6. At frequencies above the plasma frequency, the real part of the permittivity is positive while the positive imaginary part decreases quickly with increasing frequency. Consequently, the contribution of the conduction susceptibility quickly diminishes. Then the medium behaves optically like an insulator, allowing a high-frequency optical field to penetrate through with little attenuation except when the optical frequency comes close to a transition resonance.
7. For a perfect conductor, only free conduction electrons contribute to the optical response so that the permittivity has no contribution from bound electrons; thus, $\epsilon_{\text{bound}} = \epsilon_0$. For this reason, it is a good approximation to take $\epsilon_{\text{bound}} = \epsilon_0$ for a metal that has a high conductivity, such as Ag, Au, Cu, and Al. For such a metal, it is also a good approximation to take the effective electron mass as the free electron mass, $m^* = m_0$, when applying (2.46).
8. For a semiconductor where electrons and holes both contribute to the conduction susceptibility, the total permittivity is

$$\epsilon(\omega) = \epsilon_{\text{bound}}(\omega) - \frac{\sigma_{\text{e}}(0)}{\omega(\omega\tau_{\text{e}} + \text{i})} - \frac{\sigma_{\text{h}}(0)}{\omega(\omega\tau_{\text{h}} + \text{i})}, \tag{2.50}$$

where

$$\sigma_{\text{e}}(0) = \frac{N_{\text{e}}e^2\tau_{\text{e}}}{m_{\text{e}}^*} \text{ and } \sigma_{\text{h}}(0) = \frac{N_{\text{h}}e^2\tau_{\text{h}}}{m_{\text{h}}^*}. \tag{2.51}$$

The plasma frequency is found at $\epsilon'(\omega) = 0$ to be

$$\omega_{\text{p}}^2 = \frac{\sigma_{\text{e}}(0)}{\epsilon_{\text{bound}}\tau_{\text{e}}} - \frac{1}{\tau_{\text{e}}^2} + \frac{\sigma_{\text{h}}(0)}{\epsilon_{\text{bound}}\tau_{\text{h}}} - \frac{1}{\tau_{\text{h}}^2} \approx \frac{N_{\text{e}}e^2}{\epsilon_{\text{bound}}m_{\text{e}}^*} + \frac{N_{\text{h}}e^2}{\epsilon_{\text{bound}}m_{\text{h}}^*}. \tag{2.52}$$

EXAMPLE 2.4

Silver is one of the best conductors such that the free-electron Drude model describes its optical properties reasonably well. In this model, the free electron density of Ag is found to be $N = 5.86 \times 10^{28}\ \text{m}^{-3}$. The DC conductivity of Ag at $T = 273$ K is $\sigma(0) = 6.62 \times 10^7\ \text{S m}^{-1}$. Find the plasma frequency ω_{p} and the relaxation time τ for Ag at $T = 273$ K. Also find the cutoff optical frequency ν_{p} and the cutoff wavelength λ_{p}. For what optical wavelengths is Ag expected to be highly reflective? For what wavelengths is it expected to become transmissive?

Solution:

For Ag, it is a good approximation to take $\epsilon_{\text{bound}} = \epsilon_0$ and $m^* = m_0$. Then, using (2.46), we find that

$$\omega_{\text{p}}^2 = \frac{Ne^2}{\epsilon_0 m^*} = \frac{5.86 \times 10^{28} \times \left(1.6 \times 10^{-19}\right)^2}{8.854 \times 10^{-12} \times 9.1 \times 10^{-31}}\ \text{rad}^2\ \text{s}^{-2} = 1.86 \times 10^{32}\ \text{rad}^2\ \text{s}^{-2}$$

$$\Rightarrow \omega_{\text{p}} = 1.36 \times 10^{16}\ \text{rad s}^{-1},$$

$$\tau = \frac{\sigma(0)}{\epsilon_0\omega_{\text{p}}^2} = \frac{6.62 \times 10^7}{8.854 \times 10^{-12} \times 1.86 \times 10^{32}}\ \text{s}^{-1} = 4.02 \times 10^{-14}\ \text{s} = 40.2\ \text{fs}.$$