February 2017

Miller and Blair, *Input-Output Analysis: Foundations and Extensions, 2nd Edition*
Errata and Revisions

Page	Change
29	Table 2.7, row 7, column 7. The figure should be .0528, not .0228.
63	Problem 2.3. Replace the entire "Close the model..." sentence with: Close the model with respect to households and find the impacts on sectors 1 and 2 of the new final demands in part b of Problem 2.1, using the Leontief inverse for the new 3×3 coefficient matrix. Assume that these changes occur in the export part of final demand.
64	Problem 2.4. Part d should be the "Compute the…" part of the question. The part c question is "What would be the total…"
107	The right-hand side of Equation (A3.1.1) should be f_i^s, not f_j^s.
150	Table 4.17, row C, column B. The figure should be 4, not 3.
187	Table 5.3, row for Commodity 1, column for Industry 2. The figure should be 8, not 6. In the Value Added row, column for Industry 1, the figure should be 68, not 60.
232	Add the following text after Equations (A5.2.5): In the iterative steps in A5.2.5) we are assured that total commodity outputs for interindustry use (row sums of **U**) are preserved because row sums of $(\mathbf{I}-\mathbf{D}')$ are zero—$(\mathbf{I}-\mathbf{D}')\mathbf{i}=\mathbf{i}-\mathbf{i}=\mathbf{0}$ (since column sums of **D** are 1). So with each step $\mathbf{Z}^{(k+1)}\mathbf{i}=\mathbf{U}\mathbf{i}$.
	After the three unnumbered equations near the bottom of the page, add the following (before "To add further specificity..."): Notice that these equations are a 3×3 illustration of the relationships in the *transposed* version of (A5.2.8), namely $$({}_i\mathbf{Z}^{(k+1)})'=({}_i\mathbf{U})'+[{}_i\mathbf{Z}^{(k)}(\mathbf{I}-\mathbf{D}')]'=({}_i\mathbf{U})'+(\mathbf{I}-\mathbf{D})({}_i\mathbf{Z}^{(k)})'$$
233	The next-to-final paragraph should open with "Finally, we have (.3478) $z_{13}^{(k)}$" not (34.78) $z_{13}^{(k)}$.

234 Delete footnote 41. Insert four pages of text before the start of Section A5.2.3. This new text is at the end of these Errata pages titled "**Additions to the Almon Procedure**".

322 Eq. (7.32). Element (3, 1) in the matrix should be .0529, not .0592.

343 In the last line of Problem 7.8, the term $\tilde{\mathbf{Z}}(1)$ should be boldface.

390 Problem 8.5. Element (3, 1) of the national coefficient matrix should be .2084, not .1603.

391 Problem 8.6. The "CLQ" that appears twice should be "CIQ".

405 Row 9 from the bottom. The last equation should be $\boldsymbol{\alpha}(\mathbf{I}-\mathbf{A})=\mathbf{G}\hat{\mathbf{x}}^{-1}$. The hat is missing on the **x** in the printed equation.

430 Row 12 from the bottom. The **f** vector should be $\mathbf{f}=\begin{bmatrix}0\\30\\100\end{bmatrix}$.

507 Last sentence of the first paragraph should read: "The darkly shaded portion ...the Use matrix...and the lightly shaded portion...the Make matrix." In the printed version "darkly" and "lightly" are reversed.

516 Eq. (11.4). Insert equals sign between the **Q** and the left bracket.

541 Reinert and Roland-Holst 1992 page numbers should be 173-187.

554 Last line of text. Reference should be to Table 2.15, not 2.13.

566 Last two columns in Table 12.8, $\bar{T}_j$ and $\bar{\bar{T}}_j$, need new numbers. They are shown below.

Revised Table 12.8

Sector	$\bar{B}(t)_j$	$\bar{F}(t)_j$	$\bar{T}_j$	$\bar{\bar{T}}_j$
1	1.02	1.61	2.12	0.73
2	0.69	1.27	1.84	0.61
3	3.87	1.06	9.23	3.85
4	13.59	11.20	28.28	8.57
5	6.47	8.42	19.77	5.29
6	19.95	24.44	52.77	6.45
7	6.64	2.98	17.74	6.27

	$\tilde{B}(t)_j$	$\tilde{F}(t)_j$	$\tilde{T}_j$	$\tilde{\tilde{T}}_j$
1	0.14	0.22	0.11	0.16
2	0.09	0.17	0.10	0.13
3	0.52	0.15	0.49	0.85
4	1.82	1.54	1.50	1.89
5	0.87	1.16	1.05	1.17
6	2.67	3.36	2.80	1.42
7	0.89	0.41	0.94	1.38

625 Line 2 in Section 13.2.2. Should read "(section 6.6.2)", not 6.5.3. Also, fn. 30, last line should reference section 12.2.6, not 12.2.5.

