Problems for Chapter 23 of 'Ultra Low Power Bioelectronics'

Problem 23.1

a) Rewrite the pair of first-order differential equations shown in Equation (23.7) as functions only of s_n, the normalized frequency variable.

b) Show that the pair of equations derived in part a) can be combined into a single second-order differential equation given by

$$\frac{d^2V}{ds_n^2} = k_n^2 V + \frac{1}{Z}\left(\frac{dZ}{ds_n} - \frac{Z}{s_n}\right)\frac{dV}{ds_n}$$

Where k_n is a quantity to be determined.

c) Under what conditions can the equation derived in part b) be treated as a wave equation? Are these conditions satisfied in our cochlear model?

d) Write down an expression for k_n. Is it constant? Use physical reasoning to study the dependence of k_n on s_n, and discuss how this dependence affects wave propagation in our model.

Problem 23.2

a) Explain why the presence of zeros in Figure 23.3 (b) sharpens the frequency rolloff of a filtering stage in the cochlea. How is this rolloff affected by the Q of the complex zeros or poles (which are kept proportional to each other)?

b) Group delay is formally defined as $\tau_g = -\left(\frac{\partial \phi}{\partial \omega}\right)$, where ϕ is the phase of the frequency response. Problem P13.3 in Chapter 13 also discusses group delay. Why do the presence of zeros in Figure 23.3 (b) reduce group delay?

Problem 23.3

A model of the admittance of the basilar membrane is shown in Figure P23.3. Note that the inductances L_1 and L_2 also exhibit a mutual-inductance coupling of M amongst themselves.

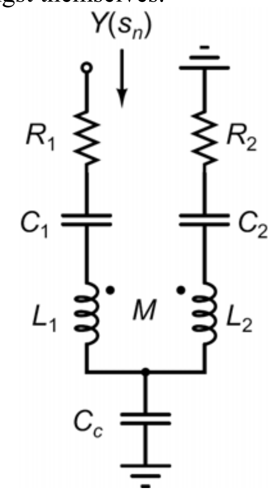

Figure P23.3: A model of the admittance of the basilar membrane.

a) Use physical reasoning to sketch the input admittance of this circuit as a function of frequency. Is it qualitatively similar to that of the actual basilar membrane in the biological cochlea?
b) Find the input admittance $Y(s)$ in terms of R_1, R_2, C_1, C_2, C_c, L_1, L_2, and M. Are there any constraints on the values of these components? [Hint: if the algebra becomes tedious, use a software package that supports symbolic mathematics.]
c) What is the resonant frequency ω_c of this portion of the basilar membrane?
d) Normalize the values of R_1, R_2, C_1, C_2, C_c, L_1, L_2, and M such that $\omega_c = 1$. Use these normalized values to rewrite the input admittance function in terms of the normalized frequency variable s_n, i.e., find $Y(s_n)$.
e) Show that the function you derived in part d) can be written in the canonical form described in Chapter 23, i.e., it can implement Equation (23.12). Express the parameters μ, Q_z, and Q_p in terms of normalized values of the circuit elements.
f) Find circuit element values that result in the following parameter values: $\mu = 0.76$, $Q_z = 3.8$, and $Q_p = 5.0$. [Hint: there are multiple solutions to this problem. Use a numerical optimization algorithm instead of hand analysis.]
g) Are the parameter values used in part f) realizable using only passive circuit elements? Explain.

Problem 23.4

a) Derive Equation (23.32) by analyzing the circuit of the retina shown in Figure 23.12.
b) Assume that V_{xyc} and V_{xyh}, i.e., solutions to the equation derived in part a), are both proportional to $\exp\left(k_x x + k_y y\right)$, where k_x and k_y are unknowns. Show that $\left(k_x^2 + k_y^2\right)$ is then given by Equation (23.34).

Problem 23.5

Consider the diode-capacitor circuit shown in P23.5, which was described in the text for fly-vision-inspired motion processing. Assume that the circuit is in steady state when the switch opens at $t = 0$.

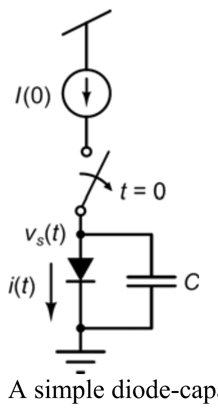

Figure P23.5: A simple diode-capacitor circuit.

a) Prove that $i(t)$, the current in the diode, and $v_S(t)$, the voltage across the capacitor, are given by Equation (23.35).
b) Show that $i(t)$ is independent of $I(0)$ for $t \gg C\phi_t / I(0)$. Find the function that describes $i(t)$ in this regime.

