## 216 **Optical Resonance**

## 6.5 FABRY–PÉROT CAVITY

The most common type of optical cavity is the Fabry–Pérot cavity, which consists of two end mirrors in the form of the Fabry–Pérot interferometer and, in the case when it is used as a laser cavity, an optical gain medium, as shown in Fig. 6.5. The radii of curvature of the left and right mirrors are  $\mathcal{R}_1$  and  $\mathcal{R}_2$ , respectively. The sign of the radius of curvature is taken to be positive for a concave mirror and negative for a convex mirror. For example, the cavity shown in Fig. 6.5 has  $\mathcal{R}_1 > 0$  and  $\mathcal{R}_2 > 0$  because it is formed with two concave mirrors.

## 6.5.1 Stability Criterion

Most of the important features of a nonwaveguiding Fabry–Pérot cavity can be obtained by applying the following simple concept. For the cavity to be a stable cavity in which a Gaussian mode can be established, the radii of curvature of both end mirrors have to match the wavefront curvatures of the Gaussian mode at the surfaces of the mirrors:  $\mathcal{R}(z_1) = -\mathcal{R}_1$  and  $\mathcal{R}(z_2) = \mathcal{R}_2$ , where  $z_1$  and  $z_2$  are, respectively, the coordinates of the left and right mirrors measured from the location of the Gaussian beam waist. Based on this concept, we have from (3.71) two relations:

$$z_1 + \frac{z_R^2}{z_1} = -\mathcal{R}_1 \text{ and } z_2 + \frac{z_R^2}{z_2} = \mathcal{R}_2.$$
 (6.31)

From these relations, we find that

$$z_{\rm R}^2 = \frac{l(\mathcal{R}_1 - l)(\mathcal{R}_2 - l)(\mathcal{R}_1 + \mathcal{R}_2 - l)}{\left(\mathcal{R}_1 + \mathcal{R}_2 - 2l\right)^2},\tag{6.32}$$

where  $l = z_2 - z_1$  is the length of the cavity defined by the separation between the two end mirrors.

Given the values of  $\mathcal{R}_1$ ,  $\mathcal{R}_2$ , and l, stable Gaussian modes exist for the cavity if both relations in (6.31) can be satisfied with a real and positive parameter of  $z_R > 0$  from (6.32) for a finite, positive beam-waist spot size  $w_0$  according to (3.69). Then the cavity is stable. If the relations in (6.31) cannot be simultaneously satisfied with a real and positive value for  $z_R$ , then the cavity is unstable because no stable Gaussian mode can be established in the cavity. Application of this concept yields the *stability criterion* for a Fabry–Pérot cavity:



**Figure 6.5** Fabry–Pérot cavity containing an optical gain medium with a filling factor  $\Gamma$ . Changes of Gaussian beam divergence at the boundaries of the gain medium are ignored in this plot.