670-674 (Section 14.2) Replace with new text at the end of these Errata pages: five pages titled "**14.2 Input-Output and Measuring Economic Productivity Growth"** and one page titled "**Appendix 14.1 More on the Derivation of TFP Growth Measures**".

734 Publisher for Dietzenbacher/Lahr is Cambridge University Press, not Palgrave.

744 Add ", 398" at the end of Szyrmer.

106, 116, 383n34, 384, 393, 740 References to Fontana should be Fontela.

Additions to the Almon Procedure (insert for page 234)

This illustrates the two issues to be addressed in converting a use table from a commodity-by-industry format to a commodity-by-commodity format. For each u_{ij} we need to:

(1) Remove from each u_{ij} the sales of commodity *i* to industry *j* that were used as inputs in production of *j*'s non-primary (secondary) products. For example, for u_{13} this means transformations across the $z_{13}^{(k+1)}$ row, except for the $z_{13}^{(k)}$ term, in (A5.2.11).

(2) Add to each u_{ij} the sales of commodity *i* to producers other than *j* who made *j* as a secondary product. For u_{13} this means transformations to the first and second terms in the $z_{13}^{(k)}$ column in (A5.2.11), and this modification is captured in the third term of this column.

The iterative routine for row-by-row creation of $\mathbf{Z}_C$ allows us to see when and how negatives first emerge, and at that time corrective interventions, like the Almon "purification" procedure, can begin. (This corrective procedure requires that **U** and **V** be square matrices.) It is instructive to look in detail the first step in the (A5.2.5) sequence—$\mathbf{Z}^{(1)} = \mathbf{U} + \mathbf{U}(\mathbf{I} - \mathbf{D}')$.

For this 3×3 example we have

$$\mathbf{U}(\mathbf{I}-\mathbf{D}') = \begin{bmatrix} 4 & 2 & 4 \\ 2 & 5 & 2 \\ 6 & 1 & 3 \end{bmatrix} \begin{bmatrix} .2857 & -.1786 & -.1071 \\ -.0690 & .1379 & -.0690 \\ -.0435 & -.3043 & .3478 \end{bmatrix} = \begin{bmatrix} .8308 & -1.6558 & .8248 \\ .1394 & -.2763 & .1364 \\ 1.5147 & -1.8466 & .3318 \end{bmatrix}$$

This leads to $\mathbf{Z}^{(1)} = \begin{bmatrix} 4.8308 & .3442 & 4.8248 \\ 2.1394 & 4.7237 & 2.1364 \\ 7.5147 & -.8466 & 3.3318 \end{bmatrix}$, and already on this first step a negative flow appears—$z_{32}^{(1)} = -.8466.$ In a small example like this, we could easily see the trouble coming, since element (3,2) in $\mathbf{U}(\mathbf{I}-\mathbf{D}')$ shows that 1.8466 units will be taken from u_{32} which has only 1 unit to give up. But in larger real-world applications this kind of visual approach is not possible. The Almon procedure is a way of identifying approaching negatives and dealing with them as they arise. It examines each iteration one row at a time.[42]

For illustration in this example we look in detail at the third row of $\mathbf{Z}^{(1)}$, as in (A5.2.10):

$${}_3\mathbf{Z}^{(1)} = {}_3\mathbf{U} + {}_3\mathbf{U}(\mathbf{I}-\mathbf{D}') = \begin{bmatrix} 6 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 1 & 3 \end{bmatrix} \begin{bmatrix} .2857 & -.1786 & -.1071 \\ -.0690 & .1379 & -.0690 \\ -.0435 & -.3043 & .3478 \end{bmatrix} =$$

$$\begin{bmatrix} 6 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1.5147 & -1.8466 & .3318 \end{bmatrix} = \begin{bmatrix} 7.5147 & -.8466 & 3.3318 \end{bmatrix}$$

