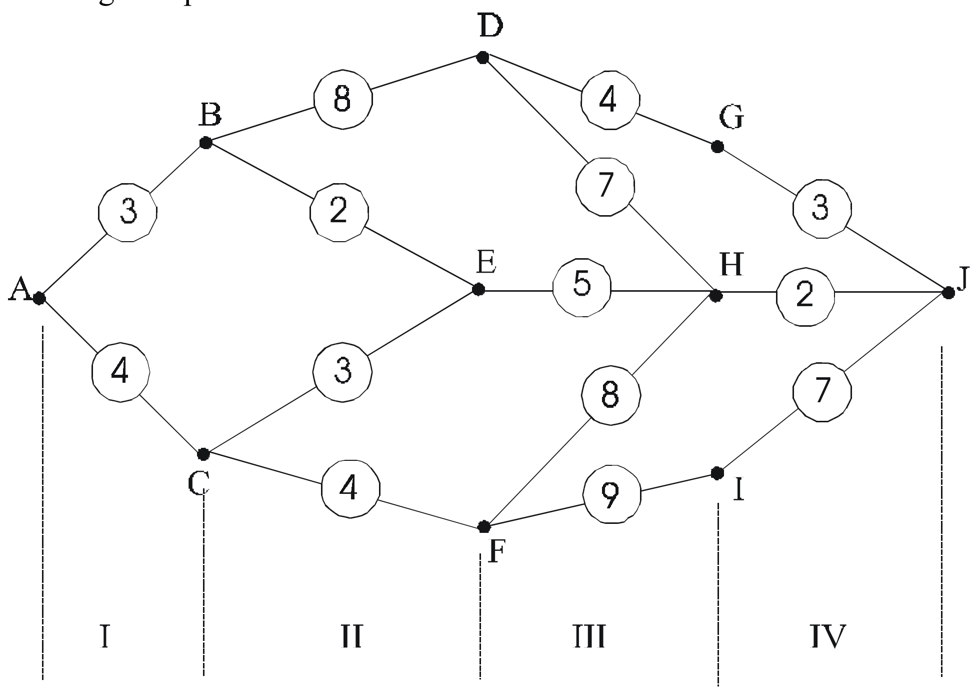
Chapter 6

```
> with(plots):
Warning, the name changecoords has been redefined
> with(DEtools):
> with(linalg):
Warning, the name adjoint has been redefined

Warning, the protected names norm and trace have been redefined and
unprotected
```

Question 1

The stages of production are as follows:



(i)

Solution paths are:

ABDGJ = 3+8+4+3 = 18
ABDHJ = 3+8+7+2 = 20
ABEHJ = 3+2+5+2 = 12
ACEHJ = 4+3+5+2 = 14
ACFHJ = 4+4+8+2 = 18
ACFIJ = 4+4+9+7 = 24

Minimum is ABEHJ = 12

(ii)

Backward solving we have path JHEBA, which implies path ABEHJ = 12. Backward solving gives the same path as minimising the route throught the system.

(iii)

Given minimising all possible solutions for solving forward is the same as backward solving, which inolves only one path, then by Occam's razor, backward solving is the

Question 2

We need to prove

$$-\int_{t_0}^{t_1} \lambda\left(\frac{\partial}{\partial t}x\right)dt = \int_{t_0}^{t_1} x\left(\frac{\partial}{\partial t}\lambda\right)dt - [\lambda(t_1)\,\mathrm{x}(t_1) - \lambda(t_0)\,\mathrm{x}(t_0)]$$

As a preliminary, we note the following,

$$\mathrm{d}(u\,v) = u\,dv + v\,du$$

$$u\,dv = \mathrm{d}(u\,v) - v\,du$$

Then

$$\int u\,dv = \int 1\,duv - \int v\,du + c$$

where c is the constant of integration. Rearranging,

$$\int u\,dv = u\,v - \int v\,du + c$$

Now let $u = \lambda$ and $v = x$, then

$$\int_{t_0}^{t_1} \lambda\,dx = [\lambda(t_1)\,\mathrm{x}(t_1) - \lambda(t_0)\,\mathrm{x}(t_0)] - \int_{t_0}^{t_1} x\,d\lambda$$

$$\int_{t_0}^{t_1} \lambda\left(\frac{\partial}{\partial t}x\right)dt = [\lambda(t_1)\,\mathrm{x}(t_1) - \lambda(t_0)\,\mathrm{x}(t_0)] - \int_{t_0}^{t_1} x\left(\frac{\partial}{\partial t}\lambda\right)dt$$

i.e.,

$$-\int_{t_0}^{t_1} \lambda\left(\frac{\partial}{\partial t}x\right)dt = \int_{t_0}^{t_1} x\left(\frac{\partial}{\partial t}\lambda\right)dt - [\lambda(t_1)\,\mathrm{x}(t_1) - \lambda(t_0)\,\mathrm{x}(t_0)]$$

Question 3

(i)

The detailed computations for each alternative is shown in the following spreadsheets. The first data series involves no discounting; the second discounting with a discount rate of 10%. For some series adjustment was made to the final figures to prevent the reserves remaining becoming negative.

