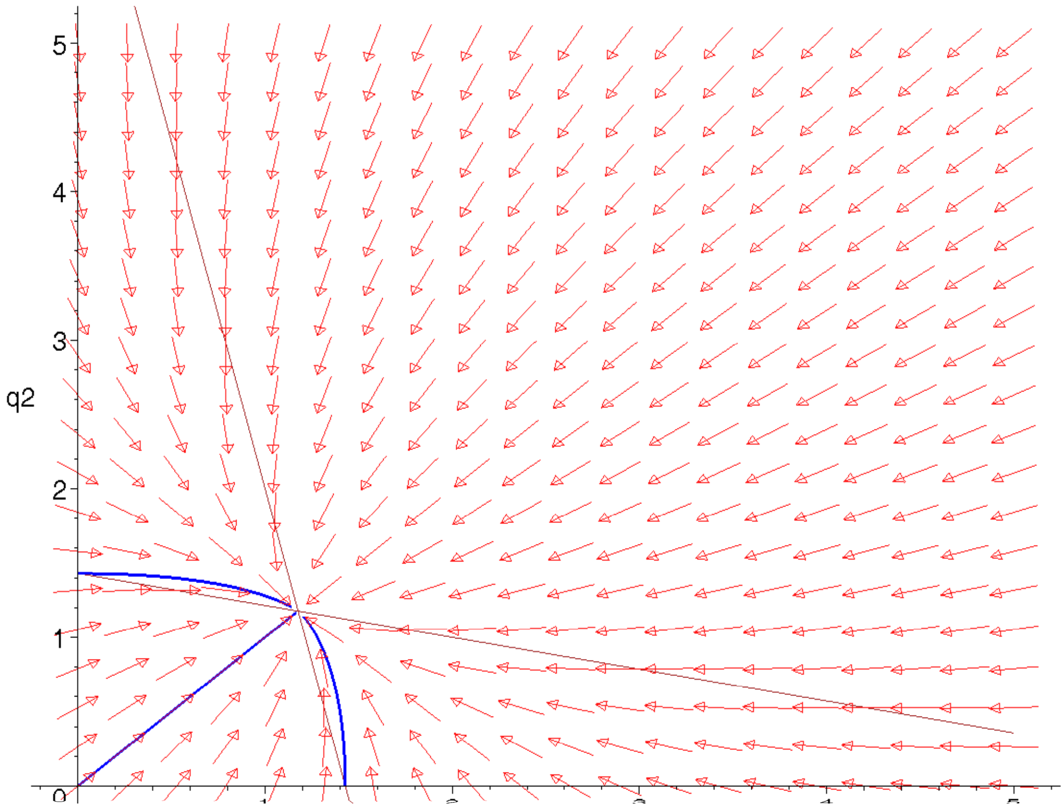
$$.00736164$$

Hence the equilibrium is dynamically stable.

(iii)

The initial points are (10/7,0) for firm 1 a monopolist and (0,10/7) for firm 2 being a monopolist and (0,0) indicating both firms are to enter the industry.

```
> curves:=DEplot({diff(q1(t),t) =
  .2*(10/7-3*q2(t)/14-q1(t)),diff(q2(t),t) =
  .2*(10/7-3*q1(t)/14-q2(t))},[q1(t),q2(t)],t=0..20,q1=0..5,
  q2=0..5,[[q1(0)=10/7,q2(0)=0],[q1(0)=0,q2(0)=10/7],[q1(0)=
  0,q2(0)=0]],stepsize=.2,arrows=medium, linecolour=blue):
> lines:=plot({20/3-14*q1/3,10/7-(3/14)*q1},q1=0..5,q2=0..5,
  colour=brown,thickness=1):
> display(curves,lines);
```



The diagram reveals the stability of the Cournot solution for the chosen initial points.

```
> evalf(dsolve({diff(q1(t),t) =
  .2*(10/7-3*q2(t)/14-q1(t)),diff(q2(t),t) =
  .2*(10/7-3*q1(t)/14-q2(t)),q1(0)=q10,q2(0)=q20},{q1(t),q2(
  t)}));
```

$$\{ \mathrm{q1}(t) = 1.176470588 + (.5000000000\, q10 - .5000000000\, q20)\, \mathbf{e}^{(-.1571428571\, t)} + (-1.176470588 + .5000000000\, q10 + .5000000000\, q20)\, \mathbf{e}^{(-.2428571429\, t)},\ \mathrm{q2}(t) = -1.\,(.5000000000\, q10 - .5000000000\, q20)\, \mathbf{e}^{(-.1571428571\, t)} + (-1.176470588 + .5000000000\, q10 + .5000000000\, q20)\, \mathbf{e}^{(-.2428571429\, t)} + 1.176470588 \}$$

Since all coefficient of q10 and q20 involve either $\mathbf{e}^{(-.1571428571\, t)}$ or $\mathbf{e}^{(-.2428571429\, t)}$ then the system converges on the Cournot equilibrium regardless of the initial values.

Questions 11-15

These questions are more easily done with a spreadsheet and are therefore not undertaken here within *Maple*.