Alternatively, when written out in detail, the individual calculations that are aggregated in the matrix multiplication operation are made clear. These are simply the first step for the equations in (A5.2.10) for $i=3$:

$$\begin{aligned}
z_{31}^{(1)} &= 6+6(.2857)+1(-.0690)+3(-.0435)=6+1.7142-.0690-.1305=7.5147 \\
z_{32}^{(1)} &= 1+6(-.1786)+1(.1379)+3(-.3043)=1-1.0716+.1379-.9129=-.8466 \\
z_{33}^{(1)} &= 3+6(-.1071)+1(-.0690)+3(.3478)=3-.6426-.0690+1.0434=3.3318
\end{aligned} \quad \text{(A5.2.12)}$$

Note that the elements in *column j* of $(\mathbf{I}-\mathbf{D}')$ show up as the elements in *row j* of these equations, because of the transposition as in (A5.2.8), and also that $z_{32}^{(1)}$ has become negative after this first step.

Viewed this way, the problem that generates a negative $z_{32}^{(1)}$ is immediately apparent. Continuing with the 1 = cheese, 2 = ice cream, 3 = other foodstuffs story, we see that other foodstuffs (commodity 3) shipped 1 unit to the ice cream *industry* ($u_{32}=1$). But the ice cream industry also made secondary products—cheese ($v_{12}=2$) and other foodstuffs ($v_{32}=2$)—and we need to remove the other foodstuffs inputs to both of these products that are non-primary to producing the *commodity* ice cream. In this case the removal suggests trouble, because we need to take away $6\times.1786=1.0716$ units to account for cheese production and $3\times.3043=.9129$ units to account for foodstuffs production—a total of 1.9845 units to be subtracted. In this case, the original allocation is 1 unit, and an additional $1\times.1379$ units are being added at this point because of the foodstuffs used in making ice cream in the cheese and foodstuffs *industries*. This means 1.9845 units are to be subtracted from 1.1379 units, leaving us with the deficit of .8466. A small algebraic rearrangement helps with visualization of the problem.

A rewrite the basic iterative equation, replacing ${}_i\mathbf{U}$ with $\mathbf{i}'({}_i\hat{\mathbf{U}})$, so ${}_i\mathbf{Z}^{(1)}={}_i\mathbf{U}+\mathbf{i}'({}_i\hat{\mathbf{U}})(\mathbf{I}-\mathbf{D}')$, generates useful results. For row 3 we now have,

$${}_3\mathbf{Z}^{(1)}={}_3\mathbf{U}+\mathbf{i}'({}_3\hat{\mathbf{U}})(\mathbf{I}-\mathbf{D}')=\begin{bmatrix}6 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}6 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 3\end{bmatrix}\begin{bmatrix}.2857 & -.1786 & -.1071\\ -.0690 & .1379 & -.0690\\ -.0435 & -.3043 & .3478\end{bmatrix}$$

and, in particular,

$$({}_3\hat{\mathbf{U}})(\mathbf{I}-\mathbf{D}')=\begin{bmatrix}1.7142 & -1.0716 & -.6426\\ -.0690 & .1379 & -.0690\\ -.1305 & -.9129 & 1.0434\end{bmatrix} \quad \text{(A5.2.13)}$$

This disaggregates the matrix product ${}_3\mathbf{U}(\mathbf{I}-\mathbf{D}')$ in such a way that the elements in column *j* of $({}_3\hat{\mathbf{U}})(\mathbf{I}-\mathbf{D}')$ show the magnitudes of each of the changes that convert u_{3j} to its corresponding $z_{3j}^{(1)}$ at this step—exactly the elements in the equations (A5.2.12) and in the text in the preceding paragraph—and the *j*-th column sum gives the total adjustment for u_{3j}. Here, as we saw earlier, $\mathbf{i}'({}_3\hat{\mathbf{U}})(\mathbf{I}-\mathbf{D}') = \begin{bmatrix} 1.5147 & -1.8466 & .3318 \end{bmatrix}$, leading to ${}_3\mathbf{Z}^{(1)} = \begin{bmatrix} 7.5147 & -.8466 & 3.3318 \end{bmatrix}$. Column 2 contains the elements that produce $z_{32}^{(1)} < 0$.