(a)

$p = 6$, $b = 2$ **and** $R = 600$

t	$\lambda(t)$	$\frac{y(t)}{x(t)}$	$x(t)$	$y(t)$
0		.1389	600.0000	83.3410
1	5.4443	.1501	516.6589	77.5911
2	5.3992	.1635	439.0678	71.8110
3	5.3457	.1796	367.2567	65.9958
4	5.2812	.1996	301.2609	60.1390
5	5.2015	.2249	241.1219	54.2327
6	5.1003	.2582	186.8891	48.2679
7	4.9669	.3046	138.6212	42.2361
8	4.7812	.3750	96.3850	36.1444
9	4.5000	1.5000	60.2406	60.2406
10	0		.0000	0

$p = 6$, $b = 2$, $r = .909091$ **and** $d = .1$

t	$\lambda(t)$	$\frac{y(t)}{x(t)}$	$x(t)$	$y(t)$
0		.4568	600.0000	274.1076
1	4.5898	.4569	325.8923	148.9059
2	4.5895	.4570	176.9863	80.8938
3	4.5889	.4573	96.0924	43.9485
4	4.5876	.4579	52.1439	23.8794
5	4.5850	.4591	28.2644	12.9780
6	4.5796	.4616	15.2863	7.0567
7	4.5688	.4667	8.2296	3.8408
8	4.5464	.4772	4.3888	2.0946
9	4.5000	1.5000	2.2941	2.2941
10	0		.0000	

(b)

$p = 3$, $b = 3$ **and** $R = 600$

t	$\lambda(t)$	$\frac{y(t)}{x(t)}$	$x(t)$	$y(t)$
0		.1389	600.0000	83.3410
1	2.1665	.1501	516.6589	77.5911
2	2.0989	.1635	439.0678	71.8110
3	2.0186	.1796	367.2567	65.9958
4	1.9218	.1996	301.2609	60.1390
5	1.8022	.2249	241.1219	54.2327
6	1.6504	.2582	186.8891	48.2679
7	1.4503	.3046	138.6212	42.2361
8	1.1718	.3750	96.3850	36.1444
9	.7500	.5000	60.2406	30.1203
10	0		30.1203	30.1203

$p = 3$, $b = 3$, $R = 600$, $r = .909091$ **and** $d = .1$

t	$\lambda(t)$	$\frac{y(t)}{x(t)}$	$x(t)$	$y(t)$
0		.2382	600.0000	142.9371
1	1.7276	.2411	457.0628	110.2075
2	1.7086	.2453	346.8553	85.0925
3	1.6808	.2514	261.7628	65.8278
4	1.6402	.2605	195.9349	51.0562
5	1.5801	.2742	144.8787	39.7313
6	1.4900	.2952	105.1474	31.0445
7	1.3513	.3288	74.1028	24.3680
8	1.1296	.3863	49.7347	19.2157
9	.7500	.5000	30.5190	15.2595
10	0		15.2595	15.2592

(c)

$p = 3$, $b = 2$ **and** $R = 800$

t	$\lambda(t)$	$\frac{y(t)}{x(t)}$	$x(t)$	$y(t)$
0		.1478	800.0000	118.3120
1	2.4084	.1608	681.6879	109.6303
2	2.3567	.1763	572.0576	100.8973
3	2.2944	.1954	471.1603	92.1037
4	2.2180	.2195	379.0565	83.2383
5	2.1216	.2511	295.8181	74.2875
6	1.9954	.2944	221.5306	65.2378
7	1.8220	.3588	156.2927	56.0914
8	1.5644	.4687	100.2013	46.9693
9	1.1250	.7500	53.2319	39.9239
10	0		13.3079	13.3079

$p = 3$, $b = 2$, $R = 800$, $r = .909091$ **and** $d = .1$

t	$\lambda(t)$	$\frac{y(t)}{x(t)}$	$x(t)$	$y(t)$
0		.3037	800.0000	242.9747
1	1.9636	.3058	557.0252	170.3761
2	1.9541	.3092	386.6490	119.5838
3	1.9391	.3147	267.0651	84.0567
4	1.9151	.3235	183.0084	59.2148
5	1.8763	.3380	123.7936	41.8500
6	1.8125	.3626	81.9435	29.7138
7	1.7044	.4064	52.2297	21.2310
8	1.5114	.4943	30.9986	15.3232
9	1.1250	.7500	15.6754	11.7566
10	0		3.9188	3.9188

(ii)

We can summarise the results as follows.

