

## Exercise: concepts from chapter 4

Reading: Fundamentals of Structural Geology, Ch 4

1) Show how to expand pascals (Pa) in terms of the fundamental units: kilograms (kg), meters (m), and seconds (s). Use a step-by-step procedure starting with the conversion of pascals to newtons per square meter. Figure 1 shows images of Newton and Pascal.



Figure 1. Issac Newton (1643-1727) and Blaise Pascal (1623-1662). Considering the remarkable similarities in some of their facial features do you suppose these two men might have been related?

2) The stress exerted by the 1 km high column of granite on one square meter is only about four times greater than the stress exerted by a person standing on your fingernail. Explain why these two stresses are nearly the same. In the process find your own weight in newtons and use this for the calculation.

3) It is easy to become confused by poor or incorrect choices of units of measure. The following statements were found in the geological literature, suggesting that not all geologists use consistent units. For each statement point out the unitary inconsistency and rewrite the statement using SI units.

"The force of gravity at sea level is approximately 980 dynes per square centimeter."

"In practice, force is often expressed in terms of grams or grams per square centimeter or in pounds or pounds per square inch."

"Many mountain belts have developed cleavages and associated shortening strains of 50% to 70% ... with rates of shortening between 0.9 cm per year and 0.25 cm per year."

4) Suppose you use the tire pressure gauge at the gas station to fill a mountain bicycle tire to about 50 psi (pounds force per square inch). Use the proper conversion factor to find this air pressure in pascals. Determine at what depth in the earth you would expect the vertical stress to be the same magnitude as the pressure in the mountain bicycle tire.

5) The following equation has been used in a structural geology textbook to describe how tensile stress,  $\sigma$ , is concentrated near the tip of a joint:

$$\sigma \cong \frac{2S}{3} \frac{(2a)^2}{b} \quad (1)$$

In this equation  $S$  is the remote tensile stress pulling the joint open,  $a$  is the length, and  $b$  is the breadth of the joint. Analyze the dimensions on both sides of this equation for dimensional homogeneity and point out the inconsistency.

6) The relative displacement (slip) across some faults varies from a maximum,  $D$ , near the center to zero at the ends. The gradient in relative displacement,  $G_D$ , measures the (spatial) rate of change of relative displacement with distance along the fault. In a recent analysis reported in the literature of structural geology this gradient was described as:

$$G_D = kD^{1/2} \quad (2)$$

One might conclude that  $k$  is a numerical constant because it is reported that  $k = 0.003$  for some sandstones (note the lack of units). Analyze the dimensions of this equation and suggest what dimensions and units should be used for  $k$ .

7) Hydraulic fractures are created in an oil reservoir by increasing the fluid pressure in a well bore until the rock breaks and a fracture is driven out into the surrounding formation. Fluid carrying sand grains is pumped into this fracture to prop it open, thereby creating a highly permeable pathway for the oil or gas to reach the well. A model for hydraulic fractures estimates the horizontal component of stress,  $\sigma_h$ , near the tip of a vertical fracture to be:

$$\sigma_h \cong \frac{(P - S_h)\sqrt{a}}{\sqrt{2\pi r}} \left\{ \cos \frac{\theta}{2} \left[ 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] \right\} \quad (3)$$

Here  $P$  is the fluid pressure,  $S_h$  is the horizontal stress far from the fracture,  $a$  is the fracture length, and  $(r, \theta)$  are polar coordinates with origin at the fracture tip and  $Ox$  is parallel to the fracture plane. Analyze the dimensions of both sides of this equation and show that it is (or is not) dimensionally homogeneous.

8) The following equation is an important differential equation from continuum mechanics that is used in models of geologic structures such as faults that slip during earthquake events. It establishes the two-dimensional dynamic equilibrium of forces acting in a rock mass in the  $x$  coordinate direction in terms of the stress components,  $\sigma_{xx}$  and  $\sigma_{yx}$ , the body force component per unit volume,  $f_x$ , mass density,  $\rho$ , the displacement component,  $u_x$ , and time,  $t$ :

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + f_x = \rho \frac{\partial^2 u_x}{\partial t^2} \quad (4)$$

In this equation  $x$  and  $y$  are the two Cartesian coordinates. You may have never seen this equation, but you should know how to describe the dimensions of each physical quantity. Use dimensional analysis to prove that this is a dimensionally homogeneous equation.

9) An example of magma flowing out of a dike near the summit of Kilauea volcano, Hawaii, is shown in Figure 2.



Figure 2. Fissure eruption on Kilauea Volcano, Hawaii.

Upward flow of magma in a vertical dike may be described by the Navier-Stokes equation written for viscous flow of a constant property fluid:

$$\rho \frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z} + \eta \frac{\partial^2 v_z}{\partial x^2} + \rho g \quad (5)$$

Here the  $x$ -coordinate is perpendicular to the dike plane and the flow direction which is  $z$ . This description of the flow field is limited to constant density,  $\rho$ , constant viscosity,  $\eta$ , and constant acceleration of gravity,  $g$ . The left-hand side is equivalent to the mass times

the acceleration of the magma, and the terms on the right-hand side are equivalent the forces per unit volume acting on any element of magma. The pressure,  $p(z, t)$ , can vary with position in the direction of flow and it can vary with time. The only velocity component,  $v_z(x, t)$ , can vary with position across the direction of flow and it can vary with time.

- a) Identify the independent variables, the dependent variables, and the material properties (constants) in this description of flow.
- b) Define a set of normalized independent and dependent variables and dimensionless differential operators for this version of the Navier-Stokes equation and substitute them into this equation.
- c) Identify the three groups of constants in the normalized equation for viscous flow in a dike and relate these groups to the forces per unit volume acting on the magma. Analyze the dimensions of each group of constants.
- d) Rearrange the groups of constants in the normalized equation for viscous flow in a dike so all the terms are dimensionless and, in doing so, define the Reynolds number and the Froude number. Indicate the appropriate ratio of forces that corresponds to each dimensionless grouping.
- e) Consider a dike that is 1 m wide and filled with magma of viscosity of  $\sim 10^5 \text{ Pa} \cdot \text{s}$  and density  $\sim 2.75 \times 10^3 \text{ kg/m}^3$ . These values are typical of basaltic dikes and magmas observed on Hawaiian volcanoes. Magma flow rates in propagating dikes of  $\sim 0.1 \text{ m/s}$  have been inferred from the migration of earthquakes along the rift zones of Hawaiian volcanoes. Show how you convert to base units and write a homogeneous equation for Reynolds number. Use this estimate of Reynolds number to infer if the magma flow is laminar or turbulent.