To concentrate on the potential difficulty with the negative off-diagonal elements in $({}_3\hat{\mathbf{U}})(\mathbf{I}-\mathbf{D}')$, we create a variant of this matrix, replacing each on-diagonal element by the absolute value of the sum of the negative elements in that column; here for row 3 at iteration 1 this is

$${}_3\mathbf{M}^{(1)} = \begin{bmatrix} .1995 & -1.0716 & -.6426 \\ -.0690 & \mathbf{1.9845} & -.0690 \\ -.1305 & -.9129 & .7116 \end{bmatrix}$$

The Almon procedure then compares each on-diagonal element ${}_3m_{jj}^{(1)}$ with its corresponding u_{3j}. When $u_{3j} < {}_3m_{jj}^{(1)}$, the recommendation is to scale back the elements in column *j*. This is the case here with $u_{32} = 1$ and ${}_3m_{22}^{(1)} = 1.9845$ (shown in bold), and as we saw, this results in $z_{32}^{(1)} < 0$. So, from ${}_3\mathbf{M}^{(1)}$ an "adjustment" matrix, ${}_3\tilde{\mathbf{M}}^{(1)}$, is created:

(1) In those columns *j* where $u_{3j} < {}_3m_{jj}^{(1)}$ the current elements are multiplied by a *scaling* factor $s_j^{(1)} = u_{3j} / m_{jj}^{(1)}$. In this example where $u_{32} < {}_3m_{22}^{(1)}$, we have $s_2^{(1)} = (1/1.9845) = .5039$, and each element in column 2 will be reduced to 50.39 percent of its current value. The total amount removed from $u_{32} = 1$ will be precisely 1 unit, and

(2) In those columns *k* where $u_{3k} \geq {}_3m_{kk}^{(1)}$ there is no problem, so for those elements is $s_k^{(1)} = 1$ (no adjustment is needed).

Notice that the scaling in (1) does not take into account the fact that there is also an addition to $z_{3j}^{(1)}$ along with the decreases. This appears as element (2, 2) in (A5.2.13), namely .1379.

Put compactly, ${}_3\tilde{\mathbf{M}}^{(1)} = {}_3\mathbf{M}^{(1)}\hat{\mathbf{s}}^{(1)}$ where $\mathbf{s} = \begin{bmatrix} s_1^{(1)} \\ s_2^{(1)} \\ s_3^{(1)} \end{bmatrix}$. In this case we have

$\mathbf{s}^{(1)} = \begin{bmatrix} 1 \\ .5039 \\ 1 \end{bmatrix}$ and ${}_3\tilde{\mathbf{M}}^{(1)} = \begin{bmatrix} .1995 & -.5400 & -.6426 \\ -.0690 & 1.0000 & -.0690 \\ -.1305 & -.4600 & .7116 \end{bmatrix}$. This matrix is subtracted from ${}_3\hat{\mathbf{U}}(\mathbf{I}-\mathbf{D}')$ to give a new adjustment matrix

$${}_3\Delta^{(1)} = {}_3\hat{\mathbf{U}}(\mathbf{I}-\mathbf{D}') - {}_3\tilde{\mathbf{M}}^{(1)} = \begin{bmatrix} 1.5147 & -.5316 & 0 \\ 0 & -.8621 & 0 \\ 0 & -.4529 & .3318 \end{bmatrix}$$

Elements in row *j* of ${}_3\Delta^{(1)}$ represent adjustments to u_{3j} that come about from reallocation of shipments of *j* to production of each of the commodities 1, 2 and 3, respectively. So adding across row *j* gives the total adjustment for u_{3j}. In this case, our new and final estimate of ${}_3\mathbf{Z}^{(1)}$ is

$${}_3\mathbf{Z}^{(1)} = {}_3\mathbf{U} + ({}_3\Delta^{(1)}\mathbf{i})' = \begin{bmatrix} 6 & 1 & 3 \end{bmatrix} + \begin{bmatrix} .9831 & -.8621 & -.1211 \end{bmatrix} = \begin{bmatrix} 6.9831 & .1379 & 2.8789 \end{bmatrix}$$

Notice that indeed we now have $z_{32}^{(1)} = .1379$; the original $u_{32} = 1$ has been wiped out because of the ice cream *industry's* non-primary production of cheese and other foodstuffs but at the same time .1379 units of ice cream are added because the non-primary production of the *commodity* ice cream in the cheese and other foodstuffs industries.

