

Answers to Exercises

Specimen answers are given to all the exercises. In some cases they do not necessarily represent the best technique for solving a problem but merely one which uses the material introduced at that point in the discussion.

Answers 2

Exercise 2.2

```
1 package Simple_Maths is
  function Sqrt(F: Float) return Float;
  function Log(F: Float) return Float;
  function Ln(F: Float) return Float;
  function Exp(F: Float) return Float;
  function Sin(F: Float) return Float;
  function Cos(F: Float) return Float;
end Simple_Maths;
```

The first few lines of the program Print_Roots could now become

```
with Simple_Maths, Simple_IO;
procedure Print_Roots is
  use Simple_Maths, Simple_IO;
```

Exercise 2.4

```
1 for I in 0 .. N loop
  Pascal(I) := Next(I);
end loop;

2 for N in 0 .. 10 loop
  Pascal2(N, 0) := 1;
  for I in 1 .. N-1 loop
    Pascal2(N, I) := Pascal2(N-1, I-1) +
                      Pascal2(N-1, I);
  end loop;
  Pascal2(N, N) := 1;
end loop;

3 type Month_Name is (Jan, Feb, Mar, Apr, May,
  Jun, Jul, Aug, Sep, Oct, Nov, Dec);
```

```
type Date is
  record
    Day: Integer;
    Month: Month_Name;
    Year: Integer;
  end record;

Today: Date;
...
Today := (24, May, 1819);
```

Answers 3

Exercise 3.1

```
1 package Buffer_System is           -- visible part
  type Buffer is private;
  Buffer_Error: exception;
  procedure Load(B: in out Buffer; S: in String);
  procedure Get(B: in out Buffer;
    C: out Character);
  function Is_Empty(B: Buffer) return Boolean;
private                             -- private part
  Max: constant Integer := 80;
  type Buffer is
    record
      Data: String(1 .. Max);
      Start: Integer := 1;
      Finish: Integer := 0;
    end record;
end Buffer_System;

package body Buffer_System is
  procedure Load(B: in out Buffer; S: in String) is
  begin
    if S'Length > Max or B.Start <= B.Finish then
```

2 Answers to exercises

```

    raise Buffer_Error;
end if;
B.Start := 1;
B.Finish := S'Length;
B.Data(B.Start .. B.Finish) := S;
end Load;

procedure Get(B: in out Buffer;
              C: out Character) is
begin
    if B.Start > B.Finish then
        raise Buffer_Error;
    end if;
    C := B.Data(B.Start);
    B.Start := B.Start + 1;
end Get;

function Is_Empty(B: Buffer) return Boolean is
begin
    return B.Start > B.Finish;
end Is_Empty;
end Buffer_System;

```

The parameter Buffer of Load now has to be in **out** because the original value is read. Also, we could replace the test in Get by

```
if Is_Empty(B) then
```

Exercise 3.2

```

1 package Objects is
    type Object is tagged
        record
            X_Coord: Float;
            Y_Coord: Float;
        end record;

    function Distance(O: Object) return Float;
    function Area(O: Object) return Float;
end Objects;

package body Objects is
    function Distance(O: Object) return Float is
    begin
        return Sqrt(O.X_Coord**2 + O.Y_Coord**2);
    end Distance;

    function Area(O: Object) return Float is
    begin
        return 0.0;
    end Area;
end Objects;

with Objects; use Objects;
package Shapes is
    type Circle is new Object with
        record
            Radius: Float;
        end record;

```

```

    function Area(C: Circle) return Float;
    type Point is new Object with null record;
    type Triangle is new Object with
        record
            A, B, C: Float;
        end record;

    function Area(T: Triangle) return Float;
end Shapes;

package body Shapes is
    function Area(C: Circle) return Float is
    begin
        return  $\pi$  * C.Radius**2;
    end Area;

    function Area(T: Triangle) return Float is
    S: constant Float := 0.5 * (T.A + T.B + T.C);
    begin
        return Sqrt(S * (S - T.A) * (S - T.B) * (S - T.C));
    end Area;
end Shapes;

```

Note that we can put the use clause for Objects immediately after the with clause.

Exercise 3.3

- 1

```

procedure Add_To_List(The_List: in out List;
                      Obj_Ptr: in Pointer) is
    Local: List := new Cell;
begin
    Local.Next := The_List;
    Local.Element := Obj_Ptr;
    The_List := Local;
end Add_To_List;

```

or more briefly using a form of allocation with initial values

```

procedure Add_To_List(The_List: in out List;
                      Obj_Ptr: in Pointer) is
begin
    The_List := new Cell'(The_List, Obj_Ptr);
end Add_To_List;

```
- 2

```

package body Objects is
    function Distance(O: Object) return Float is
    begin
        return Sqrt(O.X_Coord**2 + O.Y_Coord**2);
    end Distance;
end Objects;

```
- 3 We have to add the function Area for the type Point.
- 4 We cannot declare the function Moment for the abstract type Object because it contains a call of the abstract function Area.

3.3

Answers to exercises

3

- 5 **function** MO(OC: Object'Class) **return** Float **is**
begin
 return MI(OC) + Area(OC) * Distance(OC)**2;
end MO;

Answers 4

Exercise 4.2

- 1 The default field is 6 for a 16-bit type Integer and 11 for a 32-bit type Integer so
- ```
Put(123); -- "sss123" and "ssssssss123"
Put(-123); -- "ss-123" and "ssssssss-123"
```

### Exercise 4.4

- 1 **with** Ada.Text\_IO, Etc;  
**use** Ada.Text\_IO, Etc;  
**procedure** Table\_Of\_Square\_Roots **is**  
    **use** My\_Float\_IO, My\_Elementary\_Functions;  
    Last\_N: Integer;  
    Tab: Count;  
**begin**  
    Tab := 10;  
    Put("What is the largest value please? ");  
    Get(Last\_N);  
    New\_Line(2);  
    Put("Number"); Set\_Col(Tab);  
    Put("Square root");  
    New\_Line(2);  
    **for** N **in** 1 .. Last\_N **loop**  
        Put(N, 4); Set\_Col(Tab);  
        Put(Sqrt(My\_Float(N)), 3, 6, 0);  
        New\_Line;  
    **end loop**;  
**end** Table\_Of\_Square\_Roots;
- 2 **with** Ada.Text\_IO;  
**package** My\_Numerics.My\_Float\_IO **is**  
    **new** Ada.Text\_IO.Float\_IO(My\_Float);  
**with** Ada.Text\_IO;  
**package** My\_Numerics.My\_Integer\_IO **is**  
    **new** Ada.Text\_IO.Integer\_IO(My\_Integer);  
**with** Ada.Numerics.Generic\_Elementary\_Functions;  
**package** My\_Numerics.My\_Elementary\_Functions **is**  
    **new** Ada.Numerics.  
        Generic\_Elementary\_Functions(My\_Float);
- 3 **package** Objects **is** ...  
**with** Ada.Numerics.Elementary\_Functions;  
**use** Ada.Numerics.Elementary\_Functions;  
**package body** Objects **is** ...  
**with** Objects; **use** Objects;  
**package** Shapes **is** ...

```
with Ada.Numerics.Elementary_Functions;

use Ada.Numerics.Elementary_Functions;

package body Shapes is ...
```

```
with Shapes; use Shapes;

with Ada.Text_IO, Ada.Float_Text_IO;

use Ada.Text_IO, Ada.Float_Text_IO;

procedure Area_Of_Triangle is

 T: Triangle;
```

```
begin

 Get(T.A); Get(T.B); Get(T.C);

 Put(Area(T));

end Area_Of_Triangle;
```

We should really check that the sides do form a triangle, if they do not then the call of Sqrt in Area will have a negative parameter and so raise Ada.Numerics.Argument\_Error. See Program 1.

- 4 **with** Ada.Text\_IO, Ada.Integer\_Text\_IO;  
**use** Ada.Text\_IO, Ada.Integer\_Text\_IO;  
**with** Ada.Numerics.Discrete\_Random;  
**procedure** Sundays **is**  
    **type** Day **is** (Mon, Tue, Wed, Thu, Fri, Sat, Sun);  
    **package** Random\_Day **is**  
        **new** Ada.Numerics.Discrete\_Random(Day);  
    **use** Random\_Day;  
    G: Generator;  
    D: Day;  
    Number\_Of\_Sundays: Integer;  
**begin**  
    Number\_Of\_Sundays := 0;  
    **for** I **in** 1 .. 100 **loop**  
        D := Random(G);  
        **if** D = Sun **then**  
            Number\_Of\_Sundays :=  
                Number\_Of\_Sundays + 1;  
        **end if**;  
    **end loop**;  
    Put("Percentage of Sundays in selection was ");  
    Put(Number\_Of\_Sundays);  
    New\_Line;  
**end** Sundays;

- 5 **with** Ada.Text\_IO, Ada.Integer\_Text\_IO;  
**use** Ada.Text\_IO, Ada.Integer\_Text\_IO;  
**procedure** Triangle **is**  
    Size: Integer;  
**begin**  
    Put("Size of triangle please: "); Get(Size);  
    **declare**  
        Pascal: **array** (0 .. Size) **of** Integer;  
        Tab: Count;     -- *indentation at start of row*  
    **begin**  
        Tab := Count(2\*Size + 1);  
        Pascal(0) := 1;  
        **for** N **in** 1 .. Size **loop**

## 4 Answers to exercises

```

Pascal(N) := 1;
for I in reverse 1 .. N-1 loop
 Pascal(I) := Pascal(I-1) + Pascal(I);
end loop;
Tab := Tab - 2;
New_Line(2); Set_Col(Tab);
for I in 0 .. N loop
 Put(Pascal(I), 4);
end loop;
end loop;
New_Line(2);
if 2*Size > 8 then
 Set_Col(Count(2*Size - 8));
end if;
Put("The Triangle of Pascal");
New_Line(2);
end;
end Triangle;

```

It is instructive to consider how this should be written to accommodate larger values of Size in a flexible manner and so avoid the confusing repetition of the literal 2. A variable `Half_Field` might be declared with the value 2 in the above but would need to be 3 for values of Size up to 19 which will go off the screen anyway. Care is needed with variables of type `Count` which are not allowed to take negative values.

## Answers 5

### Exercise 5.3

- 1 The following are not legal identifiers
  - (b) contains &
  - (c) contains hyphens not underlines
  - (e) adjacent underlines
  - (f) does not start with a letter
  - (g) trailing underline
  - (h) this is two legal identifiers
  - (i) this is legal – but it is a reserved word and not an identifier

Note that (a) is of course a legal identifier but it would be unwise to declare our own variable called `Ada` because it would conflict with the predefined package of that name.

### Exercise 5.4

- 1
  - (a) legal – real
  - (b) illegal – no digit before point
  - (c) legal – integer
  - (d) illegal – integer with negative exponent
  - (e) illegal – closing # missing
  - (f) legal – real
  - (g) illegal – C not a digit of base 12

- (h) illegal – no number before exponent
  - (i) legal – integer – case of letter immaterial
  - (j) legal – integer
  - (k) illegal – underline at start of exponent
  - (l) illegal – integer with negative exponent

- 2
  - (a)  $224 = 14 \times 16$
  - (b)  $6144 = 3 \times 2^{11}$
  - (c) 4095.0
  - (d) 4095.0
- 3
  - (a) 32 ways  
41, 2#101001#, 3#1112#, ... 10#41#, ... 16#29#, 41E0, 2#101001#E0, ... 16#29#E0
  - (b) 40 ways. As for example (a) plus, since 150 is not prime but  $2 \times 3 \times 5^2 = 150$ , also  
2#1001011#E1, 3#1212#E1, 5#110#E1, 5#11#E2, 6#41#E1, 10#15#E1, 15#A#E1, 15E1

## Answers 6

### Exercise 6.1

- 1 `F: Float := 1.0;`
- 2 Zero: **constant** `Float := 0.0;`  
One: **constant** `Float := 1.0;`  
but it might be better to write real number declarations  
Zero: **constant** `:= 0.0;`  
One: **constant** `:= 1.0;`
- 3
  - (a) **var** is illegal – this is Ada not Pascal
  - (b) terminating semicolon is missing
  - (c) a constant declaration must have an initial value
  - (d) no multiple assignment in Ada
  - (e) nothing – assuming M and N are of integer type
  - (f) 2Pi is not a legal identifier

### Exercise 6.2

- 1 There are four errors
  - (1) no semicolon after declaration of J, K
  - (2) K used before a value is assigned to it
  - (3) = instead of := in declaration of P
  - (4) Q not declared and initialized

## 6.4

## Exercise 6.4

- 1 It is assumed that the values of all variables originally satisfy their constraints.
  - (a) the ranges of I and J are identical so no checks are required and consequently Constraint\_Error cannot be raised,
  - (b) the range of J is a subset of that of K and again Constraint\_Error cannot be raised,
  - (c) in this case a check is required since if  $K > 10$  it cannot be assigned to J in which case Constraint\_Error will be raised.

## Exercise 6.5

- 1
 

|          |             |
|----------|-------------|
| (a) -105 | (e) -3      |
| (b) -3   | (f) illegal |
| (c) 0    | (g) -1      |
| (d) -3   | (h) 2       |
- 2 All variables are of type Float
  - (a)  $M * R ** 2$
  - (b)  $B ** 2 - 4.0 * A * C$
  - (c)  $(4.0 / 3.0) * \pi * R ** 3$   
       -- parentheses not necessary
  - (d)  $(P * \pi * A ** 4) / (8.0 * L * \eta)$   
       -- parentheses are necessary

## Exercise 6.6

- 1
  - (a) Sat
  - (b) Sat   note that Succ applies to base type
  - (c) 2
- 2
  - (a) **type** Rainbow **is** (Red, Orange, Yellow, Green, Blue, Indigo, Violet);
  - (b) **type** Fruit **is** (Apple, Banana, Orange, Pear);
- 3 Groom'Val((N-1) mod 8)  
    or perhaps better  
    Groom'Val((N-1) mod (Groom'Pos(Groom'Last) + 1))
- 4  $D := \text{Day'Val}((\text{Day'Pos}(D) + N - 1) \bmod 7);$
- 5 If X and Y are both overloaded literals then  $X < Y$  will be ambiguous. We would have to use qualification such as  $T'(X) < T'(Y)$ .

## Exercise 6.7

- 1 T: **constant** Boolean := True;  
    F: **constant** Boolean := False;

## Answers to exercises

## 5

- 2 The values are True and False, not T or F which are the names of constants.
 

|           |           |
|-----------|-----------|
| (a) False | (d) True  |
| (b) True  | (e) False |
| (c) True  |           |
- 3 The expression is always True. The predefined operators **xor** and **/=** operating on Boolean values are the same. But see the note at the end of Section 11.3.

## Exercise 6.8

- 1
 

|           |         |
|-----------|---------|
| (a) False | (b) Sat |
|-----------|---------|

## Exercise 6.9

- 1 All variables are of type Float except for N in example (c) which is Integer.
  - (a)  $2.0 * \pi * \text{Sqrt}(L/G)$
  - (b)  $M\_0 / \text{Sqrt}(1.0 - (V/C) ** 2)$
  - (c)  $\text{Sqrt}(2.0 * \pi * \text{Float}(N)) * (\text{Float}(N)/E) ** N$
- 2  $\text{Sqrt}(2.0 * \pi * X) * \text{Exp}(X * \text{Ln}(X) - X)$

## Answers 7

## Exercise 7.1

- 1 **declare**  
    End\_Of\_Month: Integer;  
**begin**  
    **if** Month = Sep **or** Month = Apr  
        **or** Month = Jun **or** Month = Nov **then**  
        End\_Of\_Month := 30;  
    **elsif** Month = Feb **then**  
        **if** (Year mod 4 = 0 **and** Year mod 100 /= 0)  
            **or** Year mod 400 = 0 **then**  
            End\_Of\_Month := 29;  
        **else**  
            End\_Of\_Month := 28;  
        **end if**;  
    **else**  
        End\_Of\_Month := 31;  
    **end if**;  
    **if** Day /= End\_Of\_Month **then**  
        Day := Day + 1;  
    **else**  
        Day := 1;  
        **if** Month /= Dec **then**  
            Month := Month\_Name'Succ(Month);  
        **else**  
            Month := Jan;  
            Year := Year + 1;  
        **end if**;  
**end if**;

## 6 Answers to exercises

```
end if;
end;
```

If today is 31 Dec 2399 then Constraint\_Error will be raised on attempting to assign 2400 to Year.

```
2 if X < Y then
 declare
 T: Float := X;
 begin
 X := Y; Y := T;
 end;
end if;
```

*Exercise 7.2*

```
1 declare
 End_Of_Month: Integer;
begin
 case Month is
 when Sep | Apr | Jun | Nov =>
 End_Of_Month := 30;
 when Feb =>
 if (Year mod 4 = 0 and Year mod 100 /= 0)
 or Year mod 400 = 0 then
 End_Of_Month := 29;
 else
 End_Of_Month := 28;
 end if;
 when others =>
 End_Of_Month := 31;
 end case;
 -- then as before
 ...
end;
```

```
2 subtype Winter is Month_Name range Jan .. Mar;
 subtype Spring is Month_Name range Apr .. Jun;
 subtype Summer is Month_Name range Jul .. Sep;
 subtype Autumn is Month_Name range Oct .. Dec;
 ...
case M is
 when Winter => Dig;
 when Spring => Sow;
 when Summer => Tend;
 when Autumn => Harvest;
end case;
```

Note that if we wished to consider winter as December to February then we could not declare a suitable subtype.

```
3 case D is
 when 1 .. 10 => Gorge;
 when 11 .. 20 => Subsist;
 when others => Starve;
end case;
```

We cannot write 21 .. End\_Of\_Month because it is not a static range. In fact **others** covers all values of type Integer because although D is constrained, nevertheless the constraints are not static.

*Exercise 7.3*

```
1 declare
 Sum: Integer := 0;
 I: Integer;
begin
 loop
 Get(I);
 exit when I < 0;
 Sum := Sum + I;
 end loop;
end;

2 declare
 Copy: Integer := N;
 Count: Integer := 0;
begin
 while Copy mod 2 = 0 loop
 Copy := Copy / 2;
 Count := Count + 1;
 end loop;
 ...
end;

3 declare
 G: Float := -Ln(Float(N));
begin
 for P in 1 .. N loop
 G := G + 1.0/Float(P);
 end loop;
 ...
end;
```

We assume that Ln is the function for natural logarithm.

