Additional Exercises
Chapter 2

Exercise 2.3

Using the same material properties as in Exercise 2.2 and a magma body thickness H = 500 m, calculate the time it takes for the magma to be homogenized by convective motions (use the thermal diffusivity in place of D in Eq. 2.29). Calculate the stirring time for basalt, andesite and rhyolite magmas. First calculate the Rayleigh number (for the three different buoyancy sources in Exercise 2.1), then estimate the Lagrangian strain rate from Eq. (2.30). Compare the stirring times for the different magma compositions and different buoyancy sources. Remembering these calculations assume steady conditions (buoyancy source, material properties, thermal gradients, etc.), discuss how open-system behavior such as injection of new magma or cooling could affect your results.

Exercise 2.4

(a) In two-dimensional axisymmetric and plane-strain conditions (with no body forces), Eq. (2.18) takes the form

$$\frac{d^2\mathbf{d}_r}{dr^2} + \frac{1}{r}\frac{d\mathbf{d}_r}{dr} - \frac{\mathbf{d}_r}{r^2} = 0,$$

where $\mathbf{d}_r$ is the radial displacement, and r is a radial distance. Assuming a circular magma chamber of radius R centered at the origin, subject to boundary conditions

$$\mathbf{d}_r \,|_{r=R} = D_0,$$
$$\mathbf{d}_r \,|_{r=\infty} = 0,$$

derive the solution for radial displacements in the elastic medium surrounding the chamber.

(b) Displacements may be related to components of the strain tensor ε in this problem by:

$$\varepsilon_{rr} = \frac{d\mathbf{d}_r}{dr} \quad , \quad \varepsilon_{\theta\theta} = \frac{\mathbf{d}_r}{r} \quad , \quad \varepsilon_{r\theta} = 0 \quad .$$

Stresses σ may be found through use of the standard constitutive relationship for elasticity, Hooke's Law, that relates stresses to strains through the Young's modulus E and Poisson's ratio υ:

$$\sigma_{rr} = \frac{E}{1+\upsilon}\left[\dot{\varepsilon}_{rr} + \frac{\upsilon}{1-2\upsilon}(\dot{\varepsilon}_{rr} + \dot{\varepsilon}_{\theta\theta})\right] \ ,$$

$$\sigma_{\theta\theta} = \frac{E}{1+\upsilon}\left[\dot{\varepsilon}_{\theta\theta} + \frac{\upsilon}{1-2\upsilon}(\dot{\varepsilon}_{rr} + \dot{\varepsilon}_{\theta\theta})\right] \ ,$$

$$\sigma_{r\theta} = 0 \ .$$

Derive the stresses around a circular magma chamber, and find that a uniform pressure $\sigma_{rr} = P$ at the chamber wall $r = R$ may be related to radial displacement at the chamber wall D_0 by $D_0 = -R\,P\,(1+\upsilon)/E$.

(c) Comment on your results. How are displacements and stresses around a circular magma chamber different from those around a spherical magma chamber (Eqs. 2.19 and 2.20)? Are there natural scenarios in which this two-dimensional geometry is a reasonable model?