(a) A rise in the price from $p = 3$ to $p = 6$ leads to smaller output in earlier periods and a larger output in later periods in the case of no discounting. Where discounting is taken into

account, however, output in the first two periods is higher, and lower thereafter.
(b) A rise in the value of b from $b = 2$ to $b = 3$ leads to output sometimes rising and sometimes falling in the case of no discounting. Where discounting is taken into account, however, output is only down for the first three periods, and then exceeds that for $b = 2$.
(c) A rise in reserves from $R = 600$ to $R = 800$ leads to a rise in output in all periods under both no discounting and discounting.

Question 4

At 5% discount rate the production level is given in the final column of the following spreadsheet, where

$p = 3$, $b = 2$, $R = 600$, $r = .952381$ **and** $d = .05$

t	$\lambda(t)$	$\frac{y(t)}{x(t)}$	$x(t)$	$y(t)$
0		.2416	600.0000	144.9971
1	2.1350	.2466	455.0028	112.2343
2	2.1139	.2536	342.7685	86.9523
3	2.0845	.2636	255.8161	67.4341
4	2.0428	.2778	188.3819	52.3509
5	1.9828	.2989	136.0310	40.6719
6	1.8942	.3313	95.3590	31.5952
7	1.7584	.3842	63.7637	24.4981
8	1.5363	.4821	39.2655	18.9316
9	1.1250	.7500	20.3339	15.2504
10	0		5.0834	5.0834

At 10% the production level is given in the final column of the following spreadsheet, where

$p = 3$, $b = 2$, $R = 600$, $r = .909091$ **and** $d = .1$

t	$\lambda(t)$	$\frac{y(t)}{x(t)}$	$x(t)$	$y(t)$
0		.3037	600.0000	182.2310
1	1.9636	.3058	417.7689	127.7821
2	1.9541	.3092	289.9867	89.6878
3	1.9391	.3147	200.2988	63.0425
4	1.9151	.3235	137.2563	44.4111
5	1.8763	.3380	92.8452	31.3875
6	1.8125	.3626	61.4576	22.2853
7	1.7044	.4064	39.1722	15.9232
8	1.5114	.4943	23.2490	11.4924
9	1.1250	.7500	11.7566	8.8174
10	0		2.9391	2.9391

At 15% the production level is given in the final column of the following spreadsheet, where $p = 3$, $b = 2$, $R = 600$, $r = .8695652$ **and** $d = .15$

t	$\lambda(t)$	$\frac{y(t)}{x(t)}$	$x(t)$	$y(t)$
0		.3487	600.0000	209.2794
1	1.8455	.3497	390.7205	136.6734
2	1.8409	.3515	254.0470	89.3149
3	1.8327	.3547	164.7321	58.4329
4	1.8183	.3603	106.2991	38.3046
5	1.7924	.3705	67.9945	25.1957
6	1.7454	.3894	42.7987	16.6696
7	1.6583	.4262	26.1291	11.1378
8	1.4891	.5054	14.9912	7.5770
9	1.1250	.7500	7.4141	5.5606
10	0		1.8535	1.8535

```
> points1:=[[0,144.9971036], [1,112.2343579], [2,86.95238074],
  [3,67.43418395], [4,52.35096414], [5,40.67198555],
  [6,31.59528820], [7,24.49814663], [8,18.93162295],
```

```
[9,15.25047485], [10,5.0834]];
```

points1 := [[0, 144.9971036], [1, 112.2343579], [2, 86.95238074], [3, 67.43418395], [4, 52.35096414], [5, 40.67198555], [6, 31.59528820], [7, 24.49814663], [8, 18.93162295], [9, 15.25047485], [10, 5.0834]]

