3.2 Noise

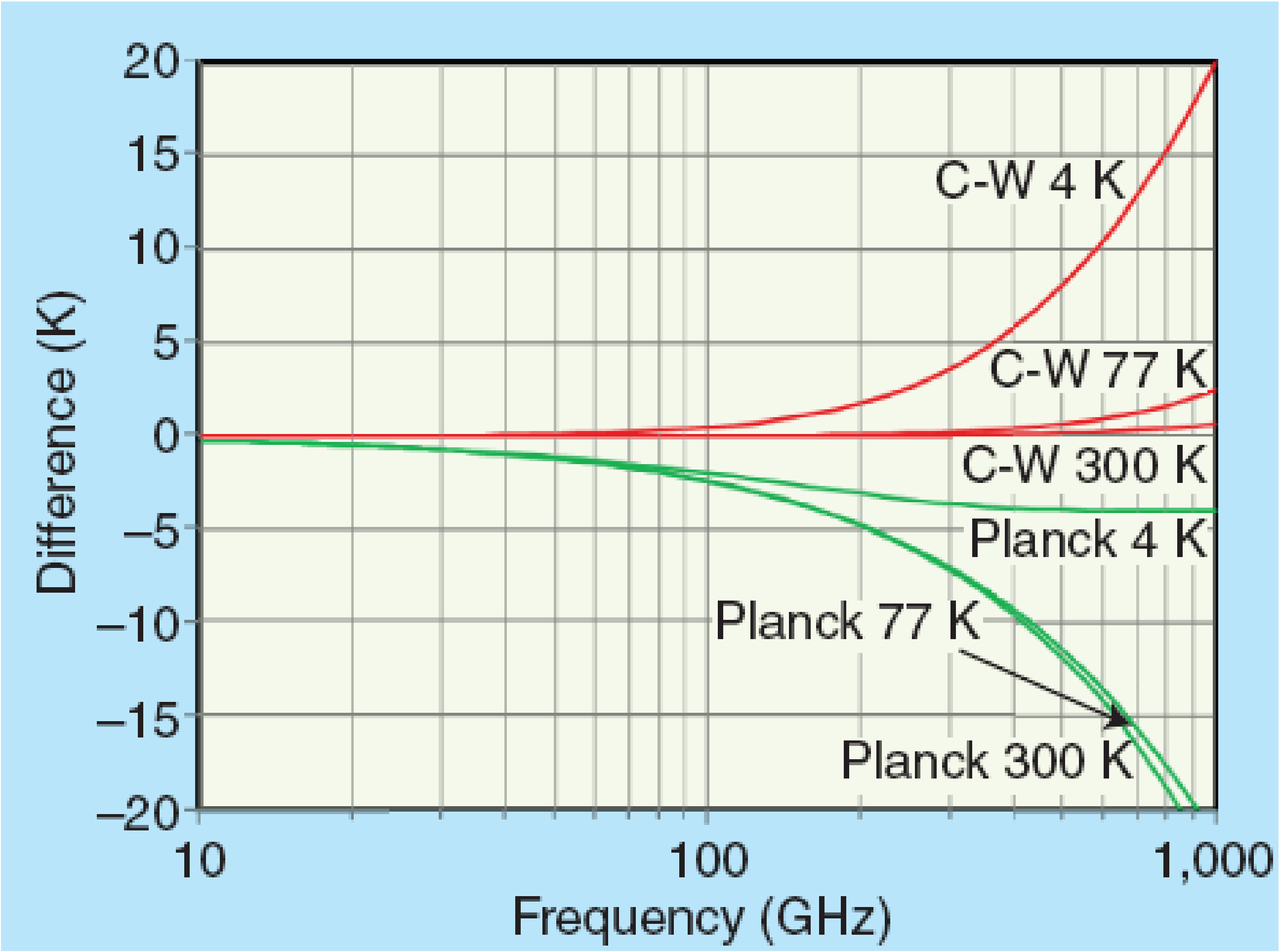
Outline

- Thermal noise, noise power and noise temperature
- Noise in RLC single-ports
- Noise in diodes and photodiodes
- 2-port and multi-port noise parameters
- Noise temperature and noise parameter measurements

Thermal noise, noise power, noise temp

- 2 definitions of T_n
  - $T_n \equiv T$ of resistor for given p_n
  - $T_n \equiv \frac{p_n}{k}$
- 3 definitions of noise power density, p_n
  - Planck, $$p_n^{Planck} = kT \left[\frac{\frac{hf}{kT}}{e^{\frac{hf}{kT}} - 1} \right]$$
  - Rayleigh-Jeans (from Planck), $p_n^{R-J} = kT$
  - Callen-Welton $$p_n^{C-W} = p_n^{Planck} + p_n^{vac} = kT \left[\frac{\frac{hf}{kT}}{e^{\frac{hf}{kT}} - 1} \right] + \frac{hf}{2}$$

Difference between T_n and T for a resistor



$$T_n^{R-J} = T$$

$$T_n^{Planck} = T\left[\frac{\frac{hf}{kT}}{e^{\frac{hf}{kT}} - 1}\right]$$

$$T_n^{C-W} = T\left[\frac{\frac{hf}{kT}}{e^{\frac{hf}{kT}} - 1}\right] + \frac{hf}{2k}$$

A.R. Kerr and J. Randa, " Thermal Noise and Noise Measurements –
A 2010 Update," IEEE Microwave Magazine, pp.40-52, October 2010.

Lossy Single-Port Noise: Resistor

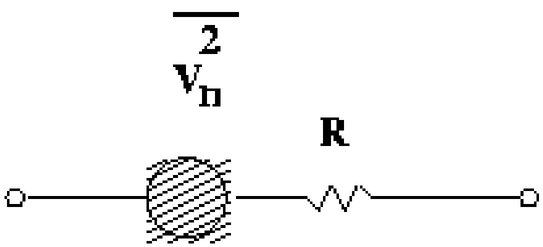
$$\overline{v_n^{\,2}} = 4kT\Delta f R$$