The interested reader can derive results for ${}_1\mathbf{Z}^{(1)}$ and ${}_2\mathbf{Z}^{(1)}$ in the same way (no other negatives will be encountered at this point), and putting it all together gives

$$\mathbf{Z}^{(1)} = \begin{bmatrix} {}_1\mathbf{Z}^{(1)} \\ {}_2\mathbf{Z}^{(1)} \\ {}_3\mathbf{Z}^{(1)} \end{bmatrix} = \begin{bmatrix} 4.8310 & .3442 & 4.8248 \\ 2.1396 & 4.7238 & 2.1365 \\ 6.9831 & .1379 & 2.8789 \end{bmatrix}$$

These first-step estimates are shown in Table 5.2.2.[43]

[42] We thank Fred Pallada, former economist of the Central Planning Bureau, The Hague (Netherlands), for his initial inquiries in 2015 regarding our coverage of this material in Appendix 5.2, for his efficient programming of the technique, for our many email discussions over the ensuing months, and for alerting us to the Vollebregt and van Dalen paper.

[43] These results are based on a **D** matrix with elements shown with four-decimal accuracy. They may sometimes differ a bit from those in Table A5.2.2 that were generated through matrix multiplications carried out with and rounded down from basic data with more significant digits.

Reference

Vollebregt, Michel and Jan van Dalen. 2001. "Deriving Homogeneous Input/Output Tables from Supply and Use Tables," Paper presented at the Fourteenth International Conference on Input-Output Techniques, Montreal, October, 2002. (Available at https://www.iioa.org/conferences/14th/papers.html.)

14.2 Input-Output and Measuring Economic Productivity Growth[1]

A key source of growth and health in an economy is the rate of growth in its economic productivity, where productivity is broadly defined as the level of output of an industry (or of the economy as a whole) per unit of input. Exploring different methods of measuring this economic productivity and its growth has been an active area of analysis and research for decades (e.g., Jorgenson and Griliches, 1967). A number of productivity measures can be expressed in input-output terms, as in Peterson (1979), Baumol and Wolff (1984), Wolff (1985, 1994, 1997) and ten Raa (2005). In this section we explore one such formulation of the concept of *total factor productivity* (TFP), which is defined generally as the growth in total output that is not attributable to growth in inputs (intermediate inputs, labor and capital).[2]

14.2.1 Total Factor Productivity

The Physical Input-Output Model (Review). In Section 2.6.6 we examined the price model based on physical data, where s_{ij} represents an interindustry flow from sector *i* to sector *j* in *physical* units [e.g., kilowatt-hours of electricity (*i*) sold to steel production (*j*)], q_j is total steel production in tons, and p_i and p_j are product prices, all for a given time period (year). Technical coefficients based on physical values were denoted, $c_{ij} = s_{ij} / q_j$ (kilowatt-hours per ton). For simplicity in what follows we assume that there is only one value-added input, s_{vj} (e.g., person-hours used in steel production), with price p_v (per person-hour) and a corresponding physical value-added coefficient $c_{vj} = s_{vj} / q_j$. The fundamental accounting expression for total industry outputs, q_j, from the input side, requires that we introduce prices in order to be able to sum down columns of a physical transactions table (as in Table 2.17 in Chapter 2). Thus,

$$p_j q_j = \sum_{i=1}^{n} p_i s_{ij} + p_v s_{vj} = \sum_{i=1}^{n} p_i c_{ij} q_j + p_v c_{vj} q_j \tag{14.1}$$

and, dividing by q_j (assumed not zero)

$$p_j = \sum_{i=1}^{n} p_i c_{ij} + p_v c_{vj} \tag{14.2}$$

as in (2.49) in Chapter 2. In matrix terms, with $\mathbf{C} = [c_{ij}]$, $\mathbf{c}'_v = [c_{v1}, \cdots, c_{vn}]$ and $\mathbf{p}' = [p_1, \cdots, p_n]$, (14.2) is $\mathbf{p}' = \mathbf{p}'\mathbf{C} + p_v \mathbf{c}'_v$ [as in (2.50)]; the extension to several value-added categories is straightforward.[3]

[1] We thank Professor Sarafeim Polyzos of Thessaly University, Greece, for bringing to our attention problems with the material in this section of the book in 2015.

[2] A special issue of *Economic Systems Research* (2007) contains articles summarizing developments in this area up until that date.

[3] A more general derivation of a measure of total factor productivity is given in Appendix 14.1 for the interested reader; it requires a small amount of differential calculus.