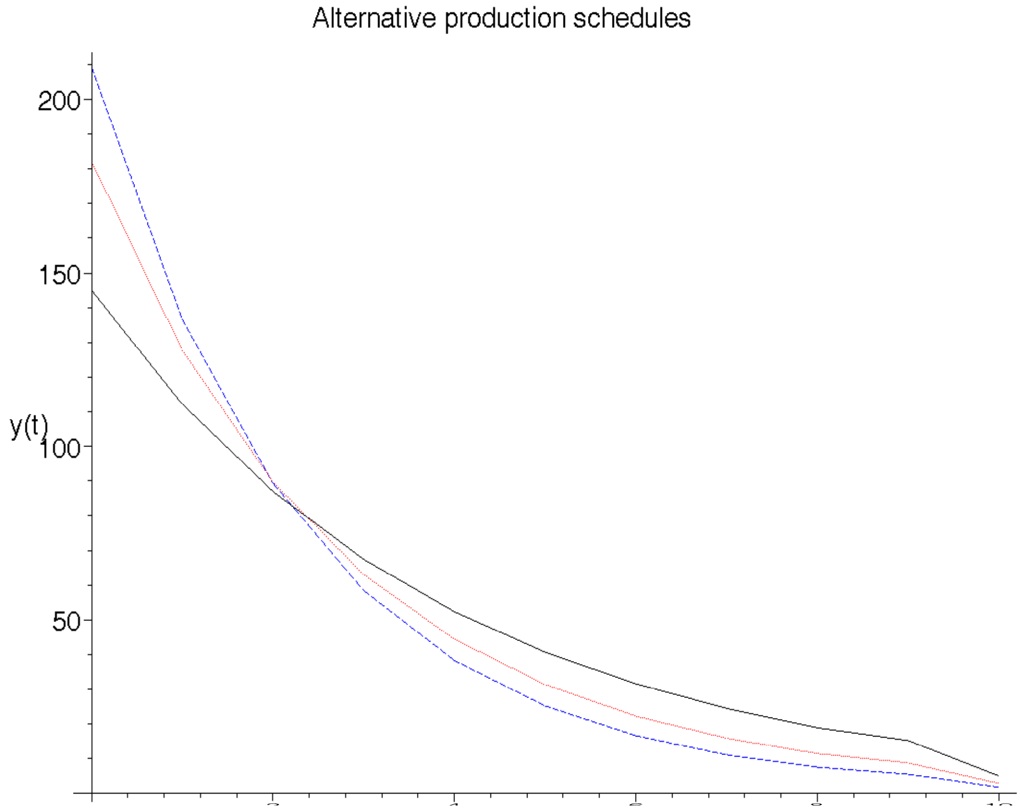
```
> points2:=[[0,182.2310763], [1,127.7821287], [2,89.68789886],
  [3,63.04255748], [4,44.41111480], [5,31.38753211],
  [6,22.28539712], [7,15.92328486], [8,11.49240762],
  [9,8.817451568], [10,2.9391]];
```

points2 := [[0, 182.2310763], [1, 127.7821287], [2, 89.68789886], [3, 63.04255748], [4, 44.41111480], [5, 31.38753211], [6, 22.28539712], [7, 15.92328486], [8, 11.49240762], [9, 8.817451568], [10, 2.9391]]

```
> points3:=[[0,209.2794831], [1,136.6734479], [2,89.31493237],
  [3,58.43297652], [4,38.30463952], [5,25.19572598],
  [6,16.66968490], [7,11.13787684], [8,7.577090595],
  [9,5.560606699], [10,1.835]];
```

points3 := [[0, 209.2794831], [1, 136.6734479], [2, 89.31493237], [3, 58.43297652], [4, 38.30463952], [5, 25.19572598], [6, 16.66968490], [7, 11.13787684], [8, 7.577090595], [9, 5.560606699], [10, 1.835]]

```
> plot1:=pointplot(points1,connect=true):
> plot2:=pointplot(points2,connect=true,colour=red,linestyle=2)
  :
> plot3:=pointplot(points3,connect=true,colour=blue,linestyle=3
  ):
> display(plot1,plot2,plot3,labels=["t","y(t)"],
  title="Alternative production schedules");
```



It is clear that as the discount rate rises, the level of production in the earlier period rises also. However, after period 3 production is higher the lower the discount rate.

Question 5

If inflation is expected to be 5% per period, then in the case of no discounting we have the following production information.

t	$p(t)$	$\lambda(t)$	$\frac{y(t)}{x(t)}$	$x(t)$	$y(t)$	*Adj* $x(t)$
0	3.0000		-.0923	600.0000	-55.3812	600.0000
1	3.1500	3.3692	-.0533	655.3812	-34.9825	600.0000
2	3.3075	3.3635	-.0139	690.3638	-9.6000	600.0000
3	3.4728	3.3631	.0278	699.9638	19.4765	600.0000
4	3.6465	3.3615	.0739	680.4873	50.3369	583.3049
5	3.8288	3.3506	.1277	630.1503	80.4753	540.1567
6	4.0202	3.3180	.1944	549.6750	106.9007	471.1743
7	4.2213	3.2423	.2854	442.7743	126.4049	379.5404
8	4.4323	3.0793	.4312	316.3693	136.4279	271.1876
9	4.6539	2.7074	1.1634	179.9413	209.3611	154.2434
10	4.8866	0		-29.4197	0	.0000

Notice in this table that we have adjusted $x(t)$ and $y(t)$. Since $y(t)$ cannot be negative, then we have zero output for periods 0, 1 and 2. Similarly, we cannot have negative reserves, so output in period 9 is set at 79.9775.

For example 6.5, where discounting takes place at 10%, we have the following production information.

t	$p(t)$	$\lambda(t)$	$\frac{y(t)}{x(t)}$	$x(t)$	$y(t)$
0	3.0000		.2162	600.0000	129.7256
1	3.1500	2.3486	.2258	470.2743	106.2020
2	3.3075	2.4713	.2371	364.0722	86.3436
3	3.4728	2.5947	.2510	277.7285	69.7223
4	3.6465	2.7155	.2688	208.0062	55.9306
5	3.8288	2.8280	.2931	152.0755	44.5842
6	4.0202	2.9217	.3286	107.4913	35.3244
7	4.2213	2.9763	.3855	72.1669	27.8250
8	4.4323	2.9469	.4927	44.3418	21.8500
9	4.6539	2.7074	1.1634	22.4918	22.4918
10	4.8866	0		0	0

No adjustment is required in this case, except to set $y(t)$ for period ten at the level of $x(t)$.