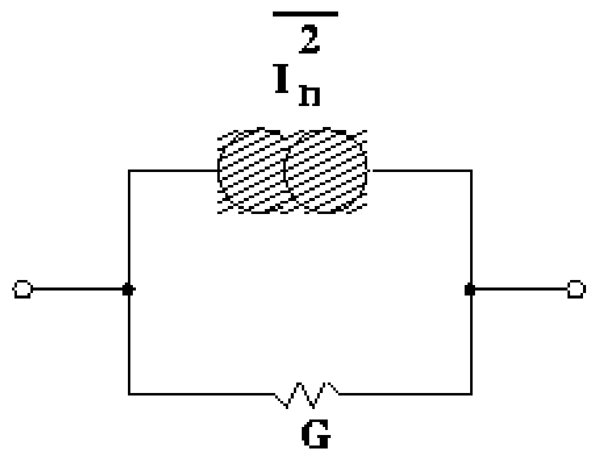
$$R = \frac{1}{G}$$

$$k = 1.38 \times 10^{-23}\ JK^{-1}$$

$$\overline{i_n^{\,2}} = 4kT\Delta f G$$

$$v_n(f) = \sqrt{\frac{R}{1k\Omega}} \times 4.06\frac{nV}{\sqrt{Hz}}$$

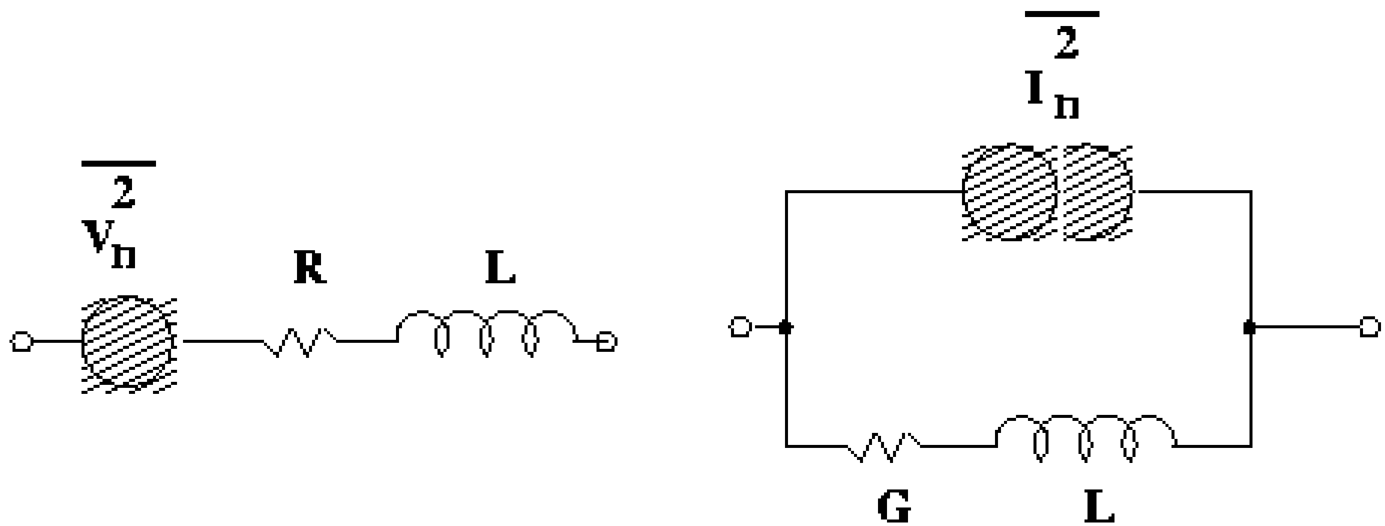




Lossy Single-Port Resistor + Inductor

$$\overline{v_n^2} = 4kT\Delta f R \qquad R = \frac{1}{G}$$

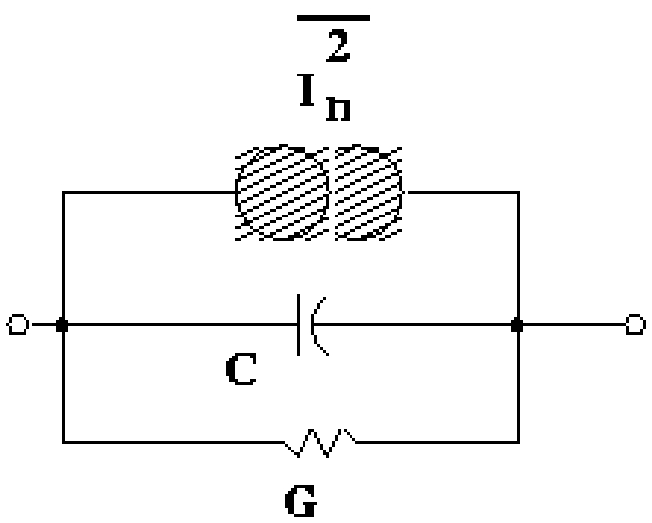
$$\overline{i_n^2} = \frac{\overline{v_n^2}}{R^2 + \omega^2 L^2} = \frac{4kT\Delta f G}{1 + \omega^2 L^2 G^2}$$

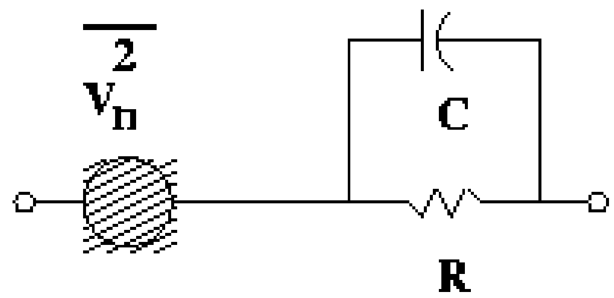


Lossy Single-Port Noise: Resistor || Capacitor

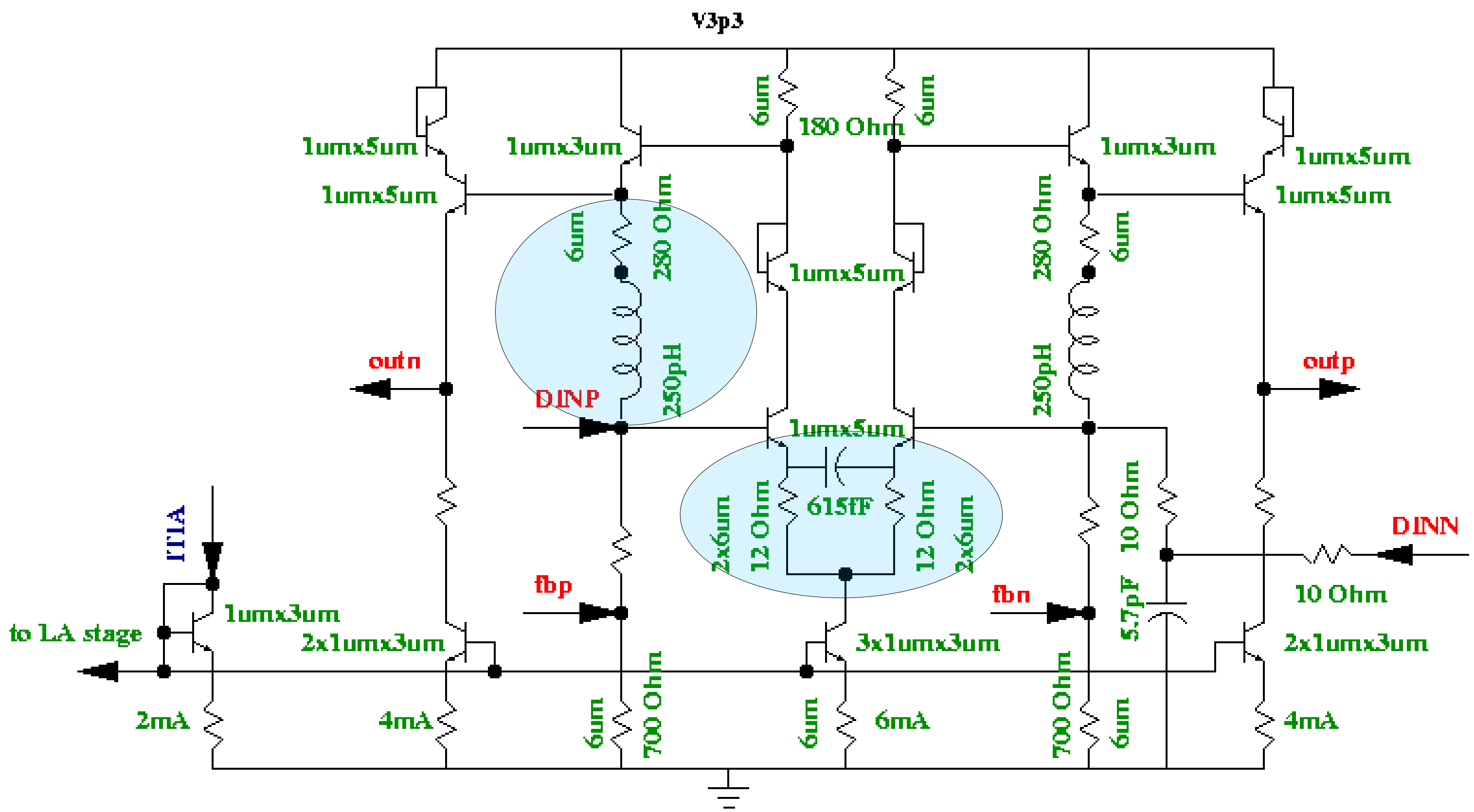
$$\overline{i_n^2} = 4kT\Delta f G \qquad R = \frac{1}{G}$$

$$\overline{v_n^2} = \frac{\overline{i_n^2}}{G^2 + \omega^2 C^2} = \frac{4kT\Delta f R}{1 + \omega^2 C^2 R^2}$$