## Answers 8

*Exercise 8.1*

```
1 declare
 F: array (0 .. N) of Integer;
begin
 F(0) := 0; F(1) := 1;
 for I in 2 .. F'Last loop
 F(I) := F(I-1) + F(I-2);
 end loop;
 ...
end;
```

## 8.1

## Answers to exercises

7

```

2 declare
 Max_I: Integer := A'First(1);
 Max_J: Integer := A'First(2);
 Max: Float := A(Max_I, Max_J);
begin
 for I in A'Range(1) loop
 for J in A'Range(2) loop
 if A(I, J) > Max then
 Max := A(I, J);
 Max_I := I; Max_J := J;
 end if;
 end loop;
 end loop;
 -- Max_I, Max_J now contain the result
end;

3 declare
 Days_In_Month: array (Month_Name) of Integer
 := (31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31);
 End_Of_Month: Integer;
begin
 if (Year mod 4 = 0 and Year mod 100 /= 0)
 or Year mod 400 = 0 then
 Days_In_Month(Feb) := 29;
 end if;
 End_Of_Month := Days_In_Month(Month);
 ... -- then as Exercise 7.1(1)
end;

4 Yesterday: constant array (Day) of Day :=
 (Sun, Mon, Tue, Wed, Thu, Fri, Sat);

5 Bor: constant array (Boolean, Boolean) of Boolean
 := ((False, True), (True, True));

6 Unit: constant array (1 .. 3, 1 .. 3) of Float :=
 ((1.0, 0.0, 0.0),
 (0.0, 1.0, 0.0),
 (0.0, 0.0, 1.0));

```

## Exercise 8.2

```

1 type Bbb is array (Boolean, Boolean) of Boolean;

2 type Ring5_Table is array (Ring5, Ring5) of Ring5;
Add: constant Ring5_Table :=
 ((0, 1, 2, 3, 4),
 (1, 2, 3, 4, 0),
 (2, 3, 4, 0, 1),
 (3, 4, 0, 1, 2),
 (4, 0, 1, 2, 3));

```

```

Mult: constant Ring5_Table :=
 ((0, 0, 0, 0, 0),
 (0, 1, 2, 3, 4),
 (0, 2, 4, 1, 3),
 (0, 3, 1, 4, 2),
 (0, 4, 3, 2, 1));

```

```

A, B, C, D: Ring5;
...
D := Mult(Add(A, B), C));

```

## Exercise 8.3

```

1 Days_In_Month: array (Month_Name) of Integer :=
 (Sep | Apr | Jun | Nov => 30,
 Feb => 28,
 others => 31);

2 Zero: constant Matrix := (1 .. N => (1 .. N => 0.0));

3 This cannot be done with the material at our
 disposal at the moment. See Exercise 9.1(6).

4 type Molecule is (Methanol, Ethanol, Propanol,
 Butanol);
type Atom is (H, C, O);
Alcohol:
 constant array (Molecule, Atom) of Integer :=
 (Methanol => (H => 4, C => 1, O => 1),
 Ethanol => (6, 2, 1),
 Propanol => (8, 3, 1),
 Butanol => (10, 4, 1));

```

Note the danger in the above. We have used named notation in the first inner aggregate to act as a sort of heading but omitted it in the others to avoid clutter. However, if we had written H, C and O in other than positional order then it would have been very confusing because the positional aggregates would not have had the meaning suggested by the heading.

## Exercise 8.4

```

1 Roman_To_Integer:
 constant array (Roman_Digit) of Integer :=
 (1, 5, 10, 50, 100, 500, 1000);

2 declare
 V: Integer := 0;
begin
 for I in R'Range loop
 if I /= R'Last and then
 Roman_To_Integer(R(I)) <
 Roman_To_Integer(R(I+1)) then
 V := V - Roman_To_Integer(R(I));
 else

```

## 8 Answers to exercises

```

 V := V + Roman_To_Integer(R(I));
 end if;
end loop;
...
end;

```

Note the use of **and then** to avoid attempting to access  $R(I+1)$  when  $I = R'Last$ .

### Exercise 8.5

- 1  $AOA(1 \dots 2) := (AOA(2), AOA(1));$
- 2 Farmyard: String\_3\_Array := ("pig", "cat", "dog", "cow", "rat", "hen");  
...  
Farmyard(4)(1) := 's';
- 3 if  $R'Last \geq 2$  and then  
     $R(R'Last-1 \dots R'Last) = "IV"$  then  
     $R(R'Last-1 \dots R'Last) := "VI";$   
end if;

### Exercise 8.6

- 1 White, Blue, Yellow, Green, Red, Purple, Orange, Black
- 2 (a) Black (c) Red  
(b) Green
- 3 **not** (True **xor** True) = True  
**not** (True **xor** False) = False  
The result follows.
- 4 An aggregate of length one must be named.
- 5 "123", "ABC", "Abc", "aBc", "abC", "abc"
- 6 (a) 1 (c) 5  
(b) 5  
Note that the lower bound of the result of & may depend upon the order of the operands; the same applies to **and**, **or** and **xor**.
- 7 (a) 1 .. 10 (c) 6 .. 15  
(b) 1 .. 10 (d) 0 .. 9

### Exercise 8.7

- 1 C1, C2, C3: Complex;  
(a)  $C3 := (C1.RI+C2.RI, C1.Im+C2.Im);$   
(b)  $C3 := (C1.RI*C2.RI - C1.Im*C2.Im,$   
     $C1.RI*C2.Im + C1.Im*C2.RI);$

- 2 **declare**  
    Index: Integer;  
**begin**  
    **for** S in PeopleRange **loop**  
        **if** People(S).Birth.Year  $\geq 1980$  **then**  
            Index := S;  
            **exit**;  
        **end if**;  
    **end loop**;  
    -- we assume that there is such a student  
**end**;

### Exercise 8.8

- 1 In a straightforward manner 8 as follows  
     $((1, 0), (0, 1)), ((1, 0), [0, 1]), ([1, 0], (0, 1)),$   
     $[(1, 0), (0, 1)], [[1, 0], (0, 1)], [(1, 0), [0, 1]],$   
     $[1, 0], [0, 1]), [[1, 0], [0, 1]]$
- 2 (**for** K in 1 .. 10  $\Rightarrow K*(K+1)/2$ )

## Answers 9

### Exercise 9.1

- 1  $B := N$  in 3 | 5 | 7 | 11 | 13 | 17 | 19;
- 2 Letter in 'a' .. 'e' | 'A' .. 'E' | 'v' .. 'z' | 'V' .. 'Z'

### Exercise 9.2

- 1 Days\_In\_Month := (if M in Sep | Apr | Jun | Nov  
    **then** 30 **elsif** M = Feb **then** (if Year mod 4 = 0  
    **then** 29 **else** 28) **else** 31);

### Exercise 9.3

- 1 L := (**case** Today is  
    **when** Monday | Friday | Sunday  $\Rightarrow 6$   
    **when** Tuesday  $\Rightarrow 7$   
    **when** Thursday | Saturday  $\Rightarrow 8$   
    **when** Wednesday  $\Rightarrow 9$ );
- 2 Pension := Integer(  
    (if Age in 50 .. 69 **then** 50.0  
    **elsif** Age in 70 .. 79 **then** 60.0  
    **elsif** Age in 80.. 100 **then** 70.0  
    **else** 0.0)  
    \*  
    (if Gender = Female **then** 0.9 **else** 1.0)  
    \*  
    (if Disabled **then** 1.05 **else** 1.0)  
    +  
    (if Age = 100 **then** 100.0 **else** 0.0));

It is probably better to use a case expression for the first part thus

```
(case Age is
 when 50 .. 69 => 50.0
 when 70 .. 79 => 60.0
 when 80 .. 100 => 70.0
 when others => 0.0)
```

assuming that Age is a static subtype of Integer.

#### Exercise 9.4

- 1 (for all K in A'First .. A'Last - 1 =>  
A(K) <= A(K+1))

This assumes that the index type of the array is an integer type. In the general case we have to use T'Pred and T'Succ where T is the type of the index thus

```
(for all K in A'First .. T'Pred(A'Last) =>
 A(K) <= A(T'Succ(K)))
```

#### Exercise 9.6

- 1 Tn := ([for J in 1..N => J\*(J+1)/2]'Reduce("+", 0));

## Answers 10

#### Exercise 10.1

- 1 function Even(X: Integer) return Boolean is  
begin  
  return X mod 2 = 0;  
end Even;
- 2 function Factorial(N: Natural) return Positive is  
begin  
  if N = 0 then  
    return 1;  
  else  
    return N \* Factorial(N-1);  
  end if;  
end Factorial;
- 3 function Outer(A, B: Vector) return Matrix is  
  C: Matrix(A'Range, B'Range);  
begin  
  for I in A'Range loop  
    for J in B'Range loop  
      C(I, J) := A(I) \* B(J);  
    end loop;  
  end loop;  
  return C;  
end Outer;

- 4 type Primary\_Array is  
  array (Integer range <>) of Primary;  
function Make\_Colour(P: Primary\_Array)  
  return Colour is

```
 C: Colour := (F, F, F);
begin
 for I in P'Range loop
 C(P(I)) := T;
 end loop;
 return C;
end Make_Colour;
```

Note that multiple values are allowed so that Make\_Colour((R, R, R)) = Red.

- 5 function Inner(A, B: Vector) return Float is  
begin  
  if A'Length /= B'Length then  
    raise Constraint\_Error;  
  end if;  
  return Result: Float := 0.0 do  
    for I in A'Range loop  
      Result := Result + A(I) \* B(I+B'First-A'First);  
    end loop;  
  end return;  
end Inner;

- 6 function Make\_Unit(N: Natural) return Matrix is  
begin  
  return M: Matrix(1 .. N, 1 .. N) do  
    for I in 1 .. N loop  
      for J in 1 .. N loop  
        if I = J then  
          M(I, J) := 1.0;  
        else  
          M(I, J) := 0.0;  
        end if;  
      end loop;  
    end loop;  
  end return;  
end Make\_Unit;

We can then declare

Unit: constant Matrix := Make\_Unit(N);

- 7 function GCD(X, Y: Natural) return Natural is  
begin  
  if Y = 0 then  
    return X;  
  else  
    return GCD(Y, X mod Y);  
  end if;  
end GCD;  
or

## 10 Answers to exercises

```

function GCD(X, Y: Natural) return Natural is
 XX: Integer := X;
 YY: Integer := Y;
 ZZ: Integer;
begin
 while YY /= 0 loop
 ZZ := XX mod YY; XX := YY; YY := ZZ;
 end loop;
 return XX;
end GCD;

```

Note that X and Y have to be copied because the formal parameters behave as constants.

### Exercise 10.2

- 1 **function** "<" (X, Y: Roman\_Number) **return** Boolean **is**  
**function** Value(R: Roman\_Number) **return** Integer **is**  
V: Integer := 0;  
**begin**  
... -- then loop as in Exercise 8.4(2)  
**return** V;  
**end** Value;  
**begin**  
**return** Value(X) < Value(Y);  
**end** "<";
- 2 **function** "+" (X, Y: Complex) **return** Complex **is**  
**begin**  
**return** (X.RI + Y.RI, X.Im + Y.Im);  
**end** "+";  
**function** "\*" (X, Y: Complex) **return** Complex **is**  
**begin**  
**return** (X.RI\*Y.RI - X.Im\*Y.Im,  
X.RI\*Y.Im + X.Im\*Y.RI);  
**end** "\*";
- 3 **function** "<" (P: Primary; C: Colour) **return** Boolean **is**  
**begin**  
**return** C(P);  
**end** "<";
- 4 **function** "<=" (X, Y: Colour) **return** Boolean **is**  
**begin**  
**return** (X and Y) = X;  
**end** "<=";
- 5 **function** "<" (X, Y: Date) **return** Boolean **is**  
**begin**  
**if** X.Year /= Y.Year **then**  
**return** X.Year < Y.Year;  
**elsif** X.Month /= Y.Month **then**  
**return** X.Month < Y.Month;  
**else**  
**return** X.Day < Y.Day;

```

end if;

end "<";

```

### Exercise 10.3

- 1 **procedure** Swap(X, Y: **in out** Float) **is**  
T: Float;  
**begin**  
T := X; X := Y; Y := T;  
**end** Swap;
- 2 **procedure** Rev(A: **in out** Vector) **is**  
R: Vector(A'Range);  
**begin**  
**for** I **in** A'Range **loop**  
R(I) := A(A'First + A'Last - I);  
**end loop**;  
A := R;  
**end** Rev;

We might then write

```
Rev(Vector(R));
```

If we had two parameters and built the result directly in the out parameter thus

```

procedure Rev(A: in Vector; R: out Vector) is

begin

for I in A'Range loop

R(I) := A(A'First + A'Last - I);

end loop;

end Rev;

```

then a call with both parameters denoting the same array would result in a mess if passed by reference because the result would overwrite the data. Both parameters denote the same object and are said to be aliased. This is a bounded error.

- 3 The fragment has a bounded error because the outcome depends upon whether the parameter is passed by copy or by reference. If by copy then A(1) ends up as 2.0; if by reference then A(1) ends up as 4.0. There is aliasing because A and V both refer to the same object.

### Exercise 10.4

- 1 **function** Add(X: Integer; Y: Integer := 1) **return** Integer **is**  
**begin**  
**return** X + Y;  
**end** Add;

The following 6 calls are equivalent

|                     |                     |
|---------------------|---------------------|
| Add(N)              | Add(N, 1)           |
| Add(X => N, Y => 1) | Add(X => N)         |
| Add(N, Y => 1)      | Add(Y => 1, X => N) |

## 10.4

```

2 function Favourite_Spirit return Spirit is
 begin
 case Today is
 when Mon .. Fri => return Gin;
 when Sat | Sun => return Vodka;
 end case;
 end Favourite_Spirit;

...

procedure Dry_Martini(Base: Spirit := Favourite_Spirit;
 How: Style := On_The_Rocks;
 Plus: Trimming := Olive);

```

This example illustrates that defaults are evaluated each time they are required and can therefore be changed from time to time. Incidentally, we could just declare the specification of Favourite\_Spirit first and then declare the bodies of both subprograms.

## Answers 11

### Exercise 11.2

```

1 procedure Append(First: in out Cell_Ptr;
 Second: in Cell_Ptr) is
 L: Cell_Ptr := First;
begin
 if First = null then
 First := Second;
 else
 while L.Next /= null loop
 L := L.Next;
 end loop;
 L.Next := Second;
 end if;
end Append;

2 function Size(T: Node_Ptr) return Integer is
 begin
 if T = null then
 return 0;
 else
 return Size(T.Left) + Size(T.Right) + 1;
 end if;
 end Size;

3 function Copy(T: Node_Ptr) return Node_Ptr is
 begin
 if T = null then
 return null;
 else
 return new Node'(Copy(T.Left),
 Copy(T.Right), T.Value);
 end if;
 end Copy;

```

## Answers to exercises

11

```

4 function "+" (A: A_String) return String is
 begin
 return A.all;
 end "+";

 and then Put(+Zoo(3)); will output the string
 "camel".

5 function "&" (X, Y: A_String) return A_String is
 begin
 return new String'(X.all & Y.all);
 end "&";

```

### Exercise 11.4

```

1 type G_String is access constant String;
type G_String_Array is
 array (Positive range <>) of G_String;

 Aardvark: aliased constant String := "aardvark";
 Baboon: aliased constant String := "baboon";
 ...
 Zebra: aliased constant String := "zebra";

 Zoo: constant G_String_Array :=
 (Aardvark'Access, Baboon'Access, ...,
 Zebra'Access);

2 N: Integer := ... ;
 M: Integer := ... ;
 World: array (1 .. N, 1 .. M) of Cell;
 Abyss: constant Cell := (0, 0, (1 .. 8 => null));
 ...
 -- offsets of 8 neighbours starting at North
 type Offset is array (1 .. 8) of Integer;
 H_Off: Offset := (+0, +1, +1, +1, +0, -1, -1, -1);
 V_Off: Offset := (+1, +1, +0, -1, -1, -1, +0, +1);
 -- now link up the cells
 for I in 1 .. N loop
 for J in 1 .. M loop
 -- link to eight neighbours except on boundary
 declare
 H_Index, V_Index: Integer;
 begin
 for N_Index in 1 .. 8 loop
 H_Index := I + H_Off(N_Index);
 V_Index := J + V_Off(N_Index);
 if H_Index in 1 .. N and
 V_Index in 1 .. M then
 World(I, J).Neighbour_Count(N_Index) :=
 World(H_Index, V_Index).
 Life_Count'Access;
 else
 -- edge of world, link to abyss
 World(I, J).Neighbour_Count(N_Index) :=
 Abyss.Life_Count'Access;
 end if;
 end loop;
 end loop;
 end loop;
 end loop;

```

## 12 Answers to exercises

```
end;
end loop;
end loop;
```

Clearly the repetition of `World(I, J)` could be eliminated by introducing an access type to the cell itself. Or we could use renaming as described in Section 13.7. It would be better if we did not have so many occurrences of the literal 8. Indeed, the enthusiastic reader might like to consider how this example might be extended to three or more dimensions. In three dimensions of course the number of neighbours is  $3^3 - 1 = 26$ .

```
3 type Cell;
 type Ref_Cell is access constant Cell;
 type Ref_Cell_Array is
 array (Integer range <>) of Ref_Cell;
 type Cell is
 record
 Life_Count: Integer range 0 .. 1;
 Total_Neighbour_Count: Integer range 0 .. 8;
 Neighbour: Ref_Cell_Array(1 .. 8);
 end record;
 ...
 C.Total_Neighbour_Count := 0;
 for I in C.Neighbour'Range loop
 C.Total_Neighbour_Count :=
 C.Total_Neighbour_Count +
 C.Neighbour(I).Life_Count;
 end loop;
```

The other changes are that `World` and `Abyss` have to be declared as aliased

```
World: array (1 .. N, 1 .. M) of aliased Cell;
Abyss: aliased constant Cell := (0, 0, (1 .. 8 => null));
```

and the expressions assigned to the neighbours omit `Life_Count` as in

```
World(I, J).Neighbour(N_Index) := Abyss'Access;
```

### Exercise 11.6

- 1 The conversion to `Ref1` is checked dynamically; it passes for the call of `P` with `X1'Access` and fails with `X2'Access`. The conversion to `Ref2` is checked statically and passes.
- 2 The conversion to `Ref1` is checked dynamically; it passes for `X1` and fails for `X2` and `X3`. The conversion to `Ref2` is checked dynamically and passes in all cases. The conversion to `Ref3` is checked statically and passes.