Question 6

Here we consider once again example 6.4 in which there is no discounting. Price inflation is assumed to be 10% while cost inflation is assumed to be higher at 15%. The resulting production path is given in the following spreadsheet.

t	p(t)	b(t)	$\lambda(t)$	$\frac{y(t)}{x(t)}$	x(t)	y(t)	*Adj*
0	3.0000	2.0000		-.1501	600.0000	-90.0924	600
1	3.3000	2.3000	3.6006	-.0633	690.0924	-43.7139	600
2	3.6300	2.6450	3.5913	.0073	733.8063	5.3759	600
3	3.9930	3.0417	3.5912	.0683	728.4304	49.8085	595
4	4.3923	3.4980	3.5770	.1242	678.6219	84.3213	554
5	4.8315	4.0227	3.5230	.1785	594.3006	106.1344	485
6	5.3146	4.6261	3.3947	.2351	488.1661	114.7993	399
7	5.8461	5.3200	3.1388	.2992	373.3668	111.7117	305
8	6.4307	6.1180	2.6626	.3802	261.6550	99.4934	213
9	7.0738	7.0357	1.7780	.5027	162.1616	81.5197	132
10	7.7812	8.0911	0		80.6418	0	65.

Question 7

First we specify the vote function $v = \mathrm{v}(u, \pi)$, which is to be maximised over the period 0 to 5. This function is weighted by the factor $\mathbf{e}^{(.05\,t)}$. Our control problme is then,

$$\text{Maximise} \int_0^5 \mathbf{e}^{(.05\,t)}\, \mathrm{v}(u, \pi)\, dt$$

$$\text{s.t.} \qquad \pi = -\alpha\,(u - u_n) + \pi^e$$

$$\frac{\partial}{\partial t}\pi^e = \beta\,(\pi - \pi^e)$$

$$\pi(0) = \pi_0 \qquad \pi(5) \text{ free}$$

where π_0 is the initial rate of inflation, which is assumed known.

$$0 = \beta\,(\pi - \pi^e)$$

Question 8

The Hamiltonian is given by

$$H = -\left(\frac{x^2}{4} - \frac{u^2}{9}\right) + \lambda\,(-x + u)$$

```
> diff(-(x^2/4-u^2/9)+lambda*(-x+u),u);
```

$$\frac{2}{9}u + \lambda$$

```
> -diff(-(x^2/4-u^2/9)+lambda*(-x+u),x);
```

$$\frac{1}{2}x+\lambda$$

```
> solve(2/9*u+lambda,u);
```

$$-\frac{9}{2}\lambda$$

```
> subs(u=-9/2*lambda,-x+u);
```

$$-x-\frac{9}{2}\lambda$$

```
> subs(u=-9/2*lambda,-x+u);
```

$$-x-\frac{9}{2}\lambda$$

Our two differential equations are therefore

$$\frac{\partial}{\partial t}x=-x-\frac{9\,\lambda}{2}$$

$$\frac{\partial}{\partial t}\lambda=\frac{1\,x}{2}+\lambda$$

```
> solve(-x-9/2*lambda=0,lambda);
```

$$-\frac{2}{9}x$$

```
> solve(1/2*x+lambda=0,lambda);
```

$$-\frac{1}{2}x$$

```
> dsolve({diff(x(t),t)=-x(t)-9*lambda(t)/2,diff(lambda(t),t)=x(
  t)/2+lambda(t)});
```

$$\{\mathrm{x}(t)=\_C1\cos\left(\frac{1}{2}\sqrt{5}\,t\right)+\_C2\sin\left(\frac{1}{2}\sqrt{5}\,t\right),\lambda(t)=\frac{1}{9}\_C1\sin\left(\frac{1}{2}\sqrt{5}\,t\right)\sqrt{5}$$
$$-\frac{1}{9}\_C2\cos\left(\frac{1}{2}\sqrt{5}\,t\right)\sqrt{5}-\frac{2}{9}\_C1\cos\left(\frac{1}{2}\sqrt{5}\,t\right)-\frac{2}{9}\_C2\sin\left(\frac{1}{2}\sqrt{5}\,t\right)\}$$

```
> evalf(subs(t=0,_C1*cos(1/2*sqrt(5)*t)+_C2*sin(1/2*sqrt(5)*t))
  );
```

$$1.\ \_C1$$

```
> evalf(subs(t=1,_C1*cos(1/2*sqrt(5)*t)+_C2*sin(1/2*sqrt(5)*t))
  );
```