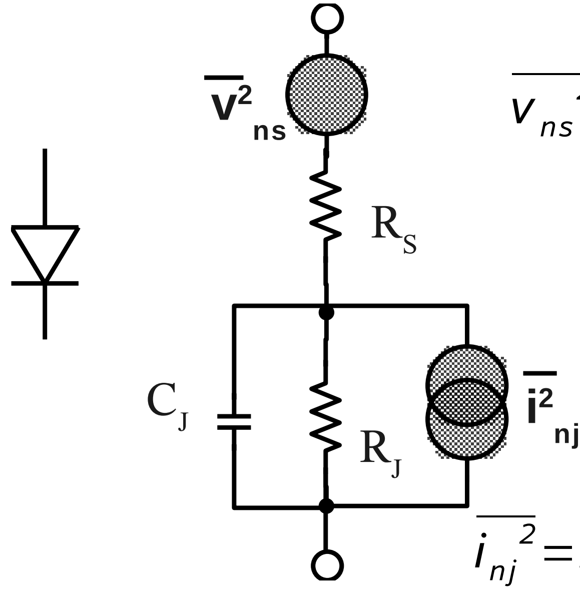
Example: series L-R series + shunt C-R



Total current = 18mA + 2mA shared with TIA and t-line transmitter

Diode Noise

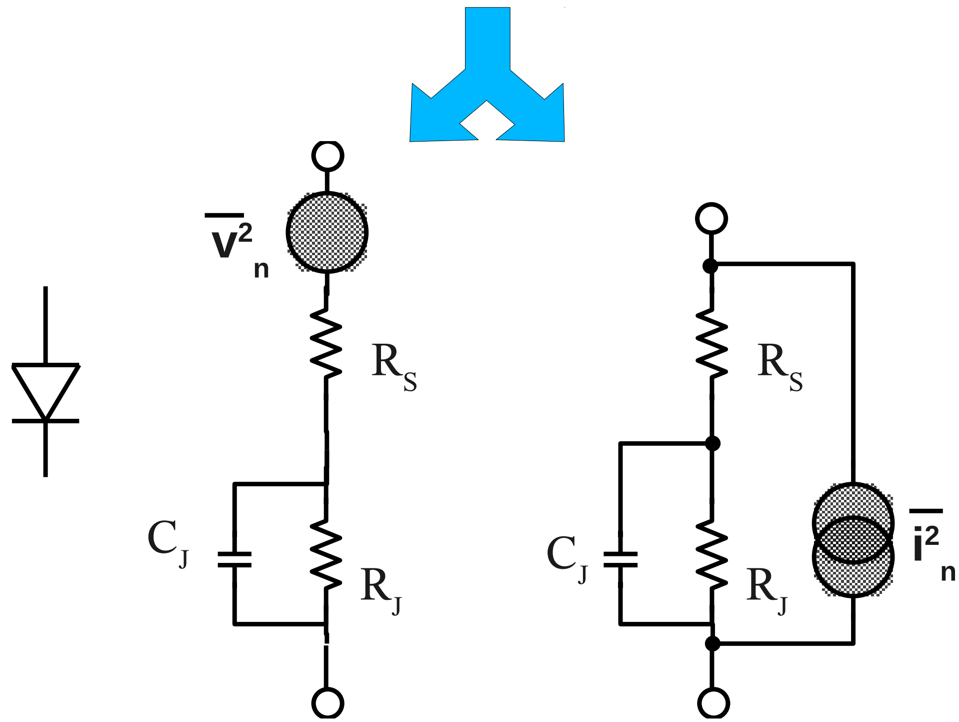
- Intrinsic noise sources i_{nj}, v_{ns}



$$\overline{v_{ns}^{\ 2}} = 4\,k\,T\,\Delta f\,R_s$$

$$\overline{i_{nj}^{\ 2}} = 2\,q\,\Delta f\,I_d + K_1\,\Delta f\,\frac{I_d^{\alpha}}{f}$$

- Equivalent input thermal noise sources: v_n or i_n
- Equivalent noise conductance



$$G_e \stackrel{\text{def}}{=} \frac{\overline{i_n^2}}{4kT\,\Delta f} \approx \frac{q\,I_d}{2\,k\,T} = \frac{1}{2\,R_j}$$

Photodetectors

- PIN diodes: M = 1, F(M) = 1
- APDs: M = 10..50, F(M) = 5 .. 10

where: *M* is the multiplication factor and

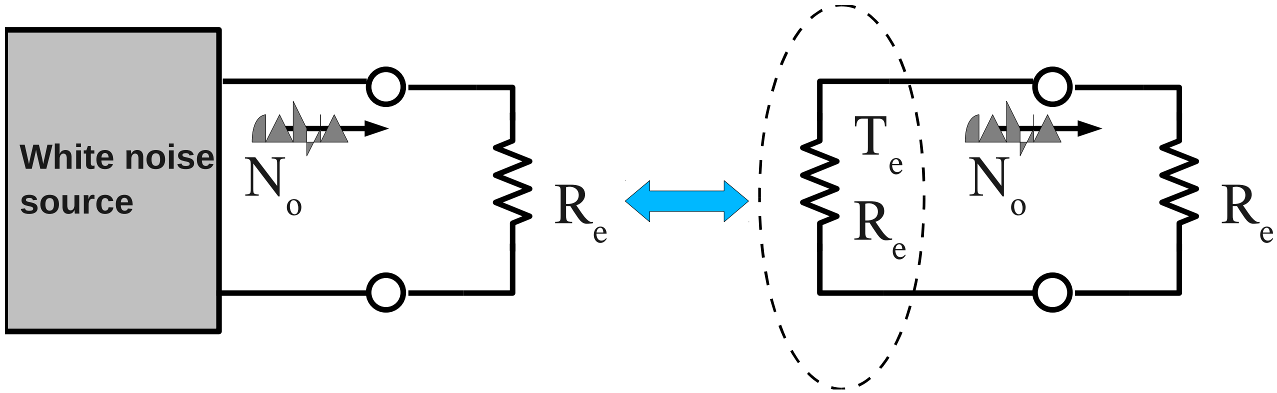
F(M) is the noise factor of the photo-diode

$I_{ph}=RMP$; $R=$ diode responsivity and $P=$ optical power

$$\overline{i_{n,ph}^{2}}=2qI\Delta f=2q\Delta f\left(RMP+MI_{dark}\right)MF(M)$$

$$G_{seq}=\frac{\left(RP+I_{dark}\right)M^{2}F(M)}{\frac{2kT}{q}} \qquad P_{min}=\frac{1}{R}\times\sqrt{2q\Delta f\,I_{dark}F(M)}$$

Noise source T_e and *ENR*



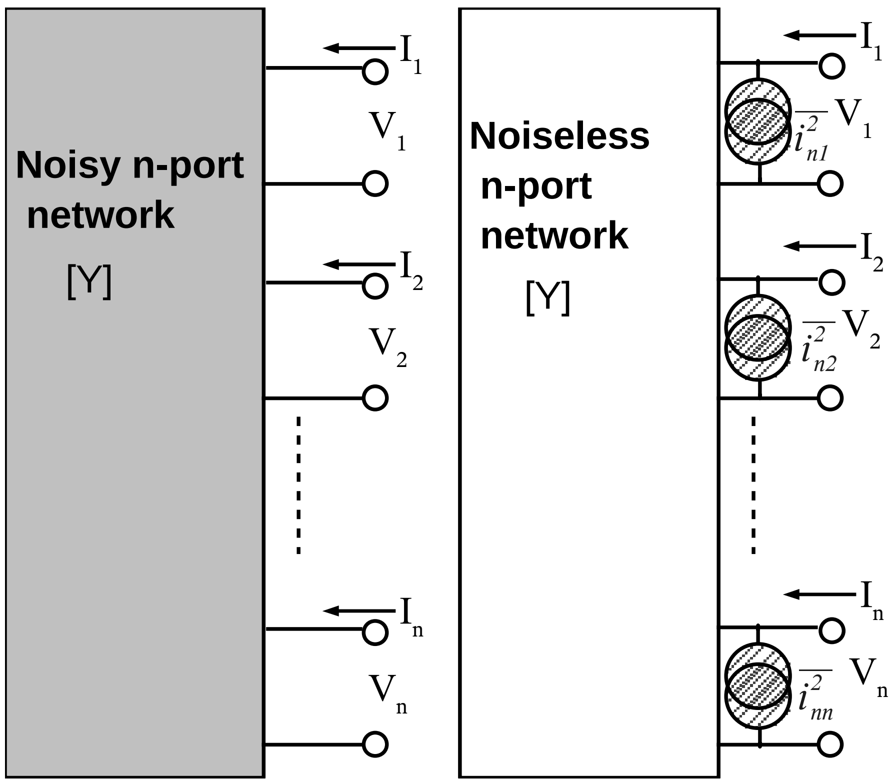
- T_e = equivalent (thermal) noise temperature of a white noise source
- *ENR* = excess noise ratio $$ENR(dB) = 10\log_{10}\left(\frac{T_e - T_0}{T_0}\right)$$
- T_0 = reference temperature, typically 290 ^oK.