Note that the case of `X3` and `Ref2` is where the accessibility is adjusted on the chained call; without this adjustment it would unnecessarily

fail. The point is that considering `P1` as a whole, since the type `A2` is inside, the conversion is always safe. But since the type `A2` is outside `P2` which actually does the conversion, it has to be checked dynamically; the adjustment ensures that it always passes.

### Exercise 11.7

- 1 The first and last assignments are legal.

### Exercise 11.8

- 1 **function** `G(T: Float) return Float is`  
**begin**  
     **return** `Exp(T) * Sin(T)`;  
**end G**;  
 ...  
`Answer: Float := Integrate(G'Access, 0.0, P)`;
- 2 **function** `Solve(F: access function (X: Float) return Float) return Float;`  
 ...  
**function** `G(X: Float) return Float is`  
**begin**  
     **return** `Exp(X) + X - 7.0`;  
**end G**;  
 ...  
`Answer := Solve(G'Access)`;
- 3 **function** `Integrate(F: access function (X, Y: Float) return Float);`  
     `LX, HX, LY, HY: Float return Float is`  
     **function** `Outer(X: Float) return Float is`  
         **function** `Inner(Y: Float) return Float is`  
             **begin**  
                 **return** `F(X, Y)`;  
             **end Inner**;  
         **begin**  
             **return** `Integrate(Inner'Access, LY, HY)`;  
         **end Outer**;  
     **begin**  
         **return** `Integrate(Outer'Access, LX, HX)`;  
     **end Integrate**;

The functions have to be nested so that the inner one can access the parameter `X` of the outer one.

## 12.1

**Answers 12***Exercise 12.1*

```

1 package Random is
 Modulus: constant := 2**13;
 subtype Small is Integer range 0 .. Modulus;
 procedure Init(Seed: in Small);
 function Next return Small;
end;

package body Random is
 Multiplier: constant := 5**5;
 X: Small;

 procedure Init(Seed: in Small) is
 begin
 X := Seed;
 end Init;

 function Next return Small is
 begin
 X := X * Multiplier mod Modulus;
 return X;
 end Next;
end Random;

2 package Complex_Numbers is
 type Complex is
 record
 Re, Im: Float := 0.0;
 end record;

 I: constant Complex := (0.0, 1.0);

 function "+" (X: Complex) return Complex;
 function "-" (X: Complex) return Complex;
 function "+" (X, Y: Complex) return Complex;
 function "-" (X, Y: Complex) return Complex;
 function "*" (X, Y: Complex) return Complex;
 function "/" (X, Y: Complex) return Complex;
end;

package body Complex_Numbers is

 function "+" (X: Complex) return Complex is
 begin
 return X;
 end "+";

 function "-" (X: Complex) return Complex is
 begin
 return (-X.Re, -X.Im);
 end "-";

 function "+" (X, Y: Complex) return Complex is
 begin
 return (X.Re + Y.Re, X.Im + Y.Im);
 end "+";

 function "-" (X, Y: Complex) return Complex is

```

## Answers to exercises

## 13

```

begin
 return (X.Re - Y.Re, X.Im - Y.Im);
end "-";

function "*" (X, Y: Complex) return Complex is
begin
 return (X.Re*Y.Re - X.Im*Y.Im,
 X.Re*Y.Im + X.Im*Y.Re);
end "*";

function "/" (X, Y: Complex) return Complex is
 D: Float := Y.Re**2 + Y.Im**2;
begin
 return ((X.Re*Y.Re + X.Im*Y.Im)/D,
 (X.Im*Y.Re - X.Re*Y.Im)/D);
end "/";

end Complex_Numbers;

```

*Exercise 12.2*

```

1 Inside the package body (or that of a child
package, see Section 13.3) we could write

function "*" (X: Float; Y: Complex) return Complex is
begin
 return (X*Y.Re, X*Y.Im);
end "*";

but outside we could only write

use Complex_Numbers;

function "*" (X: Float; Y: Complex) return Complex is
begin
 return Cons(X, 0.0) * Y;
end "*";

and similarly with the operands interchanged.

2 declare
 C, D: Complex_Numbers.Complex;
 F: Float;
begin
 C := Complex_Numbers.Cons(1.5, -6.0);
 D := Complex_Numbers."+" (C, Complex_Numbers.I);
 F := Complex_Numbers.Re_Part(D) + 6.0;
 ...
end;

3 package Rational_Numbers is
 type Rational is private;

 function "+" (X: Rational) return Rational;
 function "-" (X: Rational) return Rational;

 function "+" (X, Y: Rational) return Rational;
 function "-" (X, Y: Rational) return Rational;
 function "*" (X, Y: Rational) return Rational;
 function "/" (X, Y: Rational) return Rational;

 function "/" (X: Integer; Y: Positive) return Rational;

```

## 14 Answers to exercises

```

function Numerator(R: Rational) return Integer;
function Denominator(R: Rational) return Positive;

private
 type Rational is
 record
 Num: Integer := 0; -- numerator
 Den: Positive := 1; -- denominator
 end record;
end;

package body Rational_Numbers is

 function Normal(R: Rational) return Rational is
 -- cancel common factors
 G: Positive := GCD(abs R.Num, R.Den);
 begin
 return (R.Num/G, R.Den/G);
 end Normal;

 function "+" (X: Rational) return Rational is
 begin
 return X;
 end "+";

 function "-" (X: Rational) return Rational is
 begin
 return (-X.Num, X.Den);
 end "-";

 function "+" (X, Y: Rational) return Rational is
 begin
 return Normal((X.Num*Y.Den + Y.Num*X.Den,
 X.Den*Y.Den));
 end "+";

 function "-" (X, Y: Rational) return Rational is
 begin
 return Normal((X.Num*Y.Den - Y.Num*X.Den,
 X.Den*Y.Den));
 end "-";

 function "*" (X, Y: Rational) return Rational is
 begin
 return Normal((X.Num*Y.Num, X.Den*Y.Den));
 end "*";

 function "/" (X, Y: Rational) return Rational is
 N: Integer := X.Num*Y.Den;
 D: Integer := X.Den*Y.Num;
 begin
 if D < 0 then D := -D; N := -N; end if;
 return Normal((Num => N, Den => D));
 end "/";

 function "/" (X: Integer; Y: Positive)
 return Rational is
 begin
 return Normal((Num => X, Den => Y));
 end "/";

 function Numerator(R: Rational) return Integer is

```

```

begin
 return R.Num;
end Numerator;

function Denominator(R: Rational)
 return Positive is
begin
 return R.Den;
end Denominator;

end Rational_Numbers;

```

- 4 Although the parameter types are both Integer and therefore the same as for predefined integer division, nevertheless the result types are different. The result types are considered in the hiding rules for functions. See Section 10.5.

## Exercise 12.3

```

1 package Metrics is
 type Length is new Float;
 type Area is new Float;
 function "*" (X, Y: Length) return Length
 is abstract;
 function "*" (X, Y: Length) return Area;
 function "*" (X, Y: Area) return Area is abstract;
 function "/" (X, Y: Length) return Length
 is abstract;
 function "/" (X: Area; Y: Length) return Length;
 function "/" (X, Y: Area) return Area is abstract;
 function "*" (X: Length; Y: Integer) return Length
 is abstract;
 function "*" (X: Length; Y: Integer) return Area;
 function "*" (X: Area; Y: Integer) return Area
 is abstract;

end;

package body Metrics is

 function "*" (X, Y: Length) return Area is
 begin
 return Area(Float(X) * Float(Y));
 end "*";

 function "/" (X: Area; Y: Length) return Length is
 begin
 return Length(Float(X) / Float(Y));
 end "/";

 function "*" (X: Length; Y: Integer) return Area is
 begin
 if Y = 2 then
 return X * X;
 else
 raise Constraint_Error;
 end if;
 end "*";

end Metrics;

```

## 12.4

*Exercise 12.4*

```

1 package Stacks is
 type Stack is private;
 Empty: constant Stack;
 ...
private
 ...
 Empty: constant Stack := ((1 .. Max => 0), 0);
end;
```

Note that Empty has to be initialized because it is a **constant** despite the fact that Top which is the only component whose value is of interest is default initialized anyway. Another approach is to declare a function Empty. This has the advantage of being a primitive operation of Stack and so inherited if we derived from Stack.

```

2 function Is_Empty(S: Stack) return Boolean is
begin
 return S.Top = 0;
end Is_Empty;
```

```

function Is_Full(S: Stack) return Boolean is
begin
 return S.Top = Max;
end Is_Full;
```

Whereas Is\_Empty can test for an empty stack it cannot be used to set a stack empty. A constant or function Empty plus equality can do both.

```

3 function "=" (S, T: Stack) return Boolean is
begin
 return S.S(1 .. S.Top) = T.S(1 .. T.Top);
end "=";
```

```

4 function "=" (A, B: Stack_Array) return Boolean is
begin
 if A'Length /= B'Length then
 return False;
 end if;
 for I in A'Range loop
 if A(I) /= B(I+B'First-A'First) then
 return False;
 end if;
 end loop;
 return True;
end "=";
```

This uses the redefined = (via /=) applying to the type Stack. This pattern for array equality clearly applies to any type. Beware that we cannot use slice comparison (as in the previous answer) because that would call the function being declared and so recurse infinitely.

Equality and inequality returning Boolean arrays might be

## Answers to exercises

15

```

type Boolean_Array is
 array (Integer range <>) of Boolean;

function "=" (A, B: Stack_Array)
 return Boolean_Array is
 Result: Boolean_Array(A'Range);
begin
 if A'Length /= B'Length then
 return (A'Range => False);
 end if;
 for I in A'Range loop
 Result(I) := A(I) /= B(I+B'First-A'First);
 end loop;
 return Result;
end "=";
```

It might be better to raise Constraint\_Error in the case of arrays of unequal lengths.

```

function "/=" (A, B: Stack_Array)
 return Boolean_Array is
begin
 return not (A = B);
end "/=";
```

Recall from Section 8.6 that **not** can be applied to all one-dimensional Boolean arrays.

```

5 package Stacks is
 type Stack is private;
 procedure Push(S: in out Stack; X: in Integer);
 procedure Pop(S: in out Stack; X: out Integer);
private
 Max: constant := 100;
 Dummy: constant Integer := 0;
 type Integer_Vector is
 array (Integer range <>) of Integer;
 type Stack is
 record
 S: Integer_Vector(1 .. Max) :=
 (1 .. Max => Dummy);
 Top: Integer range 0 .. Max := 0;
 end record;
end;
```

```

package body Stacks is
 procedure Push(S: in out Stack; X: in Integer) is
 begin
 S.Top := S.Top + 1;
 S.S(S.Top) := X;
 end;
```

```

 procedure Pop(S: in out Stack; X: out Integer) is
 begin
 X := S.S(S.Top);
 S.S(S.Top) := Dummy;
 S.Top := S.Top - 1;
 end;
```

```

end Stacks;
```

## 16 Answers to exercises

Note the use of Dummy as a default Integer value for unused components of the stack.

- 6 **function** "=" (X, Y: Rational) **return** Boolean **is**  
**begin**  
    **return** X.Num \* Y.Den = X.Den \* Y.Num;  
**end** "=";

Overflow would soon occur without reduction after each operation; this would not be sensible.

### Exercise 12.5

- 1 In the case of the access formulation, although Is\_Empty is straightforward, it is difficult to write an appropriate function Is\_Full; we will return to this when exceptions are discussed in detail in Chapter 14.

**function** Is\_Empty(S: Stack) **return** Boolean **is**  
**begin**  
    **return** S = null;  
**end** Is\_Empty;

- 2 **function** "=" (S, T: Stack) **return** Boolean **is**  
    SL: Cell\_Ptr := Cell\_Ptr(S);  
    TL: Cell\_Ptr := Cell\_Ptr(T);  
**begin**  
    **while** SL /= null **and** TL /= null **loop**  
        **if** SL.Value /= TL.Value **then**  
            **return** False;  
        **end if**;  
        SL := SL.Next;  
        TL := TL.Next;  
    **end loop**;  
    **return** SL = TL;  
**end** "=";

- 3 **package** Queues **is**  
    Empty: **exception**;  
    **type** Queue **is limited private**;  
    **procedure** Join(Q: **in out** Queue; X: **in** Item);  
    **procedure** Remove(Q: **in out** Queue; X: **out** Item);  
    **function** Length(Q: Queue) **return** Integer;  
**private**  
    **type** Cell;  
    **type** Cell\_Ptr **is access** Cell;  
    **type** Cell **is**  
        **record**  
            Next: Cell\_Ptr;  
            Data: Item;  
        **end record**;  
    **type** Queue **is**  
        **record**  
            First, Last: Cell\_Ptr;  
            Count: Integer := 0;  
        **end record**;  
**end**;

### package body Queues is

**procedure** Join(Q: **in out** Queue; X: **in** Item) **is**  
    L: Cell\_Ptr;  
**begin**  
    L := **new** Cell'(Next => null, Data => X);  
    **if** Q.Count = 0 **then**     -- queue was empty  
        Q.First := L;  
        Q.Last := L;  
    **else**  
        Q.Last.Next := L;  
        Q.Last := L;  
    **end if**;  
    Q.Count := Q.Count + 1;  
**end** Join;

**procedure** Remove(Q: **in out** Queue; X: **out** Item) **is**  
**begin**  
    **if** Q.Count = 0 **then**  
        **raise** Empty;  
    **end if**;  
    X := Q.First.Data;  
    Q.First := Q.First.Next;  
    Q.Count := Q.Count - 1;  
**end** Remove;

**function** Length(Q: Queue) **return** Integer **is**  
**begin**  
    **return** Q.Count;  
**end** Length;

**end** Queues;

It would be tidy to assign **null** to Q.Last in the case when the last item is removed but it is not strictly necessary. See also Section 25.4 and in particular Exercise 25.4(1).

### Exercise 12.6

- 1 **private**  
    Max: **constant** := 1000;     -- no of accounts  
    **type** Key\_Code **is new** Integer **range** 0 .. Max;  
    **subtype** Key\_Range **is**  
        Key\_Code **range** 1 .. Key\_Code'Last;  
    **type** Key **is**  
        **record**  
            Code: Key\_Code := 0;  
        **end record**;  
**end**;
- package body** Bank **is**  
    Balance: **array** (Key\_Range) **of** Money :=  
        (others => 0);  
    Free: **array** (Key\_Range) **of** Boolean :=  
        (others => True);  
  
    **function** Valid(K: Key) **return** Boolean **is**  
**begin**  
    **return** K.Code /= 0;  
**end** Valid;

```

procedure Open_Account(K: in out Key;
 M: in Money) is
begin
 if K.Code = 0 then
 for I in Free'Range loop
 if Free(I) then
 Free(I) := False;
 Balance(I) := M;
 K.Code := I;
 return;
 end if;
 end loop;
 end if;
end Open_Account;

procedure Close_Account(K: in out Key;
 M: out Money) is
begin
 if Valid(K) then
 M := Balance(K.Code);
 Free(K.Code) := True;
 K.Code := 0;
 end if;
end Close_Account;

procedure Deposit(K: in Key;
 M: in Money) is
begin
 if Valid(K) then
 Balance(K.Code) := Balance(K.Code) + M;
 end if;
end Deposit;

procedure Withdraw(K: in out Key;
 M: in out Money) is
begin
 if Valid(K) then
 if M > Balance(K.Code) then
 Close_Account(K, M);
 else
 Balance(K.Code) := Balance(K.Code) - M;
 end if;
 end if;
end Withdraw;

function Statement(K: Key) return Money is
begin
 if Valid(K) then
 return Balance(K.Code);
 end if;
end Statement;
end Bank;

```

Various alternative formulations are possible. It might be neater to declare a record type representing an account containing the two components *Free* and *Balance*.

Note that the function *Statement* will raise

*Program\_Error* if the key is not valid. Alternatively we could return a dummy value of zero but it might be better to raise our own exception as described in Chapter 15.

- 2 An alternative formulation which represents the home savings box could be that where the limited private type is given by

```

type Box is
 record
 Code: Box_Code := 0;
 Balance: Money;
 end record;

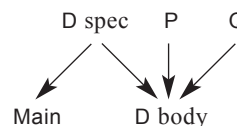
```

In this case the money is kept in the variable declared by the user. The bank only knows which boxes have been issued but does not know how much is in a particular box. The details are left to the reader.

- 3 Since the parameter is of an explicitly limited type, it is always passed by reference and nothing can go wrong. However, if the record type had not been explicitly limited then it might be passed by copy or by reference. If it is passed by copy then the call of *Action* will succeed whereas if it is passed by reference it will not.
- 4 Again, since the parameter is of an explicitly limited type it will be passed by reference and so cannot be changed. However, even if it had not been explicitly limited and passed by copy then he would still have been thwarted because of the rule mentioned in Section 10.3 that a record type with any default initialized components is always copied in precisely so that junk values cannot be created.

## Answers 13

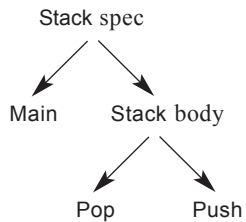
### Exercise 13.1



- 1 The number of possible orders of compilation is (a) 120, (b) 18. The source model thus gives the programmer much more flexibility.

## 18 Answers to exercises

### Exercise 13.2

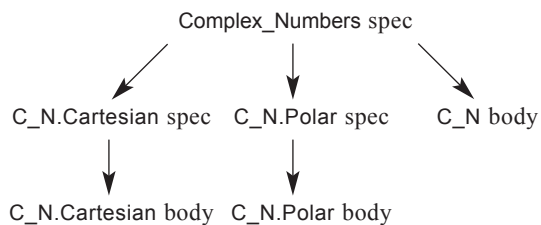


- 1 The number of possible orders of compilation is (a) 120, (b) 8. The source model again is much more flexible. Indeed the number of combinations for the source model is always  $n!$  (where  $n$  is the number of units) irrespective of the dependency structure.

### Exercise 13.3

- 1 **package** Complex\_Numbers **is**  
**type** Complex **is private**;  
 ...  
**function** "+" (X, Y: Complex) **return** Complex;  
 ...  
**private**  
 ...  
**end** Complex\_Numbers;  
**package** Complex\_Numbers.Cartesian **is**  
**function** Cons(R, I: Float) **return** Complex;  
**function** Re\_Part(X: Complex) **return** Float;  
**function** Im\_Part(X: Complex) **return** Float;  
**end** Complex\_Numbers.Cartesian;  
**package** Complex\_Numbers.Polar **is**  
 ... -- as before  
**end** Complex\_Numbers.Polar;  
 The bodies are as expected.

- 2 We have used the abbreviation C\_N for Complex\_Numbers.