$$.4374512105\ \_C1+.8992421467\ \_C2$$

```
> solve({5=c[1],10=.4374512105*c[1]+.8992421467*c[2]},{c[1],c[2
  ]});
>
```

$$\{c_1=5.,c_2=8.688142539\}$$

```
> evalf(subs(t=0,1/9*_C1*sin(1/2*sqrt(5)*t)*sqrt(5)-1/9*_C2*cos
```

```
(1/2*sqrt(5)*t)*sqrt(5)-2/9*_C1*cos(1/2*sqrt(5)*t)-2/9*_C2*si
n(1/2*sqrt(5)*t)));
```

$$-.2484519975\ \_C2 - .2222222222\ \_C1$$

```
> subs({c[1]=5,c[2]=8.688142539},-.2484519975*c[2]-.2222222222*
c[1]);
```

$$-3.269697479$$

so $\lambda(0) = -3.2697$.

```
> solx:=simplify(subs({c[1]=5,c[2]=8.688142539},c[1]*cos(1/2*sq
rt(5)*t)+c[2]*sin(1/2*sqrt(5)*t)));
```

$$solx := 5.\ \cos(1.118033989\ t) + 8.688142539\ \sin(1.118033989\ t)$$

```
> sollambda:=simplify(subs({c[1]=5,c[2]=8.688142539},1/9*c[1]*s
in(1/2*sqrt(5)*t)*sqrt(5)-1/9*c[2]*cos(1/2*sqrt(5)*t)*sqrt(5)
-2/9*c[1]*cos(1/2*sqrt(5)*t)-2/9*c[2]*sin(1/2*sqrt(5)*t)));
```

$$sollambda := -.6884383540\ \sin(1.118033989\ t) - 3.269697480\ \cos(1.118033989\ t)$$

```
> eq1:={diff(x(t),t)=-x(t)-(9/2)*lambda(t),diff(lambda(t),t)=x(
t)/2+lambda(t)};
```

$$eq1 := \{\frac{\partial}{\partial t}\,\mathrm{x}(t) = -\mathrm{x}(t) - \frac{9}{2}\,\lambda(t), \frac{\partial}{\partial t}\,\lambda(t) = \frac{1}{2}\,\mathrm{x}(t) + \lambda(t)\}$$

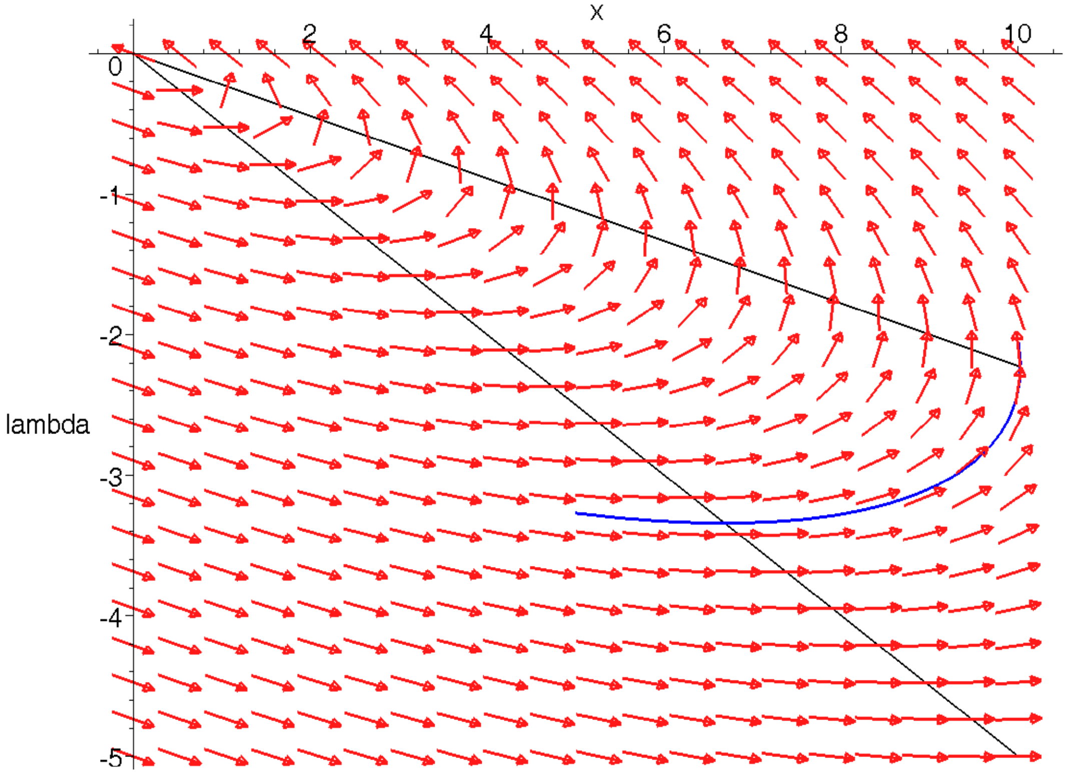
```
> var:=[x(t),lambda(t)];
```

$$var := [\mathrm{x}(t), \lambda(t)]$$

```
> init:=[[x(0)=5,lambda(0)=-3.2697]];
```