Multi-port noise representations: Y-params

- One noise current source exiting each port
- Y correlation matrix

$$\begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & . & . & Y_{1n} \\ Y_{21} & Y_{22} & . & . & Y_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ Y_{n1} & Y_{n2} & . & . & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix} + \begin{bmatrix} i_{n1} \\ i_{n2} \\ . \\ . \\ i_{nn} \end{bmatrix}$$

$$[C_y] = \begin{bmatrix} \langle i_{n1}, i_{n1}^* \rangle & \langle i_{n1}, i_{n2}^* \rangle & . & . & \langle i_{n1}, i_{nn}^* \rangle \\ \langle i_{n2}, i_{n1}^* \rangle & \langle i_{n2}, i_{n2}^* \rangle & . & . & \langle i_{n2}, i_{nn}^* \rangle \\ . & . & . & . & . \\ . & . & . & . & . \\ \langle i_{nn}, i_{n1}^* \rangle & \langle i_{nn}, i_{n2}^* \rangle & . & . & \langle i_{nn}, i_{nn}^* \rangle \end{bmatrix}$$



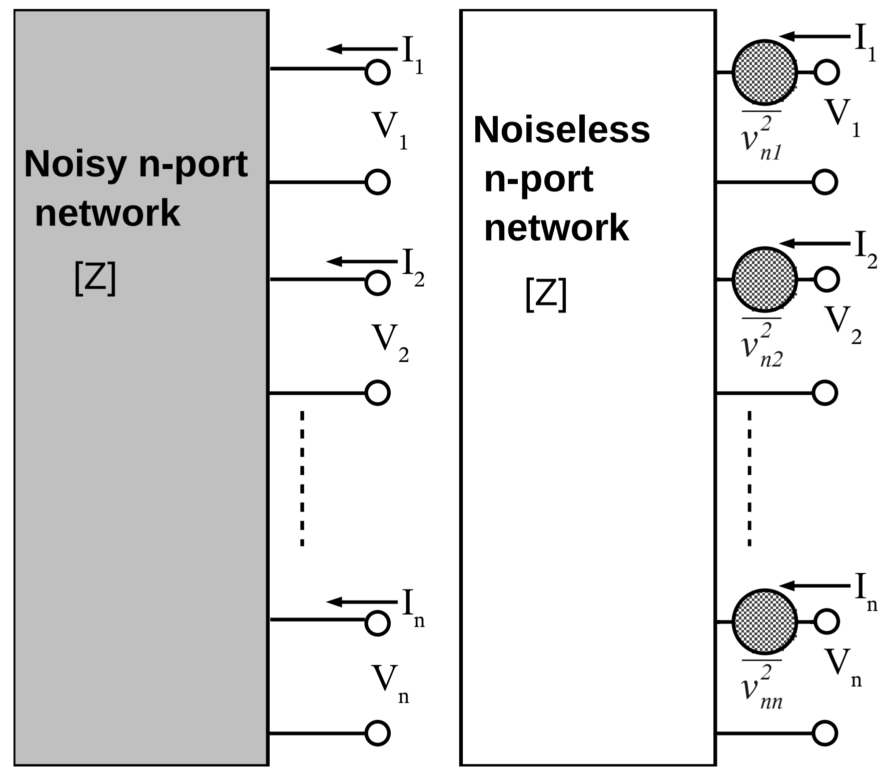
For passive network: $[C_y] = 4\,kT\,\Delta f\,\mathrm{Re}\{[Y]\}$

Multi-port noise representations: Z-params

- One noise voltage source at each port
- Z correlation matrix

$$\begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & . & . & Z_{1n} \\ Z_{21} & Z_{22} & . & . & Z_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ Z_{n1} & Z_{n2} & . & . & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix} + \begin{bmatrix} v_{n1} \\ v_{n2} \\ . \\ . \\ .v_{nn} \end{bmatrix}$$

$$[C_z] = \begin{bmatrix} \langle v_{n1}, v_{n1}^* \rangle & \langle v_{n1}, v_{n2}^* \rangle & . & . & \langle v_{n1}, v_{nn}^* \rangle \\ \langle v_{n2}, v_{n1}^* \rangle & \langle v_{n2}, v_{n2}^* \rangle & . & . & \langle v_{n2}, v_{nn}^* \rangle \\ . & . & . & . & . \\ . & . & . & . & . \\ \langle v_{nn}, v_{n1}^* \rangle & \langle v_{nn}, v_{n2}^* \rangle & . & . & \langle v_{nn}, v_{nn}^* \rangle \end{bmatrix}$$



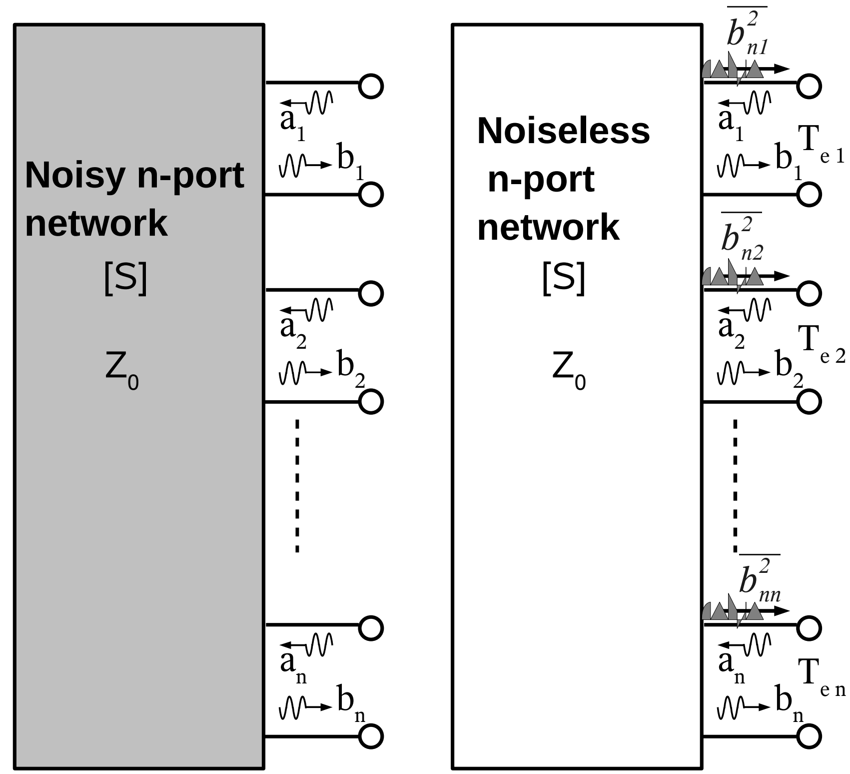
For passive network: $[C_Z] = 4\,kT\,\Delta f\,\mathrm{Re}\{[Z]\}$

Multi-port noise representations: noise wave

- One noise wave emanating from each port
- wave correlation matrix

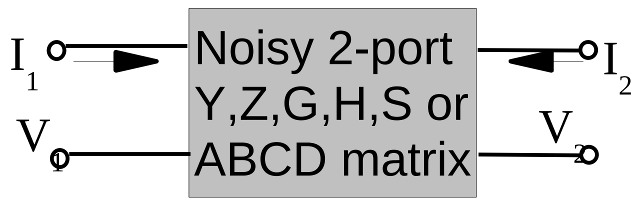
$$\begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & . & . & S_{1n} \\ S_{21} & S_{22} & . & . & S_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ S_{n1} & S_{n2} & . & . & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ a_n \end{bmatrix} + \begin{bmatrix} b_{n1} \\ b_{n2} \\ . \\ . \\ .\, b_{nn} \end{bmatrix}$$

$$[C_S] = \begin{bmatrix} \langle b_{n1}, b_{n1}^* \rangle & \langle b_{n1}, b_{n2}^* \rangle & . & . & \langle b_{n1}, b_{nn}^* \rangle \\ \langle b_{n2}, b_{n1}^* \rangle & \langle b_{n2}, b_{n2}^* \rangle & . & . & \langle b_{n2}, b_{nn}^* \rangle \\ . & . & . & . & . \\ . & . & . & . & . \\ \langle b_{nn}, b_{n1}^* \rangle & \langle b_{nn}, b_{n2}^* \rangle & . & . & \langle b_{nn}, b_{nn}^* \rangle \end{bmatrix}$$