A Measure of Total Factor Productivity Growth using Input-Output Data. Broadly speaking, the object is to examine inputs and output for each sector at two different points in time and to find the differences in the growth rates of those inputs and outputs,

> ...both evaluated at constant prices: this difference, or 'residual'...can...be thought of as showing the contribution of unidentified variables, such as economies of scale and advances in technical knowledge, to the process of economic growth. (Peterson, 1979, p. 212).

To this end, consider what we generally know about an economy at years 0 and 1 in an input-output context, namely the respective technologies represented by coefficient matrices $\mathbf{A}^0 = [a_{ij}^0]$ and $\mathbf{A}^1 = [a_{ij}^1]$. The values of intermediate inputs to sector *j* per dollar's worth of *j*'s output in a given year *t* are represented in $\mathbf{A}_j^t$, the *j*-th column of $\mathbf{A}^t$. If a sector produces the same amount of (physical) output in year 1 as it did in year 0, but with fewer (physical) inputs, or if it produces more output in year 1 than year 0 with the same amount of inputs, it can be viewed as having become more productive. But the coefficients in $\mathbf{A}^t$ matrices are derived from interindustry flows and gross outputs expressed in value terms that reflect prices specific to year *t*; $a_{ij}^t = z_{ij}^t / x_j^t = p_i^t s_{ij} / p_j^t q_j$. In assessing *real* productivity change from one period to another it is important to remove price effects (as noted by Peterson). If $z_{25}^0 =$ \$40 and $z_{25}^1 =$ \$80, we cannot conclude that sector 5 used twice as much physical input from sector 2 in year 1—for example, if p_2 doubled in that time period the physical amount of sector 2 input would have been the same.

One straightforward approach is to deflate year 1 prices back to year 0.[4] Multiplying numerator and denominator in $a_{ij}^1 = z_{ij}^1 / x_j^1$ by the (known) price ratios p_i^0 / p_i^1 and p_j^0 / p_j^1 generates a_{ij}^{1c}, a year-1 technical coefficient evaluated in constant (year-0) prices.

$$a_{ij}^{1c} = [(p_i^0 / p_i^1) z_{ij}^1] / [(p_j^0 / p_j^1) x_j^1] = (p_i^0 / p_i^1)(p_j^1 / p_j^0) a_{ij}^1 \tag{14.3}$$

It is clear that this procedure values the year-1 *physical* amounts at year-0 prices, since

$$a_{ij}^{1c} = [(p_i^0 / p_i^1) p_i^1 s_{ij}^1] / [(p_j^0 / p_j^1) p_j^1 q_j^1] = p_i^0 s_{ij}^1 / p_j^0 q_j^1 \tag{14.4}$$

Similarly, the value-added coefficient in year 1 is $v_j^1 = z_{vj}^1 / x_j^1 = p_v^1 s_{vj}^1 / p_j^1 q_j^1$, which leads to

$$v_j^{1c} = [(p_v^0 / p_v^1) / (p_j^0 / p_j^1)] v_j^1 = (p_v^0 / p_v^1)(p_j^1 / p_j^0) v_j^1 \tag{14.5}$$

In Section 2.6.6 of Chapter 2 we saw that the relationship between value coefficients, **A**, and physical coefficients, **C**, was easily expressed as $\mathbf{A} = \hat{\mathbf{p}} \mathbf{C} \hat{\mathbf{p}}^{-1}$ [this was (2.56)]. Similarly, the matrix representation of (14.3) and (14.4) is

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[4] There are many other possibilities, such as using some kind of average prices for the period over which productivity change is being measured (e.g., Wolff, 1997), and numerical results will depend on the particular set of prices chosen, a reflection of the general *index number* problem.

$$\mathbf{A}^{1c} = \hat{\mathbf{p}}^0\mathbf{C}^1(\hat{\mathbf{p}}^0)^{-1} = [\hat{\mathbf{p}}^0(\hat{\mathbf{p}}^1)^{-1}]\mathbf{A}^1[\hat{\mathbf{p}}^1(\hat{\mathbf{p}}^0)^{-1}]$$

In the **A** matrix, all input coefficients are on a "per dollar's worth of output" basis, and hence the *total* value of all inputs producing a dollar's worth of output must be \$1. However, if we remove the impact of inflation on the prices of all inputs in year 1, then a productivity increase in sector *j* between years 0 and 1 would be reflected in a total deflated value of year-1 inputs to sector *j* that was less than 1. For example (Aulin-Ahmavaara, 1999):

> The traditional direct measure of sectoral TFP growth...is equivalent to the relative *decrease* in the production price (unit production cost) of the output of sector *j* ...when all the input prices are treated as exogenous constants (p. 353, emphasis added).