- 3 The only subprograms inherited are the arithmetic operations in the package Complex\_Numbers. Those in the child packages are not primitive operations of the type Complex and so are not inherited.

### Exercise 13.4

- 1 **private package** Rational\_Numbers.Slave **is**  
**function** Normal(R: Rational) **return** Rational;  
**end**;  
**package body** Rational\_Numbers.Slave **is**  
**function** GCD(X, Y: Natural) **return** Natural **is**  
 ...  
**end** GCD;  
**function** Normal(R: Rational) **return** Rational **is**  
 G: Positive := GCD(abs R.Num, R.Den);  
**begin**  
**return** (R.Num/G, R.Den/G);  
**end** Normal;  
**end** Rational\_Numbers.Slave;  
**with** Rational\_Numbers.Slave;  
**package body** Rational\_Numbers **is**  
**use** Slave;  
 -- as before without the function Normal  
 ...  
**end** Rational\_Numbers;  
 An alternative is to make Normal a private child function.  
 2 **package** Complex\_Numbers.Trig **is**  
**function** Sin(X: Complex) **return** Complex;  
**function** Cos(X: Complex) **return** Complex;  
 ...  
**end**;  
**private function** Complex\_Numbers.Trig.  
 Sin\_Cos(X: Complex) **return** Complex **is**  
**begin**  
 ...  
**end**;  
**with** Complex\_Numbers.Trig.Sin\_Cos;  
**package body** Complex\_Numbers.Trig **is**  
 ...  
**end** Complex\_Numbers.Trig;

The function Sin\_Cos can be called directly as such within the body of Trig without a use clause.

## 13.7

## Answers to exercises 19

*Exercise 13.7*

- 1 **function** Monday **return** Diurnal.Day  
    **renames** Diurnal.Mon;
- 2 This cannot be done because Next\_Work\_Day is of an anonymous type.
- 3 Pets: String\_3\_Array **renames** Farmyard(2 .. 3);  
Note that the bounds of Pets are 2 and 3.
- 4 **function** "+" (X, Y: Complex\_Numbers.Complex)  
    **return** Complex\_Numbers.Complex  
    **renames** Complex\_Numbers."+";
- 5 This\_Cell: Cell **renames** World(I, J);

*Exercise 13.8*

- 1 **package** P **is**  
    B: Boolean;  
**end**;  
**with** P;  
**package** Q **is**  
    ... -- spec of Q  
**end**;  
**package body** Q **is**  
    ...  
**begin**  
    P.B := True;  
**end** Q;  
**with** P;  
**package** R **is**  
    ... -- spec of R  
**end**;  
**with** Q;  
**pragma** Elaborate(Q);  
**package body** R **is**  
    ...  
**begin**  
    P.B := False;  
**end** R;

We need to ensure that the bodies are elaborated in a specific order. Placing the pragma Elaborate for Q on the body of R ensures that the body of Q is elaborated before that of R. Thus P.B will be finally set False. Note also that R has to have a with clause for Q. The pragma could alternatively be placed on the specification of R since a body is always elaborated after its specification.

**Answers 14***Exercise 14.1*

- 1 P: Point := (Object(C) **with null record**);
- 2 R: Reservation:  
    ...  
    R.Flight\_Number := 77;  
    R.Date\_Of\_Travel := (5, Nov, 2007);  
    ...  
    NR: Nice\_Reservation := (R **with** Window, Green);
- 3 **package** Reservation\_System.Supersonic **is**  
    **type** Supersonic\_Reservation **is**  
        **new** Reservation **with**  
            **record**  
                ...  
            **end record**;  
    **procedure** Make(SR: **in out** Supersonic\_Reservation);  
    ...  
**end** Reservation\_System.Supersonic;

*Exercise 14.2*

- 1 **procedure** Print\_Area(OC: Object'Class) **is**  
    **begin**  
        Put(Area(OC)); -- dispatch to appropriate Area  
    **end**;
- 2 **type** Person **is tagged**  
    **record**  
        Birth: Date;  
    **end record**;  
**type** Man **is new** Person **with**  
    **record**  
        Bearded: Boolean;  
    **end record**;  
**type** Woman **is new** Person **with**  
    **record**  
        Children: Integer;  
    **end record**;
- 3 **procedure** Print\_Details(P: **in** Person) **is**  
    **begin**  
        Print\_Date(P.Birth);  
    **end**;  
    **procedure** Print\_Details(M: **in** Man) **is**  
        **begin**  
            Print\_Details(Person(M));  
            Print\_Boolean(M.Beard);  
        **end**;

## 20 Answers to exercises

```
procedure Print_Details(W: in Woman) is
begin
```

```
 Print_Details(Person(W));
 Print_Integer(W.Children);
```

```
end;
```

```
procedure Analyse_Person(PC: Person'Class) is
begin
```

```
 Print_Details(PC); -- dispatch
```

```
end;
```

### 4 **package body** Queues **is**

```
 procedure Join(Q: access Queue;
 E: in Element_Ptr) is
```

```
 begin
```

```
 if E.Next /= null then -- already on a queue
```

```
 raise Queue_Error;
```

```
 end if;
```

```
 if Q.Count = 0 then -- queue was empty
```

```
 Q.First := E;
```

```
 Q.Last := E;
```

```
 else
```

```
 Q.Last.Next := E;
```

```
 Q.Last := E;
```

```
 end if;
```

```
 Q.Count := Q.Count + 1;
```

```
 end Join;
```

```
 function Remove(Q: access Queue)
```

```
 return Element_Ptr is
```

```
 begin
```

```
 if Q.Count = 0 then
```

```
 raise Queue_Error;
```

```
 end if;
```

```
 Result := Q.First;
```

```
 Q.First := Result.Next;
```

```
 Result.Next := null;
```

```
 Q.Count := Q.Count - 1;
```

```
 return Result;
```

```
 end Remove;
```

```
 function Length(Q: Queue) return Integer is
```

```
 begin
```

```
 return Q.Count;
```

```
 end Length;
```

```
end Queues;
```

### *Exercise 14.3*

### 1 **function** Further(X, Y: Object) **return** Object **is** **begin**

```
 if Distance(X) > Distance(Y) then
```

```
 return X;
```

```
 else
```

```
 return Y;
```

```
 end if;
```

```
end Further;
```

Since it returns the type Object it becomes abstract when inherited by Circle and so has to be overridden. This is very frustrating because the text is essentially unchanged. But Point is OK because the extension is null.

### 2 **package** Objects **is**

```
 type Object is abstract tagged
```

```
 record
```

```
 X_Coord: Float;
```

```
 Y_Coord: Float;
```

```
 end record;
```

```
 function Distance(O: Object'Class) return Float;
```

```
 function Area(O: Object) return Float is abstract;
```

```
end Objects;
```

```
with Objects; use Objects;
```

```
package Shapes is
```

```
 type Point is new Object with null record;
```

```
 function Area(P: Point) return Float;
```

```
 type Circle is new Object with
```

```
 record
```

```
 Radius: Float;
```

```
 end record;
```

```
 function Area(C: Circle) return Float;
```

```
 ... -- etc.
```

```
end Shapes;
```

### 3 **function** Further(X, Y: Object'Class)

```
 return Object'Class is
```

```
begin
```

```
 if Distance(X) > Distance(Y) then
```

```
 return X;
```

```
 else
```

```
 return Y;
```

```
 end if;
```

```
end Further;
```

This calls the class wide function Distance and can be applied to all objects without change.

Moreover, it can be used to compare the distances of two different types of object such as a triangle and a circle.

### 4 The function Bigger cannot be written for the type Object because it would contain a call of the abstract function Area. Functions could of course be written for Circle and Point but would need to be written out for each. A better solution is again to use class wide parameters as in

## 14.3

## Answers to exercises 21

```

function Bigger(X, Y: Object'Class)
 return Object'Class is
begin
 if Area(X) > Area(Y) then
 return X;
 else
 return Y;
 end if;
end Bigger;

```

This dispatches to the appropriate functions Area and can be used to compare the areas of any two objects in the class. However, it would be better to manipulate references to the objects rather than the objects themselves. Indeed if we wish to know which is bigger we don't really want to be given a copy of the bigger one, we want an access value referring to it. The extension problems with the function results would then all go away.

```

5 package body Reservation_System.Subsonic is
 procedure Make(BR: in out Basic_Reservation) is
 Select_Seat(BR);
 end Make;

 procedure Make(NR: in out Nice_Reservation) is
 Make(Basic_Reservation(NR));
 Order_Meal(NR);
 end Make;

 procedure Make(PR: in out Posh_Reservation) is
 Make(Nice_Reservation(PR));
 Arrange_Limo(PR);
 end Make;
 ... -- Select_Seat etc as before
end Reservation_System.Subsonic;

```

*Exercise 14.4*

```

1 function "=" (C, D: Circle) return Boolean is
begin
 return Object(C) = Object(D) and
 abs(C.Radius - D.Radius) < Epsilon;
end "=";

2 function "=" (C, D: Circle) return Boolean is
begin
 return Old_Equality(Object(C), Object(D)) and
 abs(C.Radius - D.Radius) < Epsilon;
end "=";

```

*Exercise 14.5*

1 It would fail to compile because, as mentioned in Section 14.2, Order\_Meal is not a primitive operation of all types in the class rooted at Reservation.

```

2 procedure Select_Seat(NR: in out Nice_Reservation) is
begin
 if NR.Seat_Sort = Aisle then
 -- choose aisle seat
 else
 -- choose window seat
 end if;
end Select_Seat;
...
procedure Make(R: in out Reservation) is
begin
 Select_Seat(Reservation'Class(R)); -- redispach
end Make;

```

Observe that we know that the component NR.Seat\_Sort must exist because this is a nice reservation or derived from it. Naturally enough all seats are window or aisle seats in nice and posh categories.

*Exercise 14.6*

1 We could not declare the function Further with Shape as parameter and result because it has a controlling result and would prevent further extension from the partial view. But the class wide version taking Shape'Class is acceptable.

*Exercise 14.7*

```

1 procedure Adjust(Object: in out Thing) is
begin
 The_Count := The_Count + 1;
end Adjust;

```

The procedures Initialize and Finalize are as before.

2 The type Key and the control procedures could be

```

type Key is new Limited_Controlled with
 record
 Code: Key_Code;
 end record;

```

```

procedure Initialize(K: in out Key) is
begin
 K.Code := 0;
end Initialize;

```

```

procedure Finalize(K: in out Key) is
begin
 Return_Key(K);
end Finalize;

```

We have chosen to use Initialize to set the initial value but we could have left this to be done by the default mechanism as before.

### Exercise 14.9

- ```
2 procedure Convert(From: in Set'Class;  
                  To: out Set'Class) is
```

```

3 procedure Convert(From: in Stack'Class;
                    To: out Stack'Class) is

```

The linked stack might be

with equality as in Section 12.4.

Exercise 15.1

- ```
begin
 return Slave(N);
exception
 when Constraint_Error | Storage_Error =>
 return -1;
end Factorial;
```

## 15.2

## Answers to exercises

23

*Exercise 15.2*

- 1 **package** Random **is**  
   Bad: **exception**;  
   Modulus: **constant** := 2\*\*13;  
   **subtype** Small **is** Integer **range** 0 .. Modulus;  
   **procedure** Init(Seed: **in** Small);  
   **function** Next **return** Small;  
**end**;  
**package body** Random **is**  
   Multiplier: **constant** := 5\*\*5;  
   X: Small;  
   **procedure** Init(Seed: **in** Small) **is**  
   **begin**  
     **if** Seed **mod** 2 = 0 **then**  
       **raise** Bad;  
     **end if**;  
     X := Seed;  
   **end Init**;  
   **function** Next **return** Small **is**  
   **begin**  
     X := X \* Multiplier **mod** Modulus;  
     **return** X;  
   **end Next**;  
**end Random**;  
 2 **function** Factorial(N: Integer) **return** Integer **is**  
   **function** Slave(N: Natural) **return** Positive **is**  
   **begin**  
     **if** N = 0 **then**  
       **return** 1;  
     **else**  
       **return** N \* Slave(N-1);  
     **end if**;  
   **end Slave**;  
   **begin**  
     **return** Slave(N);  
   **exception**  
     **when** Storage\_Error =>  
       **raise** Constraint\_Error;  
   **end Factorial**;  
 3 **function** "+" (X, Y: Vector) **return** Vector **is**  
   R: Vector(X'Range);  
   **begin**  
     **if** X'Length /= Y'Length **then**  
       **raise** Constraint\_Error;  
     **end if**;  
     **for** I **in** X'Range **loop**  
       R(I) := X(I) + Y(I+Y'First-X'First);  
     **end loop**;  
     **return** R;  
   **end "+"**;

- 4 No. A malevolent user could write

```
raise Stack.Error;
```

outside the package. It would be nice if the language provided some sort of 'private' exception that could be handled but not raised explicitly outside its defining package.

- 5 **procedure** Push(S: **in out** Stack; X: **in** Integer) **is**  
**begin**  
   S := **new** Cell'(S, X);  
**exception**  
   **when** Storage\_Error =>  
     **raise** Error;  
**end**;  
**procedure** Pop(S: **in out** Stack; X: **out** Integer) **is**  
**begin**  
   **if** S = null **then**  
     **raise** Error;  
   **else**  
     X := S.Value;  
     S := S.Next;  
   **end if**;  
**end**;

*Exercise 15.3*

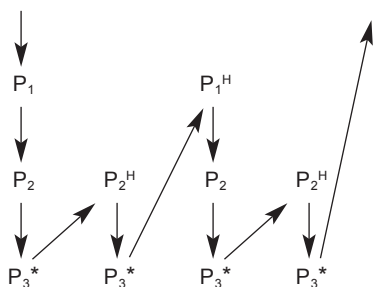
- 1 Four checks are required. The one inserted by the user, the overflow check on the assignment to Top plus the two for the assignment to S(Top) which cannot be avoided since we can say little about the value of Top (except that it is not equal to Max). So this is the worst of all worlds thus emphasizing the need to give appropriate constraints.

*Exercise 15.4*

- 1 The subprograms become  
**procedure** Push(X: Integer) **is**  
**begin**  
   **if** Top = Max **then**  
     **raise** Error **with** "stack overflow";  
   **end if**;  
   Top := Top + 1;  
   S(Top) := X;  
**end Push**;  
**function** Pop **return** Integer **is**  
**begin**  
   **if** Top = 0 **then**  
     **raise** Error **with** "stack underflow";  
   **end if**;  
   Top := Top - 1;  
   **return** S(Top + 1);  
**end Pop**;

2 Suppose  $N$  is 2. Then on the third call,  $P$  is not entered but the exception is raised and handled at the second level. The handler again calls  $P$  without success but this time, since an exception raised in a handler is not handled there but propagated up a level, the exception is handled at the first level. The pattern then repeats but the exception is finally propagated out of the first level to the originating call. In all there are three successful calls and four unsuccessful ones. The diagram opposite might help.

## 15.6



An \* indicates an unsuccessful call, H indicates a call from a handler.

More generally suppose  $C_n$  is the total number of calls for the case  $N = n$ . Then by induction

$$C_{n+1} = 2C_n + 1$$

with the initial condition  $C_0 = 1$  since in the case  $N = 0$  it is obvious that there is only one call which fails. It follows that the total number of calls  $C_N$  is  $2^{N+1} - 1$ . Of these  $2^N$  are unsuccessful and  $2^N - 1$  are successful.

I am grateful to Bob Bishop for this amusing example.

## Answers 16

### Exercise 16.1

- 1 **type** My\_Boolean **is new** Boolean  
  **with** Default\_Value => True;

In the case of most Boolean aspects such as `Inline` we can omit `True` which is then taken by default. But this does not apply to `Default_Value` since it can apply to any type and not just Boolean.

### Exercise 16.2

- 1 **overriding**  
  **function** Area(S: Square) **return** Float  
    **with** Pre => S.Side > 0.0 **and**  
       S.X\_Coord > S.Side,  
       Post => **abs** (Area'Result - S.Side\*\*2) > Eps;

The precondition ensures that all of the square is in the positive half-plane for some reason. The postcondition requires the answer to be correct within some small value Eps.

## Answers to exercises 25

### Exercise 16.3

- 1 **function** Is\_Unduplicated(S: Stack)  
  **return** Boolean **is**  
  **begin**  
    **for** I in 1 .. S.Top-1 **loop**  
      **for** J in I+1 .. S.Top **loop**  
        **if** S.S(I) = S.S(J) **then**  
          **return** False;  
        **end if**;  
      **end loop**;  
    **end loop**;  
    **return** True;  
  **end** Is\_Unduplicated;

- 2 **function** Is\_Unduplicated(S: Stack)  
  **return** Boolean **is**  
  (not(**for some** I in 1 .. S.Top-1 =>  
    (**for some** J in I+1 .. S.Top => S.S(I) = S.S(J))));

Note how this closely mimics the previous answer. Of course this expression function cannot be used as the precondition directly because it needs access to the implementation details. So it is just used as a completion. However, remember that it can be given in the private part as a completion and so is visible to the human reader of the package specification.