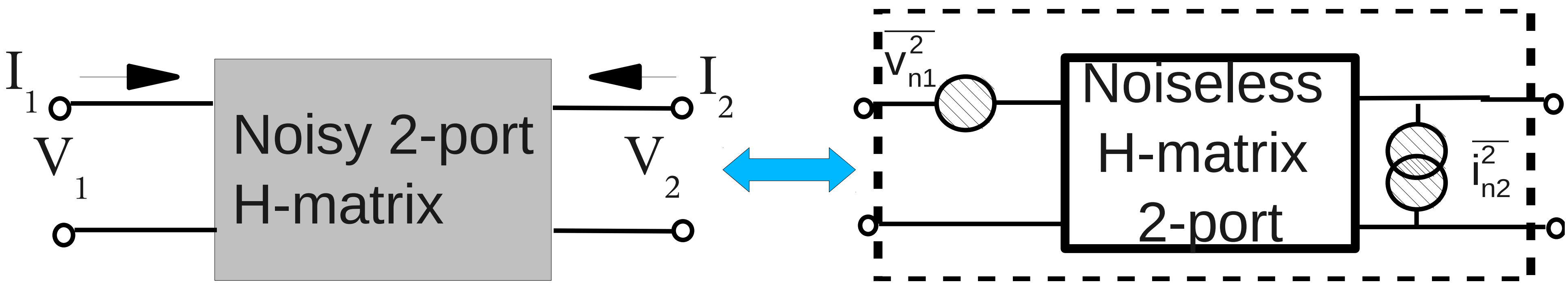


For passive network: $[C_S] = kT\Delta f([U] - [S][S]^+)$

Other Noisy 2-Port (only) Correlation Matrices



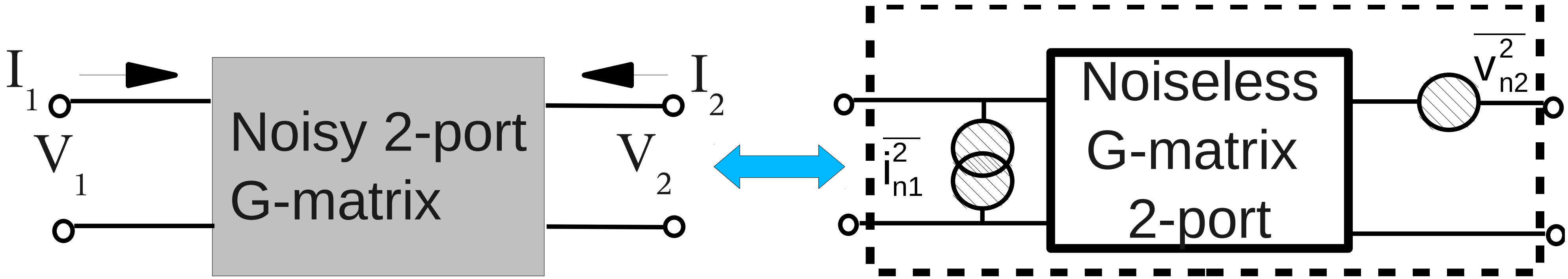
Noisy two-port with internal noise sources which can be represented as:



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} v_{n1} \\ i_{n2} \end{bmatrix}$$

$$[C_h] = \begin{bmatrix} \langle v_{n1}, \ v_{n1}^* \rangle & \langle v_{n1}, \ i_{n2}^* \rangle \\ \langle i_{n2}, \ v_{n1}^* \rangle & \langle i_{n2}, \ i_{n2}^* \rangle \end{bmatrix}$$

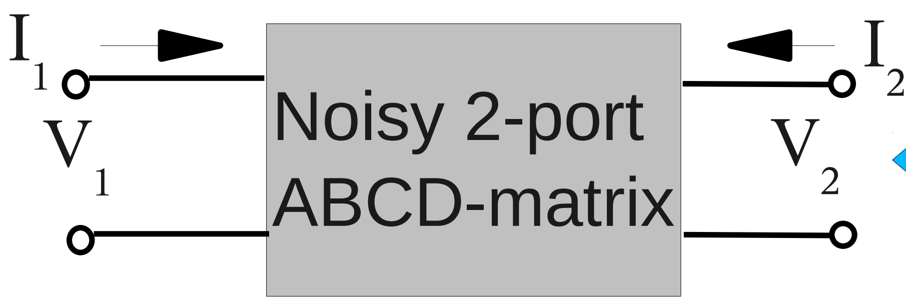
Two-port Correlation Matrices: G

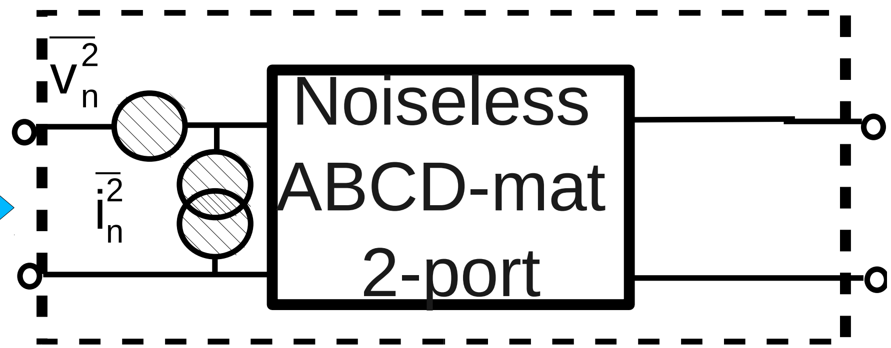


$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} i_{n1} \\ v_{n2} \end{bmatrix}$$

$$\left[C_g\right] = \begin{bmatrix} \langle i_{n1} \; i_{n1}^* \rangle & \langle i_{n1} \; v_{n2}^* \rangle \\ \langle v_{n2} \; i_{n1}^* \rangle & \langle v_{n2} \; v_{n2}^* \rangle \end{bmatrix}$$

Correlation Matrices: ABCD





$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} + \begin{bmatrix} v_n \\ i_n \end{bmatrix}$$

$$\left[C_A \right] = \begin{bmatrix} \langle v_n \ v_n^* \rangle & \langle v_n \ i_n^* \rangle \\ \langle i_n \ v_n^* \rangle & \langle i_n \ i_n^* \rangle \end{bmatrix}$$

Noise Correlation Matrix Transformations

$$[C_z]=\begin{bmatrix}1 & -Z_{11}\\ 0 & -Z_{22}\end{bmatrix}[C_A]\begin{bmatrix}1 & 0\\ -Z_{11}^{*} & -Z_{22}^{*}\end{bmatrix};\ [C_y]=\begin{bmatrix}-Y_{11} & 1\\ -Y_{22} & 0\end{bmatrix}[C_A]\begin{bmatrix}-Y_{11}^{*} & -Y_{22}^{*}\\ 1 & 0\end{bmatrix}$$