One way to operationalize this is as follows: Define $\underset{[(n+1)\times n]}{\bar{\mathbf{A}}^0} = \begin{bmatrix} \mathbf{A}^0 \\ \cdots \\ \mathbf{v}^0 \end{bmatrix}$, where $\mathbf{v}^0 = [v_1^0, v_2^0, \cdots, v_n^0]$, and, similarly, $\bar{\mathbf{A}}^{1c} = \begin{bmatrix} \mathbf{A}^{1c} \\ \cdots \\ \mathbf{v}^{1c} \end{bmatrix}$. Then $\bar{\mathbf{A}}^{1c} - \bar{\mathbf{A}}^0$ represents the changes in technology over the period at constant (year-0) prices. If productivity of sector *j* has increased over that period, we expect $\mathbf{i}'\bar{\mathbf{A}}_j^{1c} = \sum_{i=1}^{n} a_{ij}^{1c} + v_j^{1c} < 1$; since $\mathbf{i}'\bar{\mathbf{A}}_j^0 = \sum_{i=1}^{n} a_{ij}^0 + v_j^0 = 1$, one measure of sector *j*'s total factor productivity growth is taken to be

$$\pi_j = -(\mathbf{i}'\bar{\mathbf{A}}_j^{1c} - \mathbf{i}'\bar{\mathbf{A}}_j^0) = 1 - (\sum_{i=1}^{n} a_{ij}^{1c} + v_j^{1c}) \qquad (14.5)$$

[Since $(\mathbf{i}'\bar{\mathbf{A}}_j^{1c} - \mathbf{i}'\bar{\mathbf{A}}^0)$ will be negative for sectors with increased productivity, the minus sign converts this measure to a positive number for sectors with productivity *increases*.] The row vector representing total factor productivity growth for each of the sectors in the economy is thus

$$\boldsymbol{\pi} = -\mathbf{i}'(\bar{\mathbf{A}}^{1c} - \bar{\mathbf{A}}^0) = \mathbf{i}' - \mathbf{i}'\bar{\mathbf{A}}^{1c} \qquad (14.6)$$

14.2.2 Numerical Example

Consider a three-sector economy with the following input-output information:

$$\mathbf{A}^0 = \begin{bmatrix} 0.1000 & 0.2500 & 0.2500 \\ 0.1500 & 0.0625 & 0.3000 \\ 0.3000 & 0.5000 & 0.0500 \end{bmatrix}, \ \mathbf{v}^0 = [0.4500 \quad 0.1875 \quad 0.4000], \ \mathbf{p}^0 = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \ p_v^0 = 5$$

$$\mathbf{A}^1 = \begin{bmatrix} 0.1071 & 0.1500 & 0.2917 \\ 0.2143 & 0.1100 & 0.2500 \\ 0.3214 & 0.5000 & 0.0667 \end{bmatrix}, \ \mathbf{v}^1 = [0.3571 \quad 0.2400 \quad 0.3917], \ \mathbf{p}^1 = \begin{bmatrix} 2.2 \\ 3 \\ 5.4 \end{bmatrix}, \ p_v^1 = 6$$

From these, we find

$$\bar{\mathbf{A}}^{1c} = \begin{bmatrix} 0.1071 & 0.1363 & 0.2864 \\ 0.2357 & 0.1101 & 0.2700 \\ 0.3274 & 0.4629 & 0.0667 \\ \hdashline 0.3274 & 0.2000 & 0.3525 \end{bmatrix}$$

so $\mathbf{i}'\bar{\mathbf{A}}^{1c} = [0.9976 \quad 0.9092 \quad 0.9755]$ and hence $\boldsymbol{\pi} = [0.0024 \quad 0.0908 \quad 0.0245]$. Alternatively, the reader can easily verify that

$$(\bar{\mathbf{A}}^{1c} - \bar{\mathbf{A}}^{0}) = \begin{bmatrix} 0.0071 & -0.1137 & 0.0364 \\ 0.0857 & 0.0476 & -0.0300 \\ 0.0274 & -0.0371 & 0.0167 \\ \hdashline -0.1226 & 0.0125 & -0.0475 \end{bmatrix}$$

and (except for rounding) $\boldsymbol{\pi} = \mathbf{i}'(\bar{\mathbf{A}}^{1c} - \bar{\mathbf{A}}^{0})$. This suggests that the three sectors in this example were 0.24, 9.08 and 2.45 percent more productive, respectively, in year 1 than in year 0.