### Exercise 16.4

- 1 **subtype** Primary **is** Rainbow  
  **with** Static\_Predicate =>  
    Primary in Red | Yellow | Blue;
  - 2 **subtype** Curious **is** Integer  
  **with** Dynamic\_Predicate => Curious in 1 .. 999  
    **and** Curious **mod** 37 = 1;
- If we wanted to use a static predicate then we would have to write the possible values out thus
- subtype** Curious **is** Integer  
  **with** Static\_Predicate =>  
    Curious in 38 | 75 | 112 | 149 | 186 | 223 ... ;

### Exercise 16.6

- 1 **subtype** Even **is** Integer  
  **with** Dynamic\_Predicate => Even **mod** 2 = 0,  
    Predicate\_Failure => **raise** Constraint\_Error  
    **with** "something odd about even!";

## 26 Answers to exercises

### Answers 17

#### Exercise 17.1

- 1 P: on A: like Integer, on B: like Short\_Integer  
Q: on A: like Long\_Integer, on B: like Integer  
R: on A: not possible, on B: like Long\_Integer
- 2 No, the only critical case is type Q on machine A. Changing to one's complement changes the range of Integer to  
-32767 .. +32767  
and so only Integer'First is altered.
- 3 (a) Integer'Base  
(b) illegal – need explicit conversion  
(c) My\_Integer'Base  
(d) Integer'Base  
(e) *root\_integer*  
(f) My\_Integer'Base
- 4 **type** Longest\_Integer is  
    **range** System.Min\_Int .. System.Max\_Int;

#### Exercise 17.2

- 1 (a) 16#EF#                      (c) 222  
(b) 120                            (d) 239
- 2 **type** Ring5 is mod 5;  
A, B, C, D: Ring5;  
...  
D := (A + B) \* C;
- 3 (a) 4                              (b) 1  
DeMorgan's theorem that  
    **not** (A and B) = **not** A or **not** B  
does not hold if the modulus is not a power of two.
- 4 **type** Bearing is mod 360 \* 60;  
Degree: **constant** Bearing := 60;  
SE: **constant** Bearing := 135\*Degree;  
NNW: **constant** Bearing := 337\*Degree+30;  
NE\_by\_E: **constant** Bearing := 56\*Degree+15;

#### Exercise 17.3

- 1 (a) illegal                      (d) Integer'Base  
(b) Integer'Base              (e) *root\_real*  
(c) *root\_real*                  (f) *root\_integer*
- 2 R: **constant** := N \* 1.0;

#### Exercise 17.4

```
1 type Real is digits 7;
type Real_Vector is
 array (Integer range <>) of Real;

function Inner(A, B: Real_Vector) return Real is
 type Long_Real is digits 14;
 Result: Long_Real := 0.0;
begin
 for I in A'Range loop
 Result := Result +
 Long_Real(A(I)) * Long_Real(B(I));
 end loop;
 return Real(Result);
end Inner;
```

#### Exercise 17.5

```
1 The literal 0.5 is universal_real and this matches
 universal_fixed.

2 function "*" (X: Complex; N: Integer)
 return Complex is
 Result_θ: Angle := 0.0;
begin
 for I in 1 .. abs N loop
 Result_θ := Normal(Result_θ + X.θ);
 end loop;
 if N < 0 then
 Result_θ := -Result_θ;
 end if;
 return (X.R**N, Result_θ);
end "*";
```

We cannot simply write

```
return (X.R**N, Normal(X.θ * N));
```

because if **abs** N is larger than 3 the multiplication is likely to overflow; so we have to repeatedly normalize. A clever solution which is faster for all but the smallest values of **abs** N is

```
function "*" (X: Complex; N: Integer)
 return Complex is
 Result_θ: Angle := 0.0;
 Term: Angle := X.Theta;
 M: Integer := abs N;
begin
 while M > 0 loop
 if M rem 2 /= 0 then
 Result_θ := Normal(Result_θ + Term);
 end if;
 M := M / 2;
 Term := Normal(Term * 2);
 end loop;
 if N < 0 then
```

## 17.5

```

 Result_0 := -Result_0;
 end if;
 return (X.R**N, Result_0);
end "***";

```

This is a variation of the standard algorithm for computing exponentials by decomposing the exponent into its binary form and doing a minimal number of multiplications. In our case it is the multiplier N which we decompose and then do a minimal number of additions.

Recognizing that our repeated addition algorithm is essentially the same as for exponentiation, we can in parallel compute  $X.R^{**N}$  by the same method. A little manipulation soon makes us realize that we might as well write

```

function "***" (X: Complex; N: Integer)
 return Complex is
 One: constant Complex := Cons(1.0, 0.0);
 Result: Complex := One;
 Term: Complex := X;
 M: Integer := abs N;
 begin
 while M > 0 loop
 if M rem 2 /= 0 then
 Result := Result * Term; -- Complex *
 end if;
 M := M / 2;
 Term := Term * Term; -- Complex *
 end loop;
 if N < 0 then Result := One / Result; end if;
 return Result;
 end "***";

```

This brings us back full circle. This is the standard algorithm for computing exponentials applied in the abstract to our type Complex. Note the calls of "\*" and "/" applying to the type Complex. This version of "\*\*\*" can be declared outside the package Complex\_Numbers and is quite independent of the internal representation (but it will be very inefficient unless it is polar).

```

3 private
 type Angle is delta 0.05 range -720.0 .. 720.0;
 for Angle'Small use 2.0**(-5);
 type Complex is
 record
 R: Float;
 0: Angle range 0.0 .. 360.0;
 end record;
 I: constant Complex := (1.0, 90.0);
end;
...
function Normal(ϕ : Angle) return Angle is
begin

```

## Answers to exercises

27

```

if ϕ >= 360.0 then
 return ϕ - 360.0;
elsif ϕ < 0.0 then
 return ϕ + 360.0;
else
 return ϕ ;
end if;
end Normal;

```

The choice of delta and small is deduced as follows. We need 10 bits to cover the range 0 .. 720 plus one bit for the sign thus leaving 5 bits after the binary point. So *small* will be  $2^{-5}$  and thus any value of delta greater than that will do. We have chosen 0.05.

## Answers 18

## Exercise 18.1

1 Trace((M'Length, M))

If the two dimensions of M were not equal then Constraint\_Error would be raised. Note that the lower bounds of M do not have to be 1; all that matters is that the number of components in each dimension is the same since sliding is permitted when building the aggregate.

```

2 package Stacks is
 type Stack(Max: Natural) is private;
 Empty: constant Stack;
 ...
private
 type Integer_Vector is
 array (Integer range <>) of Integer;
 type Stack(Max: Natural) is
 record
 S: Integer_Vector(1 .. Max);
 Top: Integer := 0;
 end record;
 Empty: constant Stack(0) := (0, (others => 0), 0);
end;

```

We have naturally chosen to make Empty a stack whose value of Max is zero. Note that the function "=" only compares the parts of the stacks which are in use. Thus we can write  $S = \text{Empty}$  to test whether a stack S is empty irrespective of its value of Max.

```

3 function Is_Full(S: Stack) return Boolean is
begin
 return S.Top = S.Max;
end Is_Full;

4 S: constant Square := (N, Make_Unit(N));

```

## 28 Answers to exercises

### Exercise 18.2

- 1 **Z: Polynomial** := (0, (0 => 0));

The named notation has to be used because the array has only one component.

- 2 **function** "\*" (P, Q: Polynomial) **return** Polynomial **is**  
     R: Polynomial(P.N+Q.N) := (P.N+Q.N, (others => 0));  
**begin**  
     **for** I **in** P.A'Range **loop**  
         **for** J **in** Q.A'Range **loop**  
             R.A(I+J) := R.A(I+J) + P.A(I)\*Q.A(J);  
         **end loop**;  
     **end loop**;  
     **return** R;  
**end** "\*";

It is largely a matter of taste whether we write P.A'Range rather than 0 .. P.N.

- 3 **function** "-" (P, Q: Polynomial) **return** Polynomial **is**  
     Size: Integer;  
**begin**  
     **if** P.N > Q.N **then**  
         Size := P.N;  
     **else**  
         Size := Q.N;  
     **end if**;  
     **declare**  
         R: Polynomial(Size);  
     **begin**  
         **for** I **in** 0 .. P.N **loop**  
             R.A(I) := P.A(I);  
         **end loop**;  
         **for** I **in** P.N+1 .. R.N **loop**  
             R.A(I) := 0;  
         **end loop**;  
         **for** I **in** 0 .. Q.N **loop**  
             R.A(I) := R.A(I) - Q.A(I);  
         **end loop**;  
         **return** Normal(R);  
     **end**;  
**end** "-";

There are various other ways of writing this function. We could initialize R.A by using slice assignments

R.A(0 .. P.N) := P.A;  
 R.A(P.N+1 .. R.N) := (P.N+1 .. R.N => 0);

or even more succinctly by

R.A := P.A & (P.N+1 .. R.N => 0);

- 4 **procedure** Truncate(P: **in out** Polynomial) **is**  
**begin**

**if** P'Constrained **then**  
         **raise** Truncate\_Error;  
     **else**  
         P := (P.N-1, P.A(0 .. P.N-1));  
     **end if**;  
**end** Truncate;

- 5 Any unconstrained Polynomial could then include an array whose range is 0 .. Integer'Last. This will take a lot of space. Since most implementations are likely to adopt the strategy of setting aside the maximum possible space for an unconstrained record it is thus wise to keep the maximum to a practical limit by the use of a suitable subtype such as Index.

- 6 **type** Polynomial(N: Index := 0) **is**  
     **record**  
         A: Integer\_Vector(0 .. N) :=  
             (0 .. N-1 => 0) & (N => 1);  
     **end record**;

We cannot write a single aggregate because an aggregate can only have one dynamic choice which must be the only choice. An alternative approach is to declare a function thus

**function** F(N: Integer) **return** Integer\_Vector;

**type** Polynomial(N: Index := 0) **is**  
     **record**  
         A: Integer\_Vector(0 .. N) := F(N);  
     **end record**;

...

**function** F(N: Integer) **return** Integer\_Vector **is**  
     R: Integer\_Vector(0 .. N);

**begin**  
     **for** I **in** 0 .. N-1 **loop**  
         R(I) := 0;  
     **end loop**;  
     R(N) := 1;  
     **return** R;  
**end** F;

If the full type Polynomial is declared in the private part of a package then the function specification can also be in the private part with the function body in the package body. It does not matter that F is referred to before its body is elaborated provided that it is not actually called. If we declared a polynomial (without an initial value) before the body of F then Program\_Error would be raised.

Clearly any initial value can be computed this way even if it cannot be written as aggregates.

```

7 package Rational_Polynomials is
 Max: constant := 10;
 subtype Index is Integer range 0 .. Max;
 type Rational_Polynomial(N, D: Index := 0) is
 private;

 function "+" (X: Rational_Polynomial)
 return Rational_Polynomial;
 function "-" (X: Rational_Polynomial)
 return Rational_Polynomial;
 function "+" (X, Y: Rational_Polynomial)
 return Rational_Polynomial;
 function "-" (X, Y: Rational_Polynomial)
 return Rational_Polynomial;
 function "*" (X, Y: Rational_Polynomial)
 return Rational_Polynomial;
 function "/" (X, Y: Rational_Polynomial)
 return Rational_Polynomial;
 function "/" (X, Y: Polynomial)
 return Rational_Polynomial;
 function Numerator(R: Rational_Polynomial)
 return Polynomial;
 function Denominator(R: Rational_Polynomial)
 return Polynomial;
private
 type Rational_Polynomial(N, D: Index := 0) is
 record
 Num: Polynomial(N) := (N, (0 .. N => 0));
 Den: Polynomial(D) :=
 (N, (0 => 1) & (1 .. N => 0));
 end record;
end;

It might be more elegant to write functions Zero
and One taking a parameter giving the value of
N like the function F of the previous exercise.
We make no attempt to impose any special
language constraint on the denominator as we
did in the type Rational where the denominator
has subtype Positive.

8 function "&" (X, Y: V_String) return V_String is
begin
 return (X.N + Y.N, X.S & Y.S);
end "&";

```

*Exercise 18.3*

```

1 procedure Shave(P: in out Person) is
begin
 if P.Sex = Female then
 raise Shaving_Error;
 else
 P.Bearded := False;
 end if;
end Shave;

2 procedure Sterilize(M: in out Mutant) is
begin
 if M.Constrained and M.Sex /= Neuter then
 raise Sterilize_Error;
 else
 M := (Neuter, M.Birth);
 end if;
end Sterilize;

3 type Figure is (Circle, Point, Triangle);
type Object(Shape: Figure) is
 record
 X_Coord: Float;
 Y_Coord: Float;
 case Shape is
 when Circle =>
 Radius: Float;
 when Point =>
 null;
 when Triangle =>
 A, B, C: Float;
 end case;
 end record;

function Area(X: Object) return Float is
begin
 case X.Shape is
 when Circle =>
 return Pi * X.Radius**2;
 when Point =>
 return 0.0;
 when Triangle =>
 return ... ;
 end case;
end Area;

```

Note the similarity between the case statement in the function Area and the variant part of the type Object.

### 30 Answers to exercises

```

4 type Category is (Basic, Nice, Posh);
type Position is (Aisle, Window);
type Meal_Type is (Green, White, Red);
type Reservation(C: Category) is
 record
 Flight_Number: Integer;
 Date_Of_Travel: Date;
 Seat_Number: String(1 .. 3) := " ";
 case C is
 when Basic => null;
 when Nice | Posh =>
 Seat_Sort: Position;
 Food: Meal_Type;
 case C is
 when Basic | Nice => null;
 when Posh =>
 Destination: Address;
 end case;
 end case;
end record;

```

Note the curiously nested structure which is forced upon us by the rule that the individual components must have distinct names.

#### Exercise 18.4

```

1 type Boxer(W: Weight; Sex: Gender) is
 new Person(Sex => Sex) with
 record
 ...
 end record;

```

We have chosen to give the new discriminant Sex the same name as the old one.

```

2 function Geometry.Polygons.Four_Sided.
 Make_Quadrilateral(Sides, Angles: Float_Array)
 return Quadrilateral is
 P: Polygon := Make_Polygon(Sides, Angles);
begin
 if P.No_Of_Sides /= 4 then
 raise Queer_Quadrilateral;
 end if;
 return (P with null record);
end Geometry.Polygons.Four_Sided.
 Make_Quadrilateral;

3 package Geometry.Polygons.Four_Sided.
 Conversions is
 function To_Parallelogram(Q: Quadrilateral'Class)
 return Parallelogram;
 function To_Square(Q: Quadrilateral'Class)
 return Square;
end;
package body Geometry.Polygons.Four_Sided.
 Conversions is

```

```

function To_Parallelogram(Q: Quadrilateral'Class)
 return Parallelogram is
begin
 if Q.Sides(1) /= Q.Sides(3) or
 Q.Sides(2) /= Q.Sides(4) then
 raise Poor_Parallelogram;
 end if;
 return (Quadrilateral(Q) with null record);
end To_Parallelogram;

function To_Square(Q: Quadrilateral'Class)
 return Square is
 P: Parallelogram := To_Parallelogram(Q);
begin
 if P.Sides(1) /= P.Sides(2) or
 P.Angles(1) /= P.Angles(2) then
 raise Silly_Square;
 end if;
 return (P with null record);
end To_Square;

```

**end** Geometry.Polygons.Four\_Sided.Conversions;

These two functions suffice. Putting them in a child package ensures that there are no problems of going abstract on extension. Using a class wide parameter gives greater flexibility. We can convert towards the type Quadrilateral by normal type conversion. Other intermediate types such as Rectangle or Rhombus could be dealt with in a similar manner.

Note also the use of the extension aggregates in the return statements – and especially that the ancestor type must be specific and so a type conversion is required in To\_Parallelogram in order to convert the class wide formal parameter Q to the specific type Quadrilateral.

#### Exercise 18.5

```

1 function Heir(P: Person_Name) return Person_Name is
 Mother: Womans_Name;
begin
 if P.Sex = Male then
 Mother := P.Wife;
 else
 Mother := P;
 end if;
 if Mother = null
 or else Mother.First_Child = null then
 return null;
 end if;
 declare
 Child: Person_Name := Mother.First_Child;
 begin
 while Child.Sex = Female loop
 if Child.Next_Sibling = null then
 return Mother.First_Child;
 end if;
 end loop;
 end;

```

## 18.5

- ```

        end if;
        Child := Child.Next_Sibling;
    end loop;
    return Child;
end;
end Heir;

```
- 2 **procedure** Divorce(W: Womans_Name) **is**
begin
 if W.Husband = null or W.First_Child /= null then
 return; -- divorce not possible
 end if;
 W.Husband.Wife := null;
 W.Husband := null;
end Divorce;
- 3 **procedure** Marry(Bride: Womans_Name;
 Groom: Mans_Name) **is**
begin
 if Bride.Father = Groom.Father then
 raise Incest;
 end if;
 ... -- then as before
end Marry;
- Note that there is no need to check for marriage to a parent because the check for bigamy will detect this anyway. Our model does not allow remarriage if there are children.
- 4 Marry is unchanged. Note however that calls of Marry with parameters of the wrong sex will be detected at compile time whereas with variants this is detected at run time.
- 5 **function** Spouse(P: Person_Name) **return** Person_Name **is**
begin
 if P in Man then
 return Mans_Name(P).Wife;
 else
 return Womans_Name(P).Husband;
 end if;
end Spouse;
- This is not good because not only is there a check required in doing the membership test but also a conversion to the appropriate specific type so that the component can be selected; this conversion requires yet another check (which always passes and might be optimized away).
 A better solution is to make Spouse a primitive abstract operation of Person with an access parameter
- ```

function Spouse(P: access Person)
 return Person_Name is abstract;

```
- and then provide specific functions for each sex

## Answers to exercises 31

- ```

function Spouse(P: access Man)
    return Person_Name is
begin
    return P.Wife;
end Spouse;

function Spouse(P: access Woman)
    return Person_Name is
begin
    return P.Husband;
end Spouse;

```
- A call of Spouse will then resolve at compile time to the correct function if the parameter is of a specific type (such as Womans_Name) or will dispatch if it is class wide (Person_Name); this gives the best of all worlds.
- 6 **function** New_Child(Mother: Womans_Name;
 Boy_Or_Girl: Gender;
 Birthday: Date) **return** Person_Name **is**
 Child: Person_Name;
begin
 if Mother.Husband = null then
 raise Out_Of_Wedlock;
 end if;
case Boy_Or_Girl **is**
when Male => Child := new Man;
when Female => Child := new Woman;
end case;
 Child.Birth := Birthday;
 ... -- and so on as before
end New_Child;
- This feels most uncomfortable. It seems strange to have a parameter giving the sex because we have not otherwise had to introduce the type Gender. We could pass the tag (such as ManTag) as a parameter T but that seems really dirty and anyway we would still have to write a conditional statement
- ```

if T = ManTag then
 Child := new Man;
else
 Child := new Woman;
end if;

```
- This example illustrates a common problem with polymorphism; it all works fine for output when we know what we have but it is difficult for input when we do not.

## 32 Answers to exercises

### Exercise 18.6

- 1 If the user declared a constrained key with a nonzero discriminant thus
- K: Key(7);
- then he will have bypassed Get\_Key and be able to call the procedure Action without authority. Note also that if he calls Return\_Key then Constraint\_Error will be raised on the attempt to set the code to zero because the key is constrained.

Hence forged keys can be recognized since they are constrained and so we could rewrite Valid to check for this

```
function Valid(K: Key) return Boolean is
begin
 return not K'Constrained and K.Code /= 0;
end Valid;
```

We must also insert calls of Valid into Get\_Key and Return\_Key.

### Exercise 18.7

- 1 The first assignment is illegal because there is a type mismatch, the second inserts the appropriate conversion but still fails because of the dynamic accessibility check. The third is illegal because the type is limited.

