Solution:

According to (2.58), the field-induced impermeability change due to the Pockels effect is

$$\Delta\eta_\alpha(\boldsymbol{E}_0) = \sum_k r_{\alpha k} E_{0k},$$

which can be expressed in the matrix form as

$$\begin{pmatrix} \Delta\eta_1 \\ \Delta\eta_2 \\ \Delta\eta_3 \\ \Delta\eta_4 \\ \Delta\eta_5 \\ \Delta\eta_6 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{33} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} \begin{pmatrix} E_{0x} \\ E_{0y} \\ E_{0z} \end{pmatrix}.$$

Using the given nonvanishing Pockels coefficients for $LiNbO_3$, we have

$$\begin{pmatrix} \Delta\eta_1 \\ \Delta\eta_2 \\ \Delta\eta_3 \\ \Delta\eta_4 \\ \Delta\eta_5 \\ \Delta\eta_6 \end{pmatrix} = \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{42} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix} \begin{pmatrix} E_{0x} \\ E_{0y} \\ E_{0z} \end{pmatrix} = \begin{pmatrix} -r_{22}E_{0y} + r_{13}E_{0z} \\ r_{22}E_{0y} + r_{13}E_{0z} \\ r_{33}E_{0z} \\ r_{42}E_{0y} \\ r_{42}E_{0x} \\ -r_{22}E_{0x} \end{pmatrix}.$$

By the index contraction rule, $\Delta\eta_1 = \Delta\eta_{xx}$, $\Delta\eta_2 = \Delta\eta_{yy}$, $\Delta\eta_3 = \Delta\eta_{zz}$, $\Delta\eta_4 = \Delta\eta_{yz} = \Delta\eta_{zy}$, $\Delta\eta_5 = \Delta\eta_{zx} = \Delta\eta_{xz}$, $\Delta\eta_6 = \Delta\eta_{xy} = \Delta\eta_{yx}$. Using (2.62), we find

$$\Delta\epsilon_{xx} = -\epsilon_0 n_x^4 \Delta\eta_{xx} = \epsilon_0 n_o^4 r_{22} E_{0y} - \epsilon_0 n_o^4 r_{13} E_{0z},$$

$$\Delta\epsilon_{yy} = -\epsilon_0 n_y^4 \Delta\eta_{yy} = -\epsilon_0 n_o^4 r_{22} E_{0y} - \epsilon_0 n_o^4 r_{13} E_{0z},$$

$$\Delta\epsilon_{zz} = -\epsilon_0 n_z^4 \Delta\eta_{zz} = -\epsilon_0 n_e^4 r_{33} E_{0z},$$

$$\Delta\epsilon_{yz} = \Delta\epsilon_{zy} = -\epsilon_0 n_y^2 n_z^2 \Delta\eta_{yz} = -\epsilon_0 n_o^2 n_e^2 r_{42} E_{0y},$$

$$\Delta\epsilon_{zx} = \Delta\epsilon_{xz} = -\epsilon_0 n_x^2 n_z^2 \Delta\eta_{zx} = -\epsilon_0 n_o^2 n_e^2 r_{42} E_{0x},$$

$$\Delta\epsilon_{xy} = \Delta\epsilon_{yx} = -\epsilon_0 n_x^2 n_y^2 \Delta\eta_{xy} = \epsilon_0 n_o^4 r_{22} E_{0x}.$$

Expressed in the matrix form, the field-induced permittivity change is

$$\Delta\boldsymbol{\epsilon}(\boldsymbol{E}_0) = \epsilon_0 \begin{pmatrix} n_o^4 r_{22} E_{0y} - n_o^4 r_{13} E_{0z} & n_o^4 r_{22} E_{0x} & -n_o^2 n_e^2 r_{42} E_{0x} \\ n_o^4 r_{22} E_{0x} & -n_o^4 r_{22} E_{0y} - n_o^4 r_{13} E_{0z} & -n_o^2 n_e^2 r_{42} E_{0y} \\ -n_o^2 n_e^2 r_{42} E_{0x} & -n_o^2 n_e^2 r_{42} E_{0y} & -n_e^4 r_{33} E_{0z} \end{pmatrix}.$$