14.2.3 References for Section 14.2

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Appendix 14.1 More on the Derivation of TFP Growth Measures

As before, define sector *j*'s total factor productivity growth as the difference in the growth rates of *j*'s (real) output and *j*'s (real) inputs at two time periods. Using the logarithmic differentiation rule that $d\ln z = dz/z$ or $dz = z(d\ln z)$, it is clear that $d\ln z$ is just an alternative way of expressing *z*'s growth rate. Prices at any time *t* are taken to be exogenous constants. So, recalling (14.1), the growth rate for output of sector *j* is

$$d(p_j q_j)/p_j q_j = p_j dq_j / p_j q_j = dq_j / q_j = d\ln q_j \qquad \text{(A14.1.1)}$$

Notice that we started with *j*'s output in value terms ($p_j q_j$) but end up with the growth rate in physical output terms. Also from (14.1), the growth rate for the input side for sector *j* is

$$(p_j q_j)^{-1}[\sum_i p_i ds_{ij} + p_v ds_{vj}] = (p_j q_j)^{-1}[\sum_i p_i s_{ij} d\ln s_{ij} + p_v s_{vj} d\ln s_{vj}] \qquad \text{(A14.1.2)}$$

In this case we have the change in inputs per dollar's worth of output. Prices are necessary in order to be able to add together different physical inputs. Thus, from (A14.1.1) and (A14.1.2),

$$\pi_j = d\ln q_j - (p_j q_j)^{-1}[\sum_i p_i s_{ij} d\ln s_{ij} + p_v s_{vj} d\ln s_{vj}] \qquad \text{(A14.1.3)}$$

From $s_{ij} = c_{ij} q_j$, $s_{vj} = c_{vj} q_j$, $d\ln s_{ij} = d\ln c_{ij} + d\ln q_j$ and $d\ln s_{vj} = d\ln c_{vj} + d\ln q_j$. Putting this into (A14.1.3) and rearranging gives

$$\pi_j = d\ln q_j - (p_j q_j)^{-1}[\sum_i p_i s_{ij} d\ln c_{ij} + p_v s_{vj} d\ln c_{vj}] - (p_j q_j)^{-1}(\sum_i p_i s_{ij} + p_v s_{vj})(d\ln q_j)$$

(A14.1.4)

But, again from (14.1), $(p_j q_j)^{-1}(\sum_i p_i s_{ij} + p_v s_{vj}) = 1$ and so

$$\pi_j = -(p_j q_j)^{-1}[\sum_i p_i s_{ij} d\ln c_{ij} + p_v s_{vj} d\ln c_{vj}] \qquad \text{(A14.1.5)}$$

[These mathematical operations serve to eliminate the $d\ln q_j$ term from the right-hand side of (A14.1.3) or (A14.1.4), leaving an expression involving changes in *inputs* only.] Putting q_j inside the brackets, $\pi_j = -(p_j)^{-1}[\sum_i p_i (s_{ij}/q_j) d\ln c_{ij} + p_v (s_{vj}/q_j) d\ln c_{vj}] =$ $-(p_j)^{-1}[\sum_i p_i c_{ij} d\ln c_{ij} + p_v c_{vj} d\ln c_{vj}]$ or

$$\pi_j = -(p_j)^{-1}[\sum_i p_i dc_{ij} + p_v dc_{vj}] \qquad \text{(A14.1.6)}$$

Thus π_j is expressed in terms of (known) prices and changes in coefficients. It is worth noting that much of the published literature uses a_{ij} in place of c_{ij}, with the understanding that all variables are expressed in real terms. In matrix terms (A14.1.6) becomes

$$\boldsymbol{\pi} = -(\hat{\mathbf{p}}^{-1})[(d\mathbf{C}')\mathbf{p} + p_v d\mathbf{c}_v]$$

or, in finite difference form:

$$\boldsymbol{\pi} = -(\hat{\mathbf{p}}^{-1})[(\Delta\mathbf{C}')\mathbf{p} + p_v \Delta\mathbf{c}_v]$$

with an enlarged second term in the case of more than one value-added input.