## Answers 19

### Exercise 19.1

- 1 **generic**
- ```
type Item is private;
package Stacks is
  type Stack(Max: Natural) is private;
  procedure Push(S: in out Stack; X: in Item);
  procedure Pop(S: in out Stack; X: out Item);
  function "=" (S, T: Stack) return Boolean;
private
  type Item_Array is
    array (Integer range <>) of Item;
  type Stack(Max: Natural) is
    record
      S: Item_Array(1 .. Max);
      Top: Integer := 0;
    end record;
end Stacks;
```

The body is much as before. To declare a stack we must first instantiate the package thus

```
package Boolean_Stacks is
  new Stacks(Item => Boolean);
```

and then

```
use Boolean_Stacks;
S: Stack(Max => 30);
```

- 2 **generic**
- ```
type Thing is private;
package P is
 procedure Swap(A, B: in out Thing);
 procedure CAB(A, B, C: in out Thing);
end P;

package body P is
 procedure Swap(A, B: in out Thing) is
 T: Thing;
 begin
 T := A; A := B; B := T;
 end;

 procedure CAB(A, B, C: in out Thing) is
 begin
 Swap(A, B); Swap(A, C);
 end;
end P;
```

### Exercise 19.2

- 1 **function** "not" is new Next(Boolean);
- 2 **generic**
- ```
type Number is range <>;
package Rational_Numbers is
  type Rational is private;
  function "+" (X: Rational) return Rational;
  function "-" (X: Rational) return Rational;
  function "+" (X, Y: Rational) return Rational;
  function "-" (X, Y: Rational) return Rational;
  function "*" (X, Y: Rational) return Rational;
  function "/" (X, Y: Rational) return Rational;

  subtype Positive_Number is
    Number range 1 .. Number'Last;
  function "/" (X: Number; Y: Positive_Number)
    return Rational;

  function Numerator(R: Rational) return Number;
  function Denominator(R: Rational)
    return Positive_Number;

private
  type Rational is
    record
      Num: Number := 0;
      Den: Positive_Number := 1;
    end record;
end;
```
- 3 **generic**
- ```
type Index is (<>);
type Floating is digits <>;
type Vec is array (Index range <>) of Floating;
```

## 19.2

```

type Mat is array (Index range <>,
 Index range <>) of Floating;
function Outer(A, B: Vec) return Mat;
function Outer(A, B: Vec) return Mat is
 C: Mat(A'Range, B'Range);
begin
 for I in A'Range loop
 for J in B'Range loop
 C(I, J) := A(I) * B(J);
 end loop;
 end loop;
return C;
end Outer;

function Outer_Vector is
 new Outer(Integer, Float, Vector, Matrix);

4 package body Set_Of is
 function Make_Set(L: List) return Set is
 S: Set := Empty;
 begin
 for I in L'Range loop
 S(L(I)) := True;
 end loop;
 return S;
 end Make_Set;

 function Make_Set(E: Element) return Set is
 S: Set := Empty;
 begin
 S(E) := True;
 return S;
 end Make_Set;

 function Decompose(S: Set) return List is
 L: List(1 .. Size(S));
 I: Positive := 1;
 begin
 for E in Set'Range loop
 if S(E) then
 L(I) := E; I := I + 1;
 end if;
 end loop;
 return L;
 end Decompose;

 function "+" (S, T: Set) return Set is
 begin
 return S or T;
 end "+";

 function "*" (S, T: Set) return Set is
 begin
 return S and T;
 end "*";

 function "-" (S, T: Set) return Set is
 begin
 return S xor T;
 end "-";

```

## Answers to exercises 33

```

function "<" (E: Element; S: Set) return Boolean is
begin
 return S(E);
end "<";

function "<=" (S, T: Set) return Boolean is
begin
 return (S and T) = S;
end "<=";

function Size(S: Set) return Natural is
 N: Natural := 0;
begin
 for E in Set'Range loop
 if S(E) then
 N := N + 1;
 end if;
 end loop;
 return N;
end Size;

end Set_Of;

```

Sadly, we cannot use renaming as bodies for "+", "\*" and "-". This is because the functions become frozen at the end of the package specification (see Section 25.1) and are given convention Ada by default whereas the predefined operations "or" etc. have convention Intrinsic and so do not match. On the other hand, we could put such renamings in the private part of the package and then the new operations would take the convention of the renamed operations and so themselves be Intrinsic.

```

5 private
 type Element_Array is array (Element) of Boolean;
 type Set is
 record
 Value: Element_Array := (Element => False);
 end record;

 Empty: constant Set :=
 (Value => (Element => False));
 Full: constant Set :=
 (Value => (Element => True));

end;

```

We have to make the full type into a record containing the array as a single component to give it a default initial expression. Sadly, this means that the body needs rewriting and the functions become rather untidy. Also we cannot write the default expression as Empty.Value. This is because we cannot use the component name Value in its own declaration. In general, however, we can use a deferred constant as a default value before its full declaration. Note also that we have to use the named notation for the single component record aggregates.

## 34 Answers to exercises

### Exercise 19.3

- 1 First we have to declare our function "<" which we define as follows: if the polynomials have different degrees, the one with the lower degree is smaller; if the same degree, then we compare coefficients starting at the highest power. So

```
function "<" (X, Y: Polynomial) return Boolean is
begin
 if X.N /= Y.N then
 return X.N < Y.N;
 end if;
 for I in reverse 0 .. X.N loop -- or X.A'Range
 if X.A(I) /= Y.A(I) then
 return X.A(I) < Y.A(I);
 end if;
 end loop;
 return False; -- they are identical
end "<";

procedure Sort_Poly is
 new Sort(Integer, Polynomial, Poly_Array);
```

- 2 **type** Mutant\_Array **is**  
**array** (Integer range <>) **of** Mutant;

```
function "<" (X, Y: Mutant) return Boolean is
begin
 if X.Sex /= Y.Sex then
 return X.Sex > Y.Sex;
 else
 return Y.Birth < X.Birth;
 end if;
end "<";

procedure Sort_Mutant is
 new Sort(Integer, Mutant, Mutant_Array);
```

Note that the order of sexes asked for is precisely the reverse order to that in the type Gender and so we can directly use ">" applied to that type. Similarly, younger first means later birth date first and so we use the function "<" we have already defined for the type Date but with the arguments reversed.

We could not sort an array of type Person because we cannot declare such an array anyway since Person is indefinite.

- 3 We cannot do this because the array is of an anonymous type.
- 4 We might get Constraint\_Error. If C'First = Index'Base'First then the attempt to evaluate Index'Pred(C'Last) will raise Constraint\_Error. Considerable care can be required to make such extreme cases foolproof. The easy way out in this case is simply to insert

**if** C'Length < 2 **then return**; **end if**;

- 5 The generic body corresponds closely to the procedure Sort in Section 11.2. The type Vector is replaced by Collection. I is of type Index. The types Node and Node\_Ptr are declared inside Sort because they depend on the generic type Item. The incrementing of I cannot be done with "+" since the index type may not be an integer and so we have to use Index'Succ. Care is needed not to cause Constraint\_Error if the array embraces the full range of values of Index. But, the key thing is that the generic specification is completely unchanged and so we see how an alternative body can be sensibly supplied.

```
6 with Ada.Exceptions; use Ada.Exceptions;
generic
 type Index is (<>);
 type Item is limited private;
 type Collection is array (Index range <>) of Item;
 with function Is_It(X: Item) return Boolean;
 Ex: Exception_Id := Constraint_Error'Identify;
function Search(C: Collection) return Index;
function Search(C: Collection) return Index is
begin
 for J in C'Range loop
 if Is_It(C(J)) then return J; end if;
 end loop;
 Raise_Exception(Ex);
end Search;
```

```
7 generic
 type Item is private;
 type Vector is array (Integer range <>) of Item;
 with function "=" (X, Y: Item) return Boolean is <>;
function Equals(A, B: Vector) return Boolean;
function Equals(A, B: Vector) return Boolean is
begin
 ... -- body exactly as for Exercise 11.4(4)
end Equals;
```

We can instantiate by

```
function "=" is new Equals(Stack, Stack_Array, "=");
```

or simply by

```
function "=" is new Equals(Stack, Stack_Array);
```

in which case the default parameter is used.

Note that it is essential to pass "=" as a parameter otherwise predefined equality would be used and the whole point is that we have redefined "=" for the type Stack.

## 19.3

```

8 generic
 type Floating is digits <>;
 with function F(X: Floating) return Floating;
 function Solve return Floating;

 function G(X: Float) return Float is
 begin
 return Exp(X) + X - 7.0;
 end;
 ...
 function Solve_G is new Solve(Float, G);
 ...
 Answer: Float := Solve_G;

Exercise 19.4

1 package body Generic_Complex_Functions is
 use Elementary_Functions;

 function Sqrt(X: Complex) return Complex is
 begin
 return Cons_Polar(Sqrt(abs X), 0.5*Arg(X));
 end Sqrt;

 function Log(X: Complex) return Complex is
 begin
 return Cons(Log(abs X), Arg(X));
 end Log;

 function Exp(X: Complex) return Complex is
 begin
 return Cons_Polar(Exp(RI_Part(X)), Im_Part(X));
 end Exp;

 function Sin(X: Complex) return Complex is
 RI: Float_Type := RI_Part(X);
 Im: Float_Type := Im_Part(X);
 begin
 return Cons(Sin(RI)*Cosh(Im), Cos(RI)*Sinh(Im));
 end Sin;

 function Cos(X: Complex) return Complex is
 RI: Float_Type := RI_Part(X);
 Im: Float_Type := Im_Part(X);
 begin
 return Cons(Cos(RI)*Cosh(Im), -Sin(RI)*Sinh(Im));
 end Cos;

end Generic_Complex_Functions;

2 package Poly_Vector is new General_Vector
 (Integer, Polynomial, Poly_Array);

 procedure Sort_Poly is new Sort(Poly_Vector);

3 generic
 with package Signature is new Group(<>);
 use Signature;
 function Power(E: Element; N: Integer)
 return Element;

```

## Answers to exercises 35

```

function Power(E: Element; N: Integer)
 return Element is
 Result: Element := Identity;
begin
 for I in 1 .. abs N loop
 Result := Op(Result, E);
 end loop;
 if N < 0 then Result := Inverse(Result); end if;
 return Result;
end Power;
...
package Integer_Addition_Group is
 new Group(Element => Integer, Identity => 0,
 Op => "+", Inverse => "-");

function Multiply is
 new Power(Integer_Addition_Group);

4 generic
 type Element is (<>);
 Identity: in Element;
 with function Op(X, Y: Element) return Element;
 with function Inverse(X: Element) return Element;
package Finite_Group is end;

generic
 with package Signature is new Finite_Group(<>);
 use Signature;
function Is_Group return Boolean;

function Is_Group return Boolean is
begin
 -- check the operation is closed,
 -- the actual parameter could be constrained
 for E in Element'Range loop
 for F in Element'Range loop
 declare
 Result: Element;
 begin
 Result := Op(E, F);
 exception
 when Constraint_Error =>
 return False;
 end;
 end loop;
 end loop;

 -- check identity is OK
 for E in Element'Range loop
 if Op(E, Identity) /= E or
 Op(Identity, E) /= E then
 return False;
 end if;
 end loop;
end loop;

```

## 36 Answers to exercises

```

-- check inverse is OK
for E in Element'Range loop
 if Op(E, Inverse(E)) /= Identity or
 Op(Inverse(E), E) /= Identity then
 return False;
 end if;
end loop;

-- check associative law OK
for E in Element'Range loop
 for F in Element'Range loop
 for G in Element'Range loop
 if Op(E, Op(F, G)) /= Op(Op(E, F), G) then
 return False;
 end if;
 end loop;
 end loop;
end loop;
return True;
end Is_Group;

```

### Exercise 19.5

```

1 generic
 type Floating is digits <>;
package Generic_Complex_Numbers is
 ...
end Generic_Complex_Numbers;

generic
package Generic_Complex_Numbers.Cartesian is
 ...
end Generic_Complex_Numbers.Cartesian;

generic
package Generic_Complex_Numbers.Polar is
 ...
end Generic_Complex_Numbers.Polar;

with Ada.Numerics.Generic_Elementary_Functions;
use Ada.Numerics;
with Generic_Complex_Numbers;
with Generic_Complex_Numbers.Cartesian;
with Generic_Complex_Numbers.Polar;
generic
 with package Elementary_Functions is
 new Generic_Elementary_Functions(<>);
 with package Complex_Numbers is
 new Generic_Complex_Numbers
 (Elementary_Functions.Float_Type);
 with package Cartesian is
 new Complex_Numbers.Cartesian;
 with package Polar is
 new Complex_Numbers.Polar;
package Generic_Complex_Functions is
 use Complex_Numbers, Cartesian, Polar;

 function Sqrt(X: Complex) return Complex;

```

```

...
end Generic_Complex_Functions;

```

Note that the generic formal ensures that the packages passed as actuals are correctly related. For example if we did two instantiations of the hierarchy (one for Float and one for Long\_Float) then we need to ensure that we do not use the parent from one with a child from the other. However, there is no such guarantee if the hierarchy has a generic subprogram as a child since we can only express the requirement that the profile is correct; this will be enough in most cases but is not foolproof. Of course the program would not crash, just do something silly.

## Answers 20

### Exercise 20.1

```

1 procedure Shopping is
 task Get_Salad;
 task body Get_Salad is
 begin
 Buy_Salad;
 end Get_Salad;
 task Get_Wine;
 task body Get_Wine is
 begin
 Buy_Wine;
 end Get_Wine;
 task Get_Meat;
 task body Get_Meat is
 begin
 Buy_Meat;
 end Get_Meat;
begin
 null;
end Shopping;

```

### Exercise 20.2

```

1 task body Char_To_Line is
 Buffer: Line;
begin
 loop
 for I in Buffer'Range loop
 accept Put(C: in Character) do
 Buffer(I) := C;
 end;
 end loop;
 accept Get(L: out Line) do

```

## 20.2

## Answers to exercises 37

```

 L := Buffer;
 end;
end loop;
end Char_To_Line;

```

*Exercise 20.3*

```

1 with Calendar;
generic
 First_Time: Calendar.Time;
 Interval: Duration;
 Number: Integer;
 with procedure P;
procedure Call;
procedure Call is
 use type Calendar.Time;
 Next_Time: Calendar.Time := First_Time;
 Now: Calendar.Time := Calendar.Clock;
begin
 if Next_Time < Now then
 Next_Time := Now;
 end if;
 for I in 1 .. Number loop
 delay until Next_Time;
 P;
 Next_Time := Next_Time + Interval;
 end loop;
end Call;

```

2 The trouble with writing

```

 delay Next_Time - Clock;

```

is that the task might be temporarily suspended between calling Clock and issuing the delay; the delay would then be wrong by the amount of time for which the task did not have a processor.

```

3 use Calendar;
 Date: Time;
 Y: Year_Number;
 M: Month_Number;
 D: Day_Number;
 S: Day_Duration;
 High_Noon: Time;
 ...
 Split(Clock, Y, M, D, S);
 High_Noon := Time_Of(Y, M, D, 43_200.0);
 if S > 43_200.0 then -- afternoon, so add a day
 High_Noon := High_Noon + 86_400.0;
 end if;

```

*Exercise 20.4*

```

1 protected Variable is
 entry Read(Value: out Item);
 procedure Write(New_Value: in Item);

```

```

private
 Data: Item;
 Value_Set: Boolean := False;
end Variable;

protected body Variable is
 entry Read(Value: out Item) when Value_Set is
 begin
 Value := Data;
 end Read;

 procedure Write(New_Value: in Item) is
 begin
 Data := New_Value;
 Value_Set := True; -- clear the barrier
 end Write;
end Variable;

```

The problem with this solution is that it no longer allows multiple readers because the function has been replaced by an entry.

2 This is a bit of a trick question. It cannot be done because a discriminant is not static and therefore cannot be used to declare the type Index. One possible alternative is to make the index type of the array and the type of the variables In\_Ptr and Out\_Ptr to be type Integer and to use the **mod** operator to do the cyclic arithmetic. The structure would then be

```

generic
 type Item is private;
package Buffers is
 type Item_Array is array (Integer range <>) of Item;
 protected type Buffering(N: Integer) is
 -- and so on
 end Buffering;
end Buffers;

```

```

3 protected Buffer is
 entry Put(X: in Item);
 entry Get(X: out Item);
private
 V: Item;
 Is_Set: Boolean := False;
end;

protected body Buffer is
 entry Put(X: in Item) when not Is_Set is
 begin
 V := X;
 Is_Set := True;
 end Put;

 entry Get(X: out Item) when Is_Set is
 begin
 X := V;
 Is_Set := False;
 end Get;
end Buffer;

```

## 38 Answers to exercises

```

4 protected Char_To_Line is
 entry Put(C: in Character);
 entry Get(L: out Line);
private
 Buffer: Line;
 Count: Integer := 0; -- number of items in buffer
end;

protected body Char_To_Line is
 entry Put(C: in Character) when
 Count < Buffer'Last is
 begin
 Count := Count + 1;
 Buffer(Count) := C;
 end Put;

 entry Get(L: out Line) when Count = Buffer'Last is
 begin
 Count := 0;
 L := Buffer;
 end Get;
end Char_To_Line;

```

### Exercise 20.7

```

1 protected type Mailbox is
 entry Deposit(X: in Item);
 entry Collect(X: out Item);
private
 Full: Boolean := False;
 Local: Item;
end;

protected body Mailbox is
 entry Deposit(X: in Item) when not Full is
 begin
 Local := X;
 Full := True;
 end Deposit;

 entry Collect(X: out item) when Full is
 begin
 X := Local;
 Full := False;
 end Collect;
end Mailbox;

```

This mailbox is reusable whereas the task version was not (indeed the task just terminated after use). We could prevent the protected object from being reused by further state variables.

The advantages of the protected object are that there need be no concern with termination in the event of it not being used and it is of course much more efficient. A possible disadvantage is that the closely coupled form is not possible.

### Exercise 20.8

```

1 task type Buffering is
 entry Put(X: in Item);
 entry Finish;
 entry Get(X: out Item);
end;

task body Buffering is
 N: constant := 8;
 type Index is mod N;
 A: array (Index) of Item;
 In_Ptr, Out_Ptr: Index := 0;
 Count: Integer range 0 .. N := 0;
 Finished: Boolean := False;
begin
 loop
 select
 when Count < N =>
 accept Put(X: in Item) do
 A(In_Ptr) := X;
 end;
 In_Ptr := In_Ptr + 1; Count := Count + 1;
 or
 accept Finish;
 Finished := True;
 or
 when Count > 0 =>
 accept Get(X: out Item) do
 X := A(Out_Ptr);
 end;
 Out_Ptr := Out_Ptr + 1; Count := Count - 1;
 or
 when Count = 0 and Finished =>
 accept Get(X: out Item) do
 raise Done;
 end;
 end select;
 end loop;
exception
 when Done =>
 null;
end Buffering;

```

This curious example illustrates that there may be several accept statements for the same entry in the one select statement. The exception Done is propagated to the caller and also terminates the loop in Buffering before being quietly handled. Of course the exception need not be handled by Buffering because exceptions propagated out of tasks are lost, but it is cleaner to do so.