$$[C_z]=[Z][C_y][Z]^{+};\ [C_z]=([U]+[Z])[C_S]([U]+[Z])^{+}$$

$$[C_y]=[Y][C_z][Y]^{+};\ [C_y]=([U]+[Y])[C_S]([U]+[Y])^{+}$$

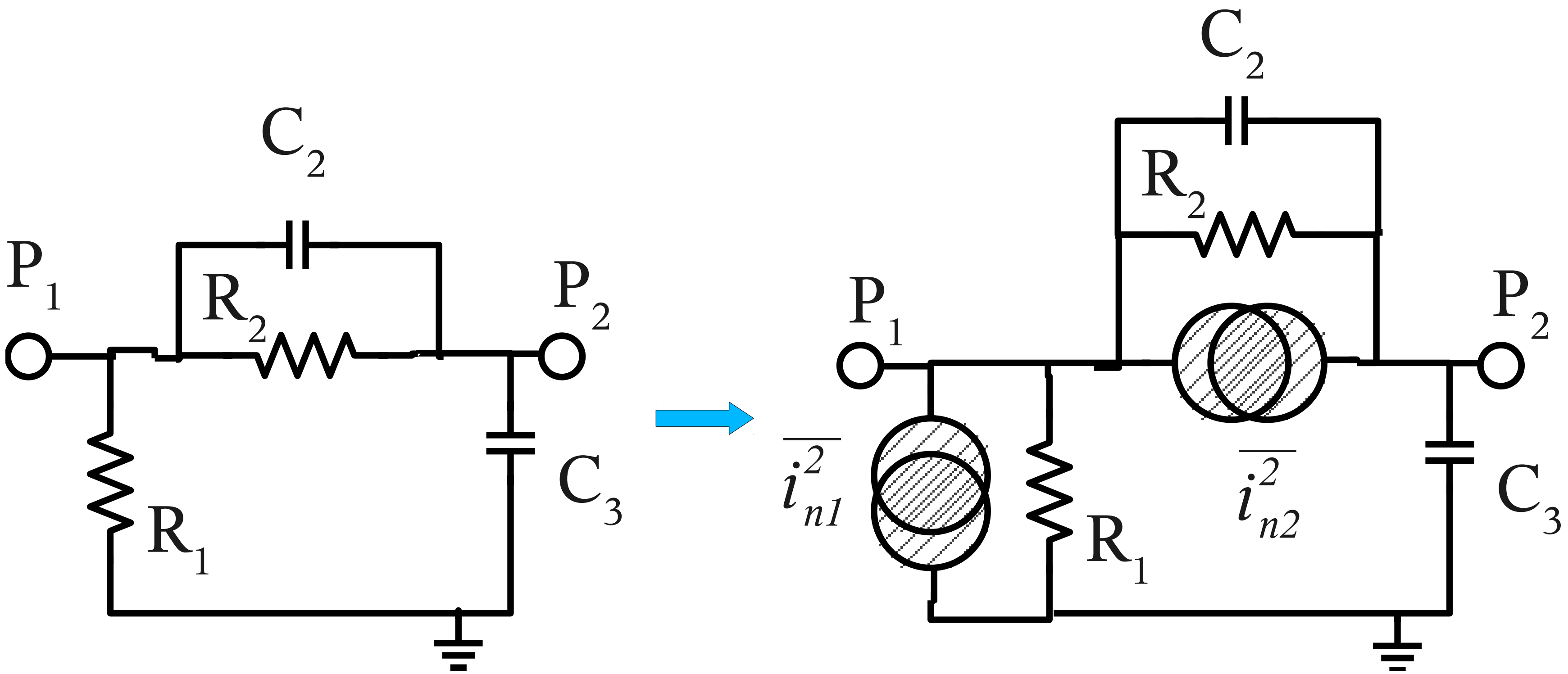
$$[C_S]=\frac{1}{4}([U]+[S])[C_y]([U]+[S])^{+};\ [C_S]=\frac{1}{4}([U]-[S])[C_Z]([U]-[S])^{+}$$

$$Im(c_{nn})=Im(\overline{v_{nn}^{2}})=Im(\overline{i_{nn}^{2}})=Im(\overline{b_{nn}^{2}})=0 \qquad c_{ki}=c_{ik}^{*}$$

Method to form the noise admittance correlation matrix of an *n*-port

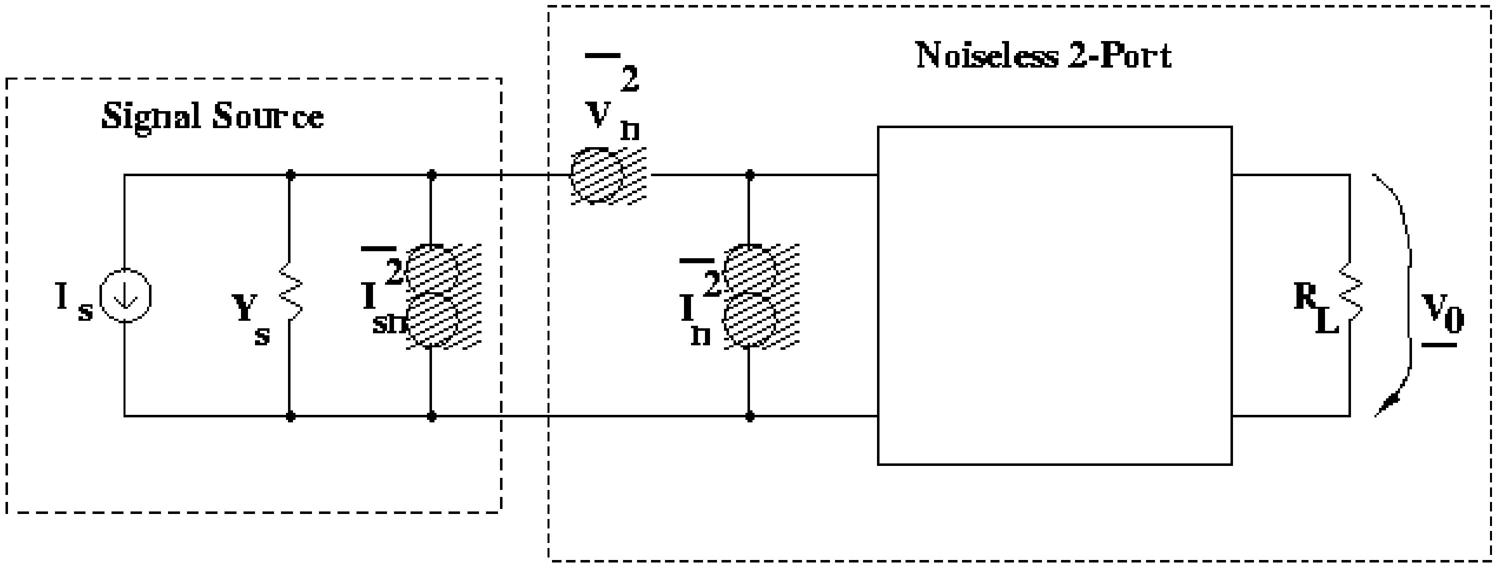
- $C_y[i,i]$ = sum of all noise currents connected to node *i*
- $C_y[i,j]$ = negative of sum of all noise currents between *i* and *j*
  - If j = GND, the noise current is only added to $C_y[i,i]$

Example:



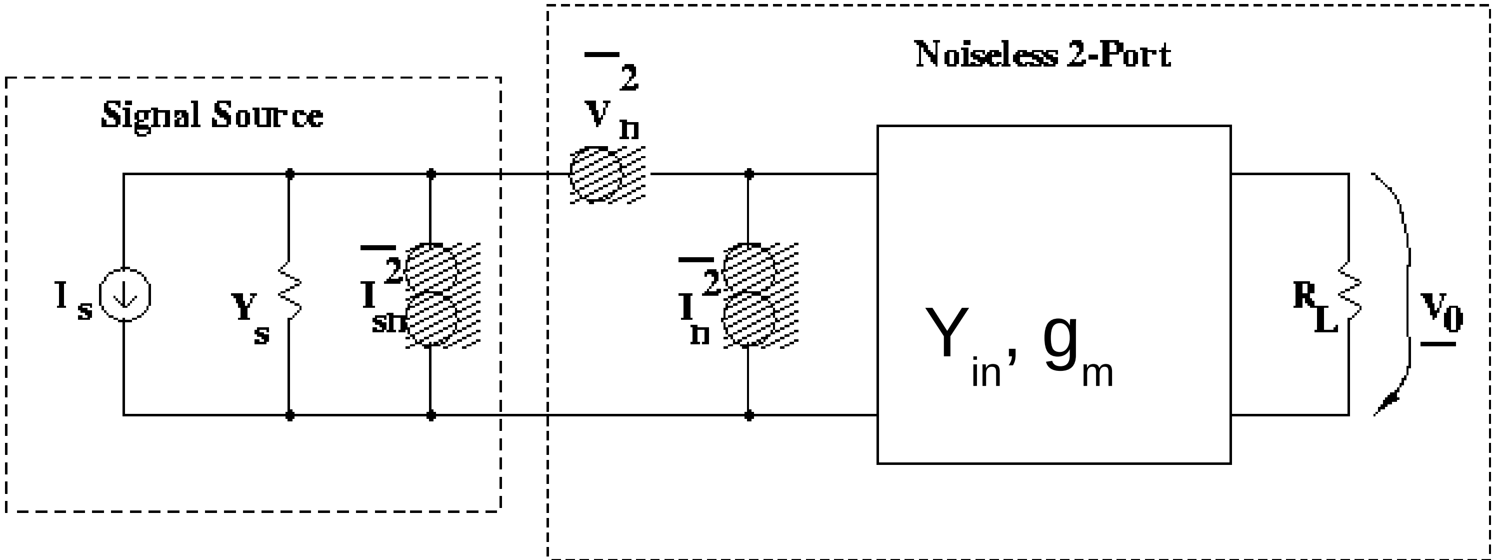
$$[C_Y] = \begin{bmatrix} \overline{i_{n1}^2} + \overline{i_{n2}^2} & -\overline{i_{n2}^2} \\ -\overline{i_{n2}^2} & \overline{i_{n2}^2} \end{bmatrix} = 4kT\,\Delta f \begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 \end{bmatrix}$$