Problem 23.6

In this problem you will study a unidirectional cochlear model that consists of N_{oct} stages per octave and that can analyze signals over β octaves. Each stage consists of a filter with transfer function given by Equation (23.27).

a) The model requires a total of N stages. What is the value of N? Also, how is N_{oct} related to N_{nat}?
b) Implement the model on a computer. You may use your favorite language, or take advantage of high-level programming environments such as MATLAB or Mathematica. Use the following default parameter values: $N_{oct} = 12$, $Q_p = 5$, $\eta_{cf} = 0.5$, and $\mu = 0.2$.
c) Plot cochlear transfer functions at various values of n. Here n describes position along the filter cascade, and is an integer that varies between 1 and N. Does the model perform a frequency-to-place transformation, as expected?
d) Plot the maximum gain and quality factor of the cochlear transfer function as a function of n. Explain important characteristics of these plots.
e) Repeat part d) for various values of N_{oct}, and comment on your results.
f) In a practical implementation of this cochlear model, which parameter would you vary as a function of signal amplitude to increase the dynamic range of the system? Explain.

Problem 23.7

a) Use the WKB solution shown in Equation (23.18) to plot the magnitude and phase of the transfer function that describes the bidirectional cochlea in normalized frequency space. Assume that the shunt admittance is described by Equation (23.12), with $\mu = 0.76$, $Q_z = 3.8$, and $Q_p = 5$. Also assume the following default values of other relevant parameters: $N_{nat} = 20$, $\omega_c(0) = 2\pi \times 10^{10}$ rad/s, $L_0 = 0.5$ nH, and $C_0 = 70$ fF.
b) Numerically differentiate the phase of the transfer function found in part a) to find its group delay. Plot your results. What is the practical significance of group delay? At what frequency does maximum group delay occur?
c) Repeat parts a) and b) for other values of N_{nat}. Summarize, in words, the resultant changes in the cochlear transfer function. Is this behavior similar to that observed for the unidirectional cochlea model analyzed in Problem 23.6? If not, what are the major differences?

Problem 23.8

Show that if the Equation (23.31), which is used to create the analog vocal tract, is to faithfully model the biological vocal tract described by Equation (23.30), then

a) The product, $L(x)C(x)$ is invariant with $A(x)$, the local areal cross section of the vocal tract. What is the physical reason for this invariance?
b) How does the local impedance of the vocal-tract transmission line, $\sqrt{(L(x)/C(x))}$ vary with $A(x)$?

c) In [43], a translinear circuit is described to implement a linear *I-V* conductance that models laminar fluid flow in the vocal tract or to implement a square-root conductance that models turbulent fluid flow in the vocal tract. Show how to create a square-law conductance using this circuit.

d) Phase locked loops and the speech locked loop shown in Figure 23.9 represent examples of an 'analysis-by-synthesis' technique. What is being synthesized in a phase locked loop?

Problem 23.9

Many systems in neurobiology, such as nerve axons and retinal cell layers, can be modeled using *RC* transmission lines. A section of an *RC* transmission line is shown in Figure P23.9. Assume that *R* and *C*, the resistance and capacitance per unit length of the line, are both independent of *x*.

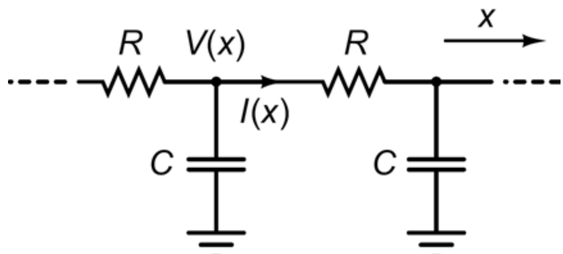

Figure P23.9: An *RC* transmission line.

a) Show that wave propagation along the line is governed by the following partial differential equation:

$$\frac{\partial^2 V}{\partial x^2} = RC\frac{\partial V}{\partial t}$$

b) Rewrite this equation in sinusoidal steady state, i.e., when the voltage and current both vary with time as $\exp(j\omega t)$.

c) Show that the amplitude of a propagating sinusoid decays exponentially with x, with the characteristic decay length being given by

$$\lambda = \sqrt{\frac{2}{\omega RC}}$$

d) Show that the characteristic impedance of the *RC* transmission line is given by

$$Z_0 = \sqrt{\frac{R}{j\omega C}}$$

e) Explain qualitatively why the characteristic impedance of the *RC* transmission line, unlike that of the *LC* transmission line, is frequency-dependent.

f) Can the characteristic impedance derived in part d) be emulated by a finite network of lumped circuit elements? Explain.

Problem 23.10

In order to implement *RC* transmission lines on-chip we may have to discretize them in space. The process is similar to that used for discretizing the *LC* transmission line that models the cochlea: lengths of line Δx long are represented by lumped circuit elements. In this case the elements consist of a series resistor of value $R(\Delta x)$ and a shunt capacitor of value $C(\Delta x)$. Assume that R and C are both independent of x.

a) Describe the discretized RC transmission line with a set of difference equations, and show that, in the limit $\Delta x \to 0$, they reduce to the same partial differential equations that describe the continuous transmission line (analyzed in Problem 23.9).

b) Show that the characteristic impedance of the discretized line in sinusoidal steady state is given by

$$Z_0 = \frac{R(\Delta x)}{2}\left[1+\sqrt{1+\frac{4}{j\omega RC(\Delta x)^2}}\right]$$

c) Under what conditions does the characteristic impedance derived in part b) become equal to that of the continuous transmission line? Use qualitative reasoning to explain your result.

d) Explain (in words) the behavior of Z_0 at very high frequencies, i.e., as $\omega \to \infty$.