- 2 The server aborts the caller during the rendezvous thereby placing the caller into an abnormal state. Although the caller cannot be

## 20.8

properly completed until after the rendezvous is finished (the server might have access to the caller's data space via a parameter), nevertheless the caller is no longer active and does not receive the exception Havoc.

```

3 select
 Trigger.Wait;
then abort
 loop
 ... -- compute next estimate in Z
 -- then store it in the protected object
 Result.Put_Estimate(Z);
 -- loop back to improve estimate
 end loop;
end select;

```

*Exercise 20.9*

1 If we wrote

**requeue Reset with abort;**

then there would be a risk that the task that called Signal was aborted before it could clear the occurred flag. The system would then be in a mess since subsequent tasks calling Wait could proceed without waiting for the next signal.

```

2 protected Event is
 entry Wait;
 procedure Signal;
private
 Occurred: Boolean := False;
end Event;

protected body Event is
 entry Wait when Occurred is
 begin
 if Wait'Count = 0 then
 Occurred := False;
 end if;
 end Wait;

 procedure Signal is
 begin
 if Wait'Count > 0 then
 Occurred := True;
 end if;
 end Signal;
end Event;

```

The last of the waiting tasks to be let go clears the occurred flag back to false (the last one out switches off the light). It is important that the procedure Signal does not set the occurred flag if there are no tasks waiting since in such a case there is no waiting task to clear it and the signal would persist (remember this is a model of a

## Answers to exercises

39

transient signal).

An amazing alternative solution is

```

protected Event is
 entry Wait;
 entry Signal;
end Event;

protected body Event is
 entry Wait when Signal'Count > 0 is
 begin
 null;
 end Wait;

 entry Signal when Wait'Count = 0 is
 begin
 null;
 end Signal;
end Event;

```

This works because joining an entry queue is a protected action and results in the evaluation of barriers (just as they are evaluated when a protected procedure or entry body finishes).

Note that there is no protected data (and hence no private part) and that both entry bodies are null; in essence the protected data is the Count attributes and these therefore behave properly. In contrast, the Count attributes of task entries are not reliable because joining and leaving task entry queues are not protected in any way.

```

3 task Controller is
 entry Sign_In(P: Priority; D: Data);
private
 entry Request(Priority) (P: Priority; D: Data);
end;

task body Controller is
 Total: Integer := 0;
begin
 loop
 if Total = 0 then
 accept Sign_In(P: Priority; D: Data) do
 Total := 1;
 requeue Request(P);
 end;
 end if;
 loop
 select
 accept Sign_In(P: Priority; D: Data) do
 Total := Total + 1;
 requeue Request(P);
 end;
 else
 exit;
 end select;
 end loop;

```

## 40 Answers to exercises

```

for P in Priority loop
 select
 accept Request(P) (P: Priority; D: Data) do 1
 Action(D);
 end;
 Total := Total - 1;
 exit;
 else
 null;
 end select;
end loop;
end loop;
end Controller;

```

The variable Total records the total number of requests outstanding. Each time round the outer loop, the task waits for a call of Sign\_In if no requests are in the system, it then services any outstanding calls of Sign\_In. The calls to Sign\_In requeue onto the appropriate member of the entry family Request. The task then deals with a request of the highest priority. Observe that we had to make P a parameter of the entry family as well as the index; this is because requeue can only be to an entry with the same parameter profile (or parameterless).

Note that the solution works if a calling task is aborted; this is because the requeue does not specify **with abort** and so is considered as part of the abort deferred region of the original rendezvous.

```

4 package Monitor is
 protected Call is
 entry Job(D: Data);
 end;
 private
 task The_Task is
 entry Job(D: Data);
 end;
 end;
 package body Monitor is
 protected body Call is
 entry Job(D: Data) when True is
 begin
 Log_The_Call(Calendar.Clock);
 requeue The_Task.Job;
 end Job;
 end Call;
 ...
 end Monitor;

```

### Exercise 20.10

```

package Cobblers is
 procedure Mend(A: Address; B: Boots);
end;

package body Cobblers is
 type Job is
 record
 Reply: Address;
 Item: Boots;
 end record;

 package P is new Buffers(Job);
 use P;
 Boot_Store: Buffer(100);

 task Server is
 entry Request(A: Address; B: Boots);
 end;

 task type Repairman;
 Tom, Dick, Harry: Repairman;

 task body Server is
 Next_Job: Job;
 begin
 loop
 accept Request(A: Address; B: Boots) do
 Next_Job := (A, B);
 end;
 Put(Boot_Store, Next_Job);
 end loop;
 end Server;

 task body Repairman is
 My_Job: Job;
 begin
 loop
 Get(Boot_Store, My_Job);
 Repair(My_Job.Item);
 My_Job.Reply.Deposit(My_Job.Item);
 end loop;
 end Repairman;

 procedure Mend(A: Address; B: Boots) is
 begin
 Server.Request(A, B);
 end;
end Cobblers;

```

We have assumed that the type Address is an access to a mailbox for handling boots. Note one anomaly; the server accepts boots from the customer before checking the store – if it turns out to be full, he is left holding them. In all, the shop can hold 104 pairs of boots – 100 in store, 1 with the server and 1 with each repairman.

## Answers 21

### Exercise 21.1

```

1 function Volume(C: Cylinder) return Float is
 begin
 return Area(C.Base) * C.Height;
 end Volume;

function Area(C: Cylinder) return Float is
 begin
 return 2.0*Area(C.Base) +
 2.0*Pi*C.Base.Radius*C.Height;
 end Area;

```

We cannot apply the function Moment to a cylinder because the type Cylinder is not in Object'Class. We are thus protected from such foolishness.

### Exercise 21.3

```

1 with Ada.Finalization; use Ada;
generic
 type Raw_Type is tagged private;
package Tracking is
 type Tracked_Type is new Raw_Type with private;
 function Identity(TT: Tracked_Type) return Integer;
private
 type Control is new Finalization.Controlled with
 record
 Identity_Number: Integer;
 end record;

 procedure Initialize(C: in out Control);
 procedure Adjust(C: in out Control);
 procedure Finalize(C: in out Control);

 type Tracked_Type is new Raw_Type with
 record
 Component: Control;
 end record;
end Tracking;

package body Tracking is
 The_Count: Integer := 0;
 Next_One: Integer := 1;

 function Identity(TT: Tracked_Type)
 return Integer is
 begin
 return TT.Component.Identity_Number;
 end Identity;

 procedure Initialize(C: in out Control) is
 begin
 The_Count := The_Count + 1;
 C.Identity_Number := Next_One;
 Next_One := Next_One + 1;
 end Initialize;

```

```

procedure Adjust ...
procedure Finalize ...
end Tracking;

2 with Objects; use Objects;
with Tracking;
package Hush_Hush is
 type Secret_Shape is new Object with private;
 function Shape_Identity(SS: Secret_Shape)
 return Integer;
 ...
private
 package Q is
 new Tracking(Raw_Type => Object);
 type Secret_Shape is new Q.Tracked_Type with
 record
 ... -- other hidden components
 end record;
end Hush_Hush;

package body Hush_Hush is
 function Shape_Identity(SS: Secret_Shape)
 return Integer is
 begin
 return Identity(SS);
 end Shape_Identity;
 ...
end Hush_Hush;

```

Note carefully that the type Secret\_Shape inherits the function Identity from Q.Tracked\_Type. Of course we could have laboriously written

```
return Q.Identity(Q.Tracked_Type(SS));
```

However, we do not actually have to write out a body for Shape\_Identity but can simply use a renaming thus

```
function Shape_Identity(SS: Secret_Shape)
 return Integer renames Identity;
```

### Exercise 21.4

```

1 package body Lists is
 procedure Insert(After: Cell_Ptr; Item: Cell_Ptr) is
 begin
 if Item = null or else Item.Next /= null then
 raise List_Error;
 end if;
 if After = null then
 raise List_Error;
 end if;
 Item.Next := After.Next;
 After.Next := Item;
 end Insert;

```

## 42 Answers to exercises

```

function Remove(After: Cell_Ptr) return Cell_Ptr is
 Result: Cell_Ptr;
begin
 if After = null then
 raise List_Error;
 end if;
 Result := After.Next;
 if Result /= null then
 After.Next := Result.Next;
 Result.Next := null;
 end if;
 return Result;
end Remove;

function Next(After: Cell_Ptr) return Cell_Ptr is
begin
 if After = null then
 raise List_Error;
 end if;
 return After.Next;
end Next;
end Lists;

```

Note that we do not have to do anything about a dummy first element. The user has to do that by declaring a list with one cell already in place by for example

The\_List: Cell\_Ptr := **new** Cell;

- 2 Using null exclusions enables most of the checks to be omitted. The subprograms become

```

procedure Insert(After: not null Cell_Ptr;
 Item: not null Cell_Ptr) is
begin
 if Item.Next /= null then
 raise List_Error;
 end if;
 Item.Next := After.Next;
 After.Next := Item;
end Insert;

function Remove(After: not null Cell_Ptr)
 return Cell_Ptr is
begin
 return Result: Cell_Ptr := After.Next do
 if Result /= null then
 After.Next := Result.Next;
 Result.Next := null;
 end if;
 end return;
end Remove;

function Next(After: not null Cell_Ptr)
 return Cell_Ptr is
begin
 return After.Next;
end Next;

```

### Exercise 21.5

```

1 package List_Iteration_Stuff is
 new Iteration_Stuff(Lists.List,
 Lists.Iterators.Iterator);

procedure Green_To_Red is
 new Generic_Green_To_Red(I_S =>
 List_Iteration_Stuff);

2 package Iterators is
 type Structure is interface;
 procedure Iterate(S: in Structure;
 Action: access procedure (C: in out Colour));
end;

package Trees is
 type Tree is new Structure with private;
 ...
 procedure Iterate(T: in Tree;
 Action: access procedure (C: in out Colour));
private
 type Node;
 type Node_Ptr is access Node;

 type Node is
 record
 Left, Right: Node_Ptr;
 C: Colour;
 end record;
 type Tree is new Structure with
 record
 Root: Node_Ptr;
 end record;
end;

package body Trees is
 ...
 procedure Iterate(T: in Tree;
 Action: access procedure (C: in out Colour)) is
 procedure Inner(N: in Node_Ptr) is
 begin
 if N /= null then
 Action(N.C);
 Inner(N.Left);
 Inner(N.Right);
 end if;
 end Inner;
 begin
 Inner(T.Root);
 end Iterate;
end Trees;

```

Note that Action is an access to procedure parameter of the primitive procedure Iterate of the interface Structure. This is called from within the body of Iterate. We then have

## 21.5

```

function Count(S: Structure'Class; C: Colour)
 return Natural is
 Result: Natural := 0;

 procedure Action(C: in out Colour) is
 begin
 if C = Count.C then
 Result := Result + 1;
 end if;
 end Action;

begin
 Iterate(S, Action'Access); -- dispatch on S
 return Result;
end Count;

Oak: Tree; -- declare some tree
... -- build the tree
N := Count(Oak, Green);

```

Note that this has a mixture of dispatching and access to subprogram calls.

*Exercise 21.9*

- 1 The tagged type case is easy; we simply write

```

package People is
 type Person_Name(<>) is private;
 type Mans_Name (<>) is private;
 type Womans_Name(<>) is private;

 function Man_Of(P: Person_Name)
 return Mans_Name;

 function Woman_Of(P: Person_Name)
 return Womans_Name;

 function Person_Of(M: Mans_Name)
 return Person_Name;

 function Person_Of(W: Womans_Name)
 return Person_Name;
 ... -- other subprograms
private
 type Person;
 ...
 type Person_Name is access all Person'Class;
 type Mans_Name is access all Man;
 ...
 type Person is abstract tagged ...
 ...
end People;

where the private part is exactly as before. The
conversion functions are simply

function Man_Of(P: Person_Name)
 return Mans_Name is
begin
 Mans_Name(P);
end;

```

## Answers to exercises

43

and so on. Remember that conversion is allowed between general access types referring to derived types in the same class. Constraint\_Error is raised if we attempt to convert a person to a name of the wrong sex. Conversion in the opposite direction (towards the root) always works.

The variant formulation requires more care. We cannot write, in the private part, something like

```

type Person_Name is access Person;
type Mans_Name is access all Person(Male);

because the full type always has to be a new
type. But we can write

type Person_Name is access all Person;
type Mans_Name is access all Person(Male);
type Womans_Name is access all Person(Female);

```

where we have again used general access types so that that we can convert between them.

## Answers 22

*Exercise 22.2*

```

1 procedure Gauss_Seidel is
 N: constant := 5;
 subtype Full_Grid is Integer range 0 .. N;
 subtype Grid is Full_Grid range 1 .. N-1;
 type Real is digits 7;
 Tolerance: constant Real := 0.0001;
 Error_Limit: constant Real := Tolerance * (N-1)**2;
 Converged: Boolean := False;
 Error_Sum: Real;

 function F(I, J: Grid) return Real is separate;

 task type Iterator is
 entry Start(I, J: in Grid);
 end;

 protected type Point is
 procedure Set_P(X: in Real);
 function Get_P return Real;
 function Get_Delta_P return Real;
 procedure Set_Converged(B: in Boolean);
 function Get_Converged return Boolean;
 private
 Converged: Boolean := False;
 P: Real;
 Delta_P: Real;
 end;

 Process: array (Grid, Grid) of Iterator;
 Data: array (Full_Grid, Full_Grid) of Point;

```

## 44 Answers to exercises

```

task body Iterator is
 I, J: Grid;
 P: Real;
begin
 accept Start(I, J: in Grid) do
 Iterator.I := Start.I;
 Iterator.J := Start.J;
 end Start;

 loop
 P := 0.25 * (Data(I-1, J).Get_P +
 Data(I+1, J).Get_P +
 Data(I, J-1).Get_P +
 Data(I, J+1).Get_P - F(I, J));
 Data(I, J).Set_P(P);
 exit when Data(I, J).Get_Converged;
 end loop;
end Iterator;

protected body Point is
 procedure Set_P(X: in Real) is
 begin
 Delta_P := X - P;
 P := X;
 end;

 function Get_P return Real is
 begin
 return P;
 end;

 function Get_Delta_P return Real is
 begin
 return Delta_P;
 end;

 procedure Set_Converged(B: in Boolean) is
 begin
 Converged := B;
 end;

 function Get_Converged return Boolean is
 begin
 return Converged;
 end;
end Point;

begin -- of main subprogram; tasks now active
 for I in Grid loop
 for J in Grid loop -- tell them who they are
 Process(I, J).Start(I, J);
 end loop;
 end loop;

 loop
 Error_Sum := 0.0;
 for I in Grid loop
 for J in Grid loop
 Error_Sum := Error_Sum +
 Data(I, J).Get_Delta_P**2;

```

```

 end loop;
 end loop;

 Converged := Error_Sum < Error_Limit;
 exit when Converged;
 end loop;

 -- tell protected objects that system has converged

 for I in Grid loop
 for J in Grid loop
 Data(I, J).Set_Converged(True);
 end loop;
 end loop;

 -- output results

end Gauss_Seidel;

Note that there are protected objects on the
boundary points but we have not shown how to
initialize them. The central computation could
be made neater by using renaming in order to
avoid repeated evaluation of Data(I, J) and so
on; this would also speed things up. Thus we
could write

function Get_P1 return Real
 renames Data(I-1, J).Get_P;
function Get_P2 return Real
 renames Data(I+1, J).Get_P;
...
procedure Set_P(X: in Real)
 renames Data(I, J).Set_P;
function Get_Converged return Boolean
 renames Data(I, J).Get_Converged;

and then

Set_P(0.25 *
 (Get_P1+Get_P2+Get_P3+Get_P4 - F(I, J)));
exit when Get_Converged;

```

## Exercise 22.3

```

1 protected A_Map is new Map with
 procedure Insert(K: in Key; V: in Value);
 procedure Find(K: in Key; V: out Value);
private
 ...
end A_Map;

```

## 22.4

## Exercise 22.4

- 1 **procedure** Handle(CD: **access** Cannon\_Data;  
E: **in** Exception\_Occurrence) **is**  
**begin**  
  **if** Exception\_Identity(E) = Bang'Identity **then**  
    Put\_Line("Cannon seems to have exploded.");  
    Put("Perhaps ");  
    Put(CD.Pounds\_Of\_Powder);  
    Put("\n pounds of powder was too much!");  
  **else**  
    Put("Some other catastrophe ...");  
  **end if**;  
**end** Handle;
- 2 The root package might be  
**package** Root\_Activity **is**  
  **type** Root\_Descriptor **is abstract tagged private**;  
    **function** No\_Of\_Cycles **return** Integer;  
    **function** No\_Of\_Cycles(D: **access**  
      Root\_Descriptor'Class) **return** Integer;  
  **private**  
    Total\_Count: Integer := 0;  
    **type** Root\_Descriptor **is tagged**  
      **record**  
        Instance\_Count: Integer := 0;  
      **end record**;  
  **end**;  
  **package body** Root\_Activity **is**  
    **function** No\_Of\_Cycles **return** Integer **is**  
      **begin**  
        **return** Total\_Count;  
      **end**;  
    **function** No\_Of\_Cycles(D: **access**  
      Root\_Descriptor'Class) **return** Integer **is**  
      **begin**  
        **return** D.Instance\_Count;  
      **end**;  
  **end** Root\_Activity;  
  
  The type Descriptor and associated operations  
  are now placed in the child package together  
  with the task type Control whose body is  
  modified to update the counts on each cycle.  
  Remember that the body of a child package can  
  see the private part of its parent. So we have  
  
  **package** Root\_Activity.Cyclic **is**  
    **type** Descriptor **is new** Root\_Descriptor **with** ...  
    ...  
    **task type** Control(Activity: **access** Descriptor'Class);  
  **end**;  
  