Noise Factor of a 2-port driven by a signal source



- 2-port with input equivalent noise sources (sign is not important)
- Noise of signal source is typically thermal: $i^2_{sn} = 4kT\Delta f \, Re\{Y_s\}$
- Signal source may be a photodiode: $i^2_{sn} = 4kT\Delta f G_{seq}$ **;** $Y_s = j\omega C_D$
- G_{seq} is due to dark current and optical power.

Deriving the Noise Factor of the 2-Port



- Find input shortcircuit current i_{scn} due to i_{sn}, i_n, v_n by superposition

- Input short circuit current i_{scsn} due to i_{sn}

- Determine F as the ratio: $$F = \frac{\overline{i_{scn}^2}}{\overline{i_{scsn}^2}}$$

Two-Port Noise Parameters (1)

Noise admittance formalism

$$v_n = v_n ; i_n = i_u + i_c = i_u + Y_{cor} v_n$$

$$R_n = \frac{\overline{v_n^2}}{4kT\Delta f} \qquad G_u = \frac{\overline{i_u^2}}{4kT\Delta f}$$

$$Y_{cor} = \frac{\overline{i_n v_n^x}}{\overline{v_n^2}} = G_{cor} + jB_{cor}$$

Noise resistance formalism

$$i_n = i_n ; v_n = v_u + v_c = v_u + Z_{cor} i_n$$

$$R_u = \frac{\overline{v_u^2}}{4kT\Delta f} \qquad G_n = \frac{\overline{i_n^2}}{4kT\Delta f}$$

$$Z_{cor} = \frac{\overline{v_n i_n^x}}{\overline{i_n^2}} = R_{cor} + jX_{cor}$$

Two-Port Noise Parameters (2)

$$F = 1 + \frac{\overline{v_n^2}\,|Y_{cor} + Y_s|^2}{\overline{i_{sn}^2}} + \frac{\overline{i_u^2}}{\overline{i_{sn}^2}}$$

$$F = 1 + \frac{\overline{i_n^2}\,|Z'_{cor} + Z_s|^2}{\overline{v_{sn}^2}} + \frac{\overline{v_u^2}}{\overline{v_{sn}^2}}$$

$$F = 1 + \frac{R_n}{G_s}|Y_{cor} + Y_s|^2 + \frac{G_u}{G_s}$$

$$F = 1 + \frac{G_n}{R_s}|Z_{cor} + Z_s|^2 + \frac{R_u}{R_s}$$

$$F = F_{MIN} + \frac{R_n}{G_s}|Y_s - Y_{sopt}|^2$$

$$F = F_{MIN} + \frac{G_n}{R_s}|Z_s - Z_{sopt}|^2$$

Constant Noise Circles on Smith Chart

Two-Port Noise Parameters (3)

$$Y_{sop} = G_{sopt} + jB_{sopt}$$

$$F_{min} = 1 + 2R_n(G_{cor} + G_{sopt})$$

$$G_{sopt} = \sqrt{G_{cor}^2 + \frac{G_u}{R_n}} ; B_{sopt} = -B_{cor}$$

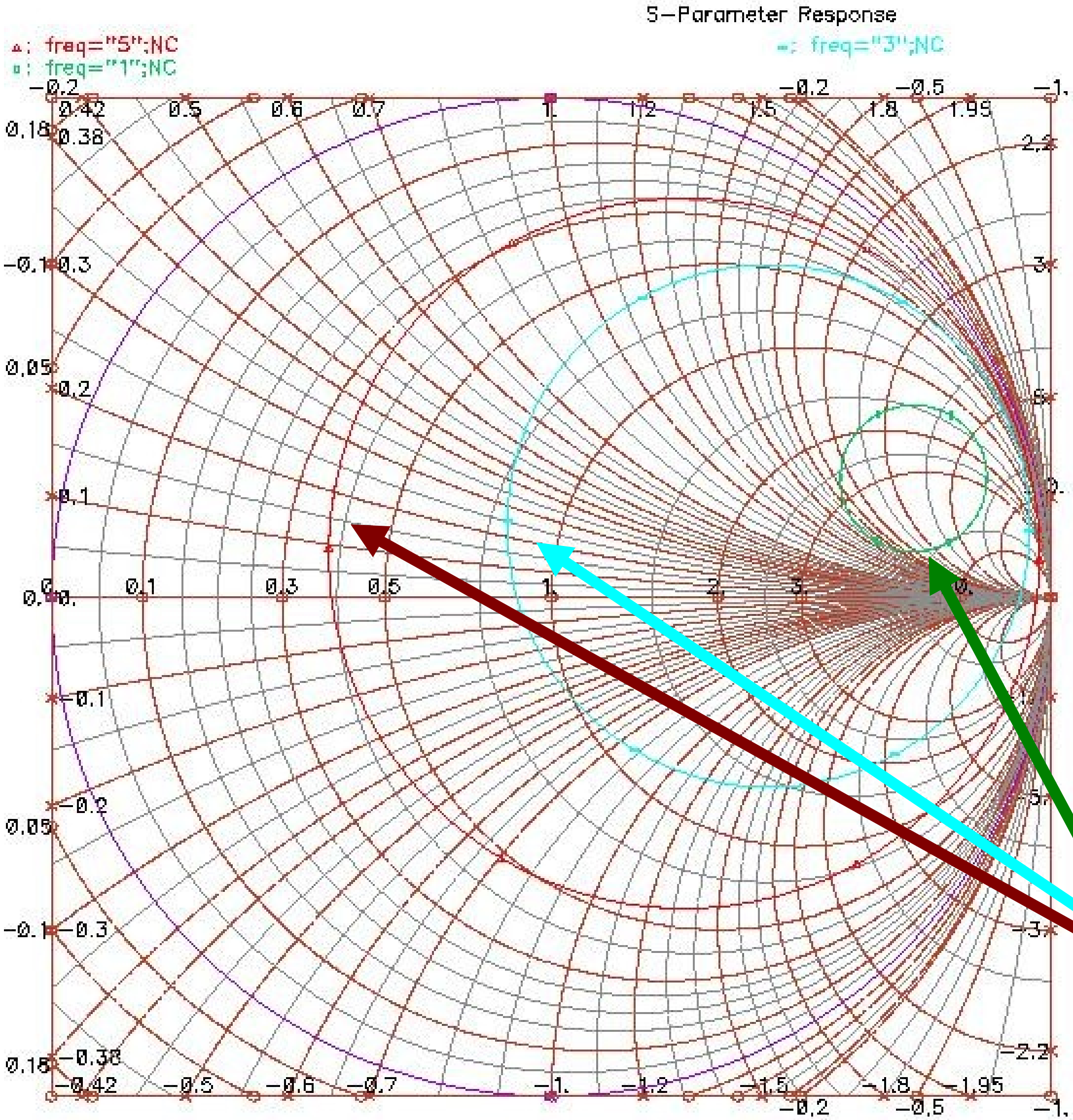
$$Z_{sopt} = R_{sopt} + jX_{sopt}$$

$$F_{min} = 1 + 2G_n(R_{cor} + R_{sopt})$$

$$R_{sopt} = \sqrt{R_{cor}^2 + \frac{R_u}{G_n}} ; X_{sopt} = -X_{cor}$$

$$Y_{sop} = \frac{1}{Z_{sopt}} \qquad Y_{cor}^{x} = \frac{Z_{cor}}{|Z_{sopt}|^2} \qquad R_n = G_n |Z_{sopt}|^2 \qquad G_u = \frac{R_u}{|Z_{sopt}|^2}$$