$$init := [[\mathrm{x}(0) = 5, \lambda(0) = \text{-}3.2697]]$$

```
> dfield:=DEplot(eq1,var,t=0..1,x=0..10,lambda=-5..0,init,steps
ize=.1,arrows=medium,linecolour=blue,thickness=3):
> lines:=plot({-(2/9)*x,-(1/2)*x},x=0..10,colour=black,thicknes
s=2):
> display(dfield,lines);
```




```
>
```

Question 9

The Hamiltonian is given by

$H = 3\,x^2 - u^2 + \lambda\,(2\,x + u)$

```
> diff((3*x^2-u^2)+lambda*(2*x+u),u);
```

$$-2\,u + \lambda$$

```
> -diff((3*x^2-u^2)+lambda*(2*x+u),x);
```

$$-6\,x - 2\,\lambda$$

```
> solve(-2*u+lambda,u);
```

$$\frac{1}{2}\lambda$$

```
> subs(u=lambda/2,2*x+u);
```

$$2\,x + \frac{1}{2}\lambda$$

```
> subs(u=lambda/2,-6*x-2*lambda);
```

$$-6\,x - 2\,\lambda$$

```
> dsolve({diff(x(t),t)=2*x(t)+lambda(t)/2,diff(lambda(t),t)=-6*
  x(t)-2*lambda(t)},{x(t),lambda(t)});
```

$$\{x(t) = \_C1\,\mathbf{e}^{t} + \_C2\,\mathbf{e}^{(-t)},\ \lambda(t) = -2\,\_C1\,\mathbf{e}^{t} - 6\,\_C2\,\mathbf{e}^{(-t)}\}$$

Then $x(0) = 10 = c_2 + c_1$ and $x(1) = 15 = c_1\,\mathbf{e} + c_2\,\mathbf{e}^{(-1)}$

Solving for c_1 and c_2 we obtain

```
> solve({10=c[1]+c[2],15=c[1]*exp(1)+c[2]*exp(-1)},{c[1],c[2]})
```

```
;
```

$$\{c_2 = 5\frac{-3+2\,\mathbf{e}}{\mathbf{e}-\mathbf{e}^{(-1)}},\ c_1 = -5\frac{2\,\mathbf{e}^{(-1)}-3}{\mathbf{e}-\mathbf{e}^{(-1)}}\}$$

```
> fsolve({10=c[1]+c[2],15=c[1]*exp(1)+c[2]*exp(-1)},{c[1],c[2]}
);
```

$$\{c_2 = 5.183290466,\ c_1 = 4.816709534\}$$

```
> subs({c[1]=4.816709534,c[2]=5.183290466},-2*c[1]-6*c[2]);
```

$$-40.73316187$$

So $\lambda(0) = -40.7332$.

```
> eq1:={diff(x(t),t)=2*x(t)+lambda(t)/2,diff(lambda(t),t)=-6*x(
t)-2*lambda(t)};
```

$$eq1 := \{\frac{\partial}{\partial t}\mathrm{x}(t) = 2\,\mathrm{x}(t) + \frac{1}{2}\lambda(t),\ \frac{\partial}{\partial t}\lambda(t) = -6\,\mathrm{x}(t) - 2\,\lambda(t)\}$$

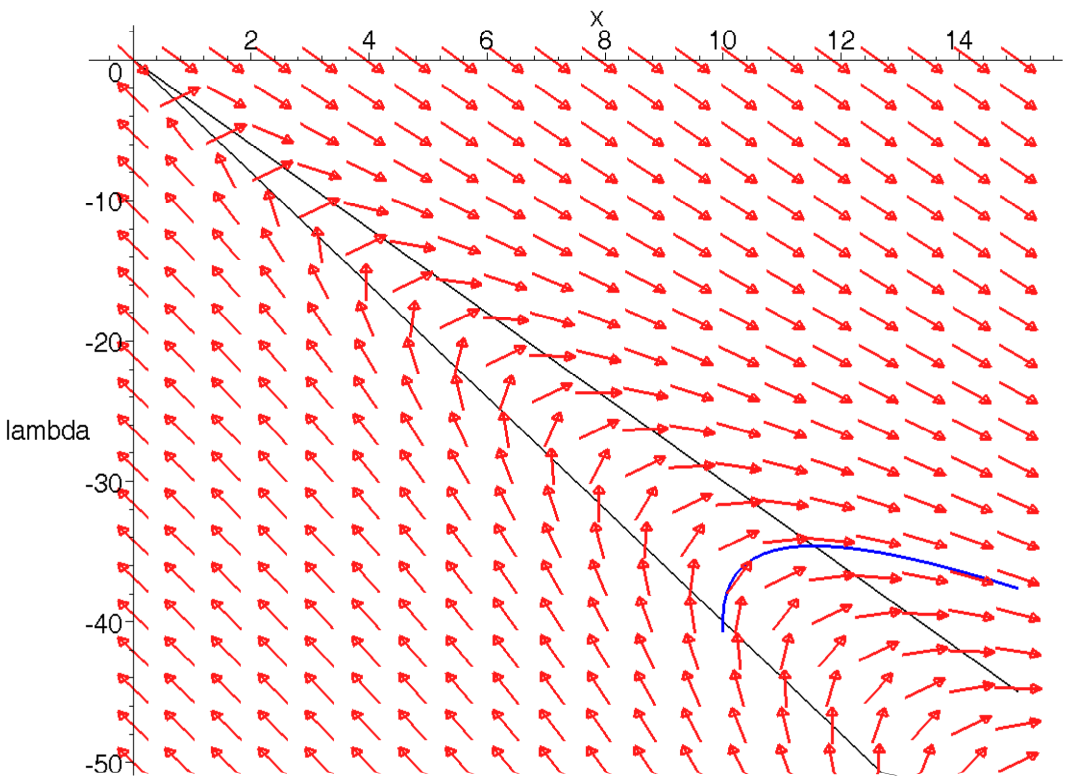
```
> var:=[x(t),lambda(t)];
```

$$var := [\mathrm{x}(t), \lambda(t)]$$

```
> init:=[[x(0)=10,lambda(0)=-40.7332]];
```

$$init := [[\mathrm{x}(0) = 10,\ \lambda(0) = -40.7332]]$$

```
> dfield:=DEplot(eq1,var,t=0..1,x=0..15,lambda=-50..0,init,step
size=.1,arrows=medium,linecolour=blue,thickness=3):
> lines:=plot({-4*x,-3*x},x=0..15,colour=black,thickness=2):
> display(dfield,lines);
```



Question 10

From the text we know that

$$\frac{\partial}{\partial t} \mathrm{c}(t) = \frac{\left(\left(\frac{\partial}{\partial k} f\right) - \delta + \beta\right) c}{\sigma(c)}$$

$$\frac{\partial}{\partial t} k = \mathrm{f}(k) - (n + \delta)\, k - c$$

Substituting we obtain

$$\frac{\partial}{\partial t} c = (.4\, k^{(-.7)} - .1067)\, c$$

$$\frac{\partial}{\partial t} k = k^{.3} - .05\, k - c$$

wih equilibrium values

```
> fsolve({.4*k^(-0.7)-0.1067=0,k^0.3-0.05*k-c=0},{k,c});
```

$$\{c = 1.431561213,\ k = 6.604665345 + 0.\, I\}$$

i.e., $kstar = 6.6047$ and $cstar = 1.4316$.

Next we need to linearize the system around $(kstar, cstar) = (6.6047, 1.4316)$.
Let

$$\mathrm{F}(c, k) = (.4\, k^{(-.7)} - .1067)\, c$$
$$\mathrm{G}(c, k) = k^{.3} - .05\, k - c$$

```
> F:=(.4*k^(-.7)-.1067)*c;
```

$$F := \left(.4 \frac{1}{k^{.7}} - .1067\right) c$$

```
> G:=k^.3-.5e-1*k-c;
```

$$G := k^{.3} - .05\, k - c$$

```
> Fc:=diff(F,c);
```

$$Fc := .4 \frac{1}{k^{.7}} - .1067$$

```
> Fk:=diff(F,k);
```

$$Fk := -.28 \frac{c}{k^{1.7}}$$

```
> Gc:=diff(G,c);
```

$$Gc := -1$$

```
> Gk:=diff(G,k);
```

$$Gk := .3 \frac{1}{k^{.7}} - .05$$

```
> subs({c=1.4316,k=6.6047},Fc);
```

$$-.3919\ 10^{-6}$$

```
> subs({c=1.4316,k=6.6047},Fk);
```

$$-.01618935171$$

```
> subs({c=1.4316,k=6.6047},Gc);
```

$$-1$$

```
> subs({c=1.4316,k=6.6047},Gk);
```

$$.03002470606$$

```
> mA:=matrix([[-.3919e-6,-.1618935171e-1],[-1,
  .3002470606e-1]]);
```

$$mA := \begin{bmatrix} -.3919\ 10^{-6} & -.01618935171 \\ -1 & .03002470606 \end{bmatrix}$$

```
> eigenvalues(mA);
```

$$-.1131078193,\ .1431321334$$

Since these are of opposite sign, then the equilibrium is a saddle-point solution.