Two-Port Noise Circles



$$F(\Gamma_s) = F_{min} + 4\frac{R_n}{Z_0}\frac{|\Gamma_s - \Gamma_{sopt}|^2}{(1-|\Gamma_S|^2)|1+\Gamma_{sopt}|^2}$$

$$r_i = \frac{\sqrt{N_i^2 + N_i(1-|\Gamma_{sopt}|^2)}}{1+N_i}$$

$$C_i = \frac{\Gamma_{sopt}}{1+N_i}$$

$$N_i = \frac{F_i - F_{min}}{4\,r_n}|1+\Gamma_{sopt}|^2$$

Constant Noise Circles on Smith Chart

Correlation Matrix and 2-Port Noise Parameters

$$[C_A]=\begin{bmatrix}\langle v_{n1}\; v_{n1}^{*}\rangle & \langle v_{n1}\; i_{n1}^{*}\rangle\\ \langle i_{n1}\; v_{n1}^{*}\rangle & \langle i_{n1}\; i_{n1}^{*}\rangle\end{bmatrix}=\begin{bmatrix}C_{uu*} & C_{ui*}\\ C_{iu*} & C_{ii*}\end{bmatrix}=\begin{bmatrix}R_n & \frac{F_{MIN}-1}{2}-R_n Y_{opt}^{*}\\ \frac{F_{MIN}-1}{2}-R_n Y_{opt} & R_n|Y_{opt}|^2\end{bmatrix}$$

$$Y_{opt}=\sqrt{\frac{C_{ii^*}}{C_{uu^*}}-\left[\Im\left(\frac{C_{ui^*}}{C_{uu^*}}\right)\right]^2}+j\,\Im\left(\frac{C_{ui^*}}{C_{uu^*}}\right)$$

$$F_{MIN}=1+\frac{C_{ui^*}+C_{uu^*}Y_{opt}^{*}}{kT}$$

$$R_n=\frac{C_{uu^*}}{kT}$$

From Stephen Maas' "Noise"

Correlation Matrix and 2-Port Noise Parameters

$$[C_y] = \begin{bmatrix} \langle i_{n1} \; i_{n1}^* \rangle & \langle i_{n1} \; i_{n2}^* \rangle \\ \langle i_{n2} \; i_{n1}^* \rangle & \langle i_{n2} \; i_{n2}^* \rangle \end{bmatrix} = \begin{bmatrix} G_u + R_n |Y_{11} - Y_{cor}|^2 & R_n Y_{21}^* (Y_{11} - Y_{cor}) \\ R_n Y_{21} (Y_{11}^* - Y_{cor}^*) & R_n |Y_{21}|^2 \end{bmatrix}$$

$$R_n = \frac{C_{y22}}{|Y_{21}|^2} \qquad Y_{cor} = Y_{11} - \frac{C_{y12}}{C_{y22}} Y_{21}$$

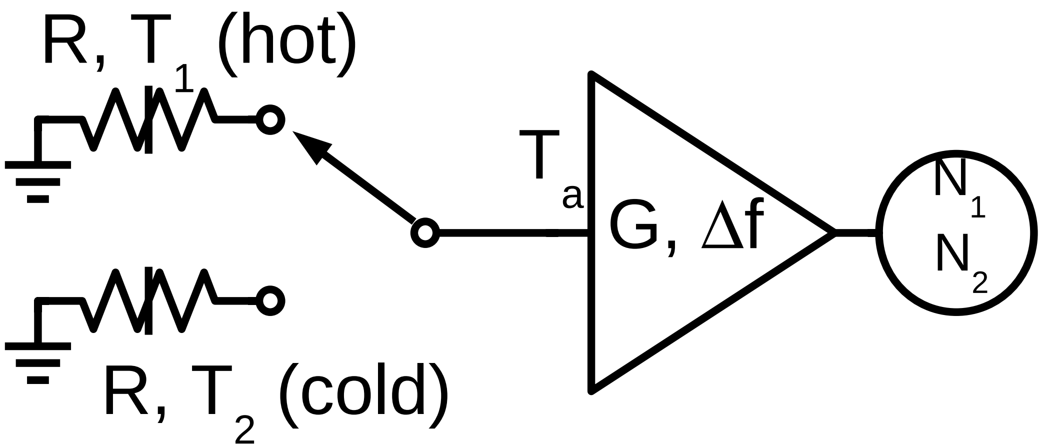
$$G_u = C_{y11} - R_n |Y_{11} - Y_{cor}|^2$$

From Stephen Maas' "Noise"

Noise in Passive 2-ports

- A lossless passive 2-port has NF_{MIN}=0 dB
- In a lossy passive 2-port the noise figure is equal to the insertion loss.
- If a noisy 2-port is cascaded with a lossless passive 2-port the minimum noise figure is preserved.
  - Hence the popularity of reactive matching networks in tuned LNA design

Two-Port Noise Temperature Measurements



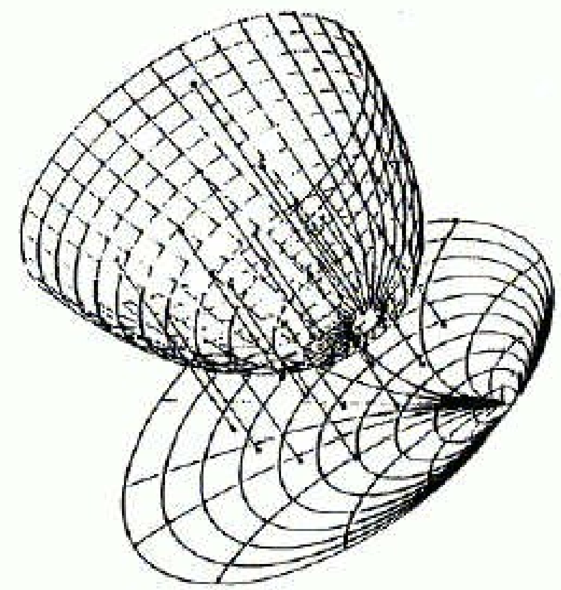
$$N_1 = GkT_1\Delta f + GkT_a\Delta f;\;\; N_2 = GkT_2\Delta f + GkT_a\Delta f$$

$$Y = \frac{N_1}{N_2} = \frac{T_1 + T_a}{T_2 + T_a};\;\; T_a = \frac{T_1 - YT_2}{Y - 1}$$

- Also know as the *Y-factor* measurement
- Measures output noise for two loads at significantly different temperatures connected at the input of the two-port

Two-Port Noise Parameter Measurements

$$F_k = \mathbf{F_{MIN}} + \frac{\mathbf{R_n}}{G_{sk}} \left| Y_{sk} - \mathbf{Y_{sopt}} \right|^2$$



Ref.: ATN Microwave NP5 System Manual

- Want small R_n for insensitivity to source noise impedance mismatch

Two-Port Noise Parameter Measurements (2)

- The 4 noise parameters are measured indirectly using a Noise Figure Meter
- Measure noise figure for 4 or more different source impedances and solve for the noise parameters:

$$F_k = \mathbf{F_{MIN}} + \frac{\mathbf{R_n}}{G_{sk}} \left| Y_{sk} - \mathbf{Y_{sopt}} \right|^2$$

k = 1..4..

- Choice of Y_{sk} is critical to meas. accuracy

Summary

- *NF* is a function of signal source impedance and of the 4 noise parameters of the 2-port
- For a 2-port there is an optimum signal source impedance which minimizes *NF*
- To achieve NF_{MIN} the 2-port must be "noise-matched"
- Noise matching can be achieved using negative feedback or (lossless) impedance transformation techniques
- The noise figure of a passive 2-port is equal to its insertion loss.