  **package body** Root\_Activity.Cyclic **is**  
    ...

## Answers to exercises 45

- task body** Control **is**  
  Next\_Time: Calendar.Time := Activity.Start\_Time;  
**begin**  
  **loop**  
    Total\_Count := Total\_Count + 1;  
    Activity.Instance\_Count :=  
      Activity.Instance\_Count + 1;  
    **delay until** Next\_Time;  
    ...  
  **end** Control;  
**end** Root\_Activity.Cyclic;
- 3 It could be rewritten as follows. Type extension  
is necessary to pass the additional data.  
**task type** Control(Activity: **access** Descriptor'Class);  
**task body** Control **is**  
  Next\_Time: Calendar.Time := Activity.Start\_Time;  
**begin**  
  **loop**  
    **delay until** Next\_Time;  
    Activity.Action(Activity);       -- *indirect call*  
    Next\_Time := Next\_Time + Activity.Interval;  
    **exit when** Next\_Time > Activity.End\_Time;  
  **end loop**;  
  Activity.Last\_Wishes(Activity);   -- *indirect call*  
**exception**  
  **when** Event: **others** =>  
    Activity.Handle(Activity, Event); -- *indirect call*  
**end** Control;
- package** Root\_Activity **is**  
  **type** Descriptor **is tagged**;  
  **type** Action\_Type **is access**  
    **procedure** (D: **access** Descriptor'Class);  
  **type** Last\_Wishes\_Type **is access**  
    **procedure** (D: **access** Descriptor'Class);  
  **type** Handle\_Type **is access**  
    **procedure** (D: **access** Descriptor'Class;  
      E: **in** Exception\_Occurrence);  
  
  **procedure** Null\_Last\_Wishes  
    (D: **access** Descriptor'Class) **is null**;  
  **procedure** Default\_Handle  
    (D: **access** Descriptor'Class;  
      E: **in** Exception\_Occurrence);  
  
  **type** Descriptor **is tagged**  
    **record**  
      Start\_Time, End\_Time: Calendar.Time;  
      Interval: Duration;  
      Action: Action\_Type;  
      Last\_Wishes: Last\_Wishes\_Type :=  
        Null\_Last\_Wishes'Access;  
      Handle: Handle\_Type :=  
        Default\_Handle'Access;  
    **end record**;  
**end**;

## 46 Answers to exercises

**package body** Root\_Activity **is**

```

procedure Default_Handle
 (D: access Descriptor'Class;
 E: in Exception_Occurrence) is

```

```

begin
 Put_Line("Unhandled exception");
 Put_Line(Exception_Information(E));
end Default_Handle;

```

**end** Root\_Activity;

**use** Root\_Activity;

```

type Cannon_Data is new Descriptor with
 record
 Pounds_Of_Powder: Integer;
 end record;

```

```

procedure Cannon_Action
 (D: access Descriptor'Class) is

```

```

begin
 Load_Cannon(Cannon_Data(D.all).
 Pounds_Of_Powder);
 Fire_Cannon;
end Cannon_Action;

```

```

The_Data: aliased Cannon_Data :=
 (Start_Time => High_Noon;
 End_Time => When_The_Stars_Fade_And_Fall;
 Interval => 24*Hours;
 Action => Cannon_Action'Access;
 ...
 Pounds_Of_Powder => 100);

```

Cannon\_Task: Control(The\_Data'Access);

So here is a deep point. The access problems are overcome by type extension itself and not by the dispatching. However, the dispatching approach is neater because the default subprograms are automatically inherited and do not clutter the record.

*Exercise 22.5*

```

1 package Start_Up
 with Elaborate_Body is
end;

with Ada.Task_Termination;
use Ada.Task_Termination;
package body Start_Up is
begin
 Set_Dependents_Fallback_Handler
 (RIP.One'Access);
end Start_Up;

with Start_Up; pragma Elaborate(Start_Up);
package Library_Tasks is
 ...
 -- declare library tasks here
end;

```

*Exercise 22.8*

- 1 Good luck with the contemplation. The website might have a solution.

*Exercise 22.9*

- 1 Presumably the optimistic programmer writes  
**pragma** Conflict\_Check\_Policy(No\_Conflict\_Checks);  
 and the pessimistic programmer writes  
**pragma** Conflict\_Check\_Policy(All\_Conflict\_Checks);

## Answers 23

*Exercise 23.3*

- 1 Index(S, Decimal\_Digit\_Set **or** To\_Set('.'))
- 2 "begins" which seems to be the longest word in English with the letters in alphabetical order. Other Ada words, "abort" and "first", are good runners-up.
- 3 **function** Make\_Map(K: String)
 **return** Character\_Mapping **is**
 Key\_Set, Non\_Key\_Set: Character\_Set;
 In\_letters, Out\_Letters: String(1 .. 26);
 **begin**
 Key\_Set := To\_Set(To\_Upper(K));
 Non\_Key\_Set := To\_Set(('A', 'Z')) - Key\_Set;
 In\_Letters := To\_Sequence(Key\_Set) &
 To\_Sequence(Non\_Key\_Set);
 Out\_Letters := To\_Sequence(To\_Set(('A', 'Z')));
 **return** To\_Mapping
 (In\_Letters & To\_Lower(In\_Letters),
 Out\_Letters & To\_Lower(Out\_Letters));
 **end** Make\_Map;
 ...
 Translate(S, Make\_Map("Byron"));
- 4 **function** Decode(M: Character\_Mapping)
 **return** Character\_Mapping **is**
**begin**
**return** To\_Mapping(To\_Range(M), To\_Domain(M));
 **end** Decode;
 ...
 Translate(S, Decode(Make\_Map("Byron")));
 The functions To\_Domain and To\_Range produce the domain and range of the original mapping and the reverse map is simply created by calling To\_Mapping with them reversed. Note that To\_Mapping raises Translation\_Error anyway if the first argument has duplicates and so no additional check is required.

## 23.4

*Exercise 23.4*

```

1 with Ada.Numerics.Elementary_Functions;
use Ada.Numerics;
package body Simple_Maths is

 function Sqrt(F: Float) return Float is
 begin
 return Elementary_Functions.Sqrt(F);
 exception
 when Argument_Error =>
 raise Constraint_Error;
 end Sqrt;

 function Log(F: Float) return Float is
 begin
 return Elementary_Functions.Log(F, 10.0);
 exception
 when Argument_Error =>
 raise Constraint_Error;
 end Log;

 function Ln(F: Float) return Float is
 begin
 return Elementary_Functions.Log(F);
 exception
 when Argument_Error =>
 raise Constraint_Error;
 end Ln;

 function Exp(F: Float) return Float
 renames Elementary_Functions.Exp;

 function Sin(F: Float) return Float
 renames Elementary_Functions.Sin;

 function Cos(F: Float) return Float
 renames Elementary_Functions.Cos;

end Simple_Maths;

We did not write a use clause for Elementary_
Functions because it would not have enabled us
to write for example return Sqrt(F); since this
would have resulted in an infinite recursion.

2 type Hand is (Paper, Stone, Scissors);
type Jacks_Hand is new Hand;
type Jills_Hand is new Hand;
type Outcome is (Jack, Draw, Jill);
Payoff: array (Jacks_Hand, Jills_Hand) of Outcome :=
 ((Draw, Jack, Jill),
 (Jill, Draw, Jack),
 (Jack, Jill, Draw));
package Random_Hand is
 new Discrete_Random(Hand);
use Random_Hand;
Jacks_Gen: Generator;
Jills_Gen: Generator;
Result: Outcome;
...

```

## Answers to exercises

## 47

```

Reset(Jacks_Gen); Reset(Jills_Gen);
loop
 Result := Payoff(Jacks_Hand(Random(Jacks_Gen)),
 Jills_Hand(Random(Jills_Gen)));
 case Result is
 when Jack =>
 ...
 when Draw =>
 ...
 when Jill =>
 ...
 end case;
end loop;

```

The types `Jacks_Hand` and `Jills_Hand` are introduced simply so that the array `Payoff` cannot be indexed incorrectly. There are clearly lots of different ways of doing this example. A more object oriented approach might be to declare a type `Player` containing the personal generator and perhaps the player's score.

*Exercise 23.5*

```

1 with Ada.Direct_IO;
generic
 type Element is private;
procedure Rev(From, To: in String);

procedure Rev(From, To: in String) is
 package IO is new Ada.Direct_IO(Element);
 use IO;
 Input: File_Type;
 Output: File_Type;
 X: Element;
begin
 Open(Input, In_File, From);
 Open(Output, Out_File, To);
 Set_Index(Output, Size(Input));
 loop
 Read(Input, X);
 Write(Output, X);
 exit when End_Of_File(Input);
 Set_Index(Output, Index(Output)-2);
 end loop;
 Close(Input);
 Close(Output);
end Rev;

```

Remember that **reverse** is a reserved word. Note also that this does not work if the file is empty (the first call of `Set_Index` will raise `Constraint_Error`).

## 48 Answers to exercises

### Exercise 23.6

- 1 The output is shown in string quotes in order to reveal the layout. Spaces are indicated by s. In reality of course, there are no quotes and spaces are spaces.
 

|                |                    |
|----------------|--------------------|
| (a) "Fred"     | (f) "ss8#170#"     |
| (b) "sss120"   | (g) "-3.80000E+01" |
| (c) "sssss120" | (h) "sssss7.00E-2" |
| (d) "120"      | (i) "3.1416E+01"   |
| (e) "-120"     | (j) "1.0E+10"      |
- 2 

```
with Ada.Text_IO;
with Ada.Float_Text_IO;
use Ada;
package body Simple_IO is
 procedure Get(F: out Float) is
 begin
 Float_Text_IO.Get(F);
 end Get;

 procedure Put(F: in Float) is
 begin
 Float_Text_IO.Put(F);
 end Put;

 procedure Put(S: in String)
 renames Text_IO.Put;

 procedure New_Line(N: in Integer := 1) is
 begin
 Text_IO.New_Line(Text_IO.Count(N));
 end New_Line;
end Simple_IO;
```

We have used the nongeneric package `Ada.Float_Text_IO`. We have to use the full dotted notation to avoid recursion. We cannot use renaming for `Get` and `Put` for the type `Float` because those in `Float_Text_IO` have additional parameters (which have defaults).

The other point of note is the type conversion in `New_Line`.

### Exercise 23.7

- 1 

```
procedure Date_Read(Stream: not null access
 Root_Stream_Type'Class;
 Item: out Date) is
 Month_Number: Integer range 1 .. 12;
begin
 Integer'Read(Stream, Item.Day);
 Integer'Read(Stream, Month_Number);
 Item.Month := Month_Name'Val(Month_Number - 1);
 Integer'Read(Stream, Item.Year);
end Date_Read;
```

...

for Date'Read use Date\_Read;

## Answers 24

### Exercise 24.2

- 1 

```
private with Ada.Containers.Doubly_Linked_Lists;
package Queues is
 Empty: exception;
 type Queue is limited private;
 procedure Join(Q: in out Queue; X: in Item);
 procedure Remove(Q: in out Queue; X: out Item);
 function Length(Q: Queue) return Integer;
private
 use Ada.Containers;
 package Q_Container is
 new Doubly_Linked_Lists(Item);
 type Queue is
 new Q_Container.List with null record;
end;

package body Queues is
 procedure Join(Q: in out Queue; X: in Item) is
 begin
 Append(Q, Item);
 end Join;

 procedure Remove(Q: in out Queue; X: out Item) is
 begin
 if Is_Empty(Q) then
 raise Empty;
 end if;
 X := First_Element(Q);
 Delete_First(Q);
 end Remove;

 function Length(Q: Queue) return Integer is
 begin
 return Integer(Count_Type'(Q.Length));
 end Length;
end Queues;
```

We have used a private with clause since there is no need for access to the container in the visible part of the package.

The type `Queue` is not visibly tagged but is visibly limited. The fact that the full type is tagged but not limited does not matter.

The function `Length` which results from the instantiation and type derivation might appear to clash with the function `Length` that we have to provide. But the new one has result of type `Count_Type` (which is declared in `Ada.Containers`). Thus the call of `Q.Length` can be qualified to select the correct one and the result is then converted.

## 24.3

*Exercise 24.3*

```

1 private with Ada.Containers.Vectors;
package Queues is
 Empty: exception;
 type Queue is limited private;
 procedure Join(Q: in out Queue; X: in Item);
 procedure Remove(Q: in out Queue; X: out Item);
 function Length(Q: Queue) return Integer;
private
 use Ada.Containers;
 package Q_Container is new Vectors(Item);
 use Q_Container;
 type Queue is new Vector with null record;
end;

```

The body remains unchanged. But it will be terribly slow because each call of Remove will result in the vector sliding.

```

2 with Ada.Containers.Vectors;
with Ada.Containers.Doubly_Linked_Lists;
use Ada.Containers;
generic
 with package DLL is
 new Doubly_Linked_Lists(<>);
 with package V is new Vectors(
 Element_Type => DLL.Element_Type;
 others => <>);
function Convert(The_Vector: V.Vector)
 return DLL.List;

function Convert(The_Vector: V.Vector)
 return DLL.List is
 The_List: DLL.List;
begin;
 for Ind in The_Vector.First_Index ..
 The_Vector.Last_Index loop
 The_List.Append(The_Vector.Element(Ind));
 end loop;
 return The_List;
end Convert;

```

We have to ensure that the element types of both containers are the same. But the Index\_Type for the vector does not matter nor do the equality operations have to be the same – so we have used the **others => <>** notation to cover them.

The function body could be written in many ways. We have chosen to use the index facilities of the vector container for illustration. We could equally have used a cursor thus

```

function Convert(The_Vector: V.Vector)
 return DLL.List is
 The_List: DLL.List;
 V_Cursor: V.Cursor;
begin;

```

## Answers to exercises

## 49

```

V_Cursor := The_Vector.First;
loop
 exit when V_Cursor = V.No_Element;
 The_List.Append(The_Vector.Element);
 V.Next(V_Cursor);
end loop;
return The_List;
end Convert;

3 with Ada.Containers.Vectors;
with Ada.Containers.Doubly_Linked_Lists;
use Ada.Containers;
generic
 with package DLL is
 new Doubly_Linked_Lists(<>);

 with package V is new Vectors(
 Index_Type => <>;
 Element_Type => DLL.Element_Type;
 "=" => DLL."=");
function Equals(The_Vector: V.Vector;
 The_List: DLL.List) return Boolean;

function Equals(The_Vector: V.Vector;
 The_List: DLL.List) return Boolean is
begin;
 if The_List.Length /= The_Vector.Length then
 return False;
 end if;
 for Ind in The_Vector.First_Index ..
 The_Vector.Last_Index loop
 if The_List.Find(The_Vector.Element(Ind)) =
 DLL.No_Element then
 return False;
 end if;
 end loop;
 return True;
end Equals;

```

In this case it is necessary for the equality operations to be the same, but naturally the Index\_Type does not matter.

This simple solution assumes that there is no duplication of elements. It checks that the two containers have the same number of elements and that for each element in the vector there is an element in the list with the same value. The reader is invited to extend the solution to avoid the assumption of no duplication.

## 50 Answers to exercises

### Exercise 24.4

```

1 with Ada.Containers.Ordered_Maps;
with Ada.Tags; use Ada.Tags;
package body Tag_Registration is
 package Map_It is
 new Ada.Containers.Ordered_Maps(
 Key_Type => Character,
 Element_Type => Tag);

 use Map_It;
 The_Map: Map;

 procedure Register(The_Tag: Tag;
 Code: Character) is
 begin
 The_Map.Insert(Code, The_Tag);
 end Register;

 function Decode(Code: Character) return Tag is
 C: Cursor := The_Map.Find(Code);
 begin
 if C = No_Element then
 return No_Tag;
 else
 return Element(C);
 end if;
 end Decode;
end Tag_Registration;

```

The test for `No_Element` in `Decode` could be done in several ways. We could use `Has_Element` or we could even crudely call the function `Element` that directly takes a `Code` and this would raise `Constraint_Error` which could then be handled to return `No_Tag`. Thus

```

function Decode(Code: Character) return Tag is
begin
 return The_Map.Element(Code);
exception
 when Constraint_Error =>
 return No_Tag;
end Decode;

```

Shorter but not sweeter.

### Exercise 24.5

```

1 with Abstract_Sets;
private with Ada.Containers.Ordered_Sets;
package Container_Sets is
 type C_Set is
 new Abstract_Sets.Set with private;
 function Empty return C_Set;
 function Unit(E: Element) return C_Set;
 function Union(S, T: C_Set) return C_Set;
 function Intersection(S, T: C_Set) return C_Set;
 procedure Take(From: in out C_Set;
 E: out Element);

```

```

private
 use Ada.Containers;
 package S_Container is
 new Ordered_Sets(Element);
 use S_Container;
 type C_Set is new Abstract_Sets.Set with
 record
 The_Set: Set; -- that is S_Container.Set
 end record;
end;

package body Container_Sets is
 function Empty return C_Set is
 begin
 return (The_Set => Empty_Set);
 end Empty;

 function Unit(E: Element) return C_Set is
 begin
 return R: C_Set := (The_Set => <>) do
 R.The_Set.Insert(E);
 end return;
 end;

 function Union(S, T: C_Set) return C_Set is
 begin
 return (The_Set => S.The_Set or T.The_Set);
 end Union;

 function Intersection(S, T: C_Set) return C_Set is
 begin
 return (The_Set => S.The_Set and T.The_Set);
 end Intersection;

 procedure Take(From: in out C_Set;
 E: out Element) is
 begin
 E := From.The_Set.First_Element;
 From.The_Set.Delete_First;
 end Take;
end Container_Sets;

```

In this case we have used a wrapper and have to remember that aggregates of one element must be named. We have chosen to use an extended return for `Unit` – there is no need to initialize `R` but it helps to emphasize the structure.

If we use a hashed set then `Take` will need to be rewritten in terms of cursors because `First_Element` and `Delete_First` do not exist for hashed sets. Thus

```

procedure Take(From: in out C_Set;
 E: out Element) is
 C: Cursor;
begin
 C := From.The_Set.First;
 E := Element(C);
 From.The_Set.Delete(C);
end Take;

```

## 24.5

A more important point is that we can use multiple inheritance and so avoid the wrapper.

```

with Abstract_Sets;
private with Ada.Containers.Ordered_Sets;
package Container_Sets is
 type C_Set is
 new Abstract_Sets.Set with private;
 function Empty return C_Set;
 function Unit(E: Element) return C_Set;
 function Union(S, T: C_Set) return C_Set;
 function Intersection(S, T: C_Set) return C_Set;
 procedure Take(From: in out C_Set;
 E: out Element);
private
 use Ada.Containers;
 package S_Container is
 new Ordered_Sets(Element);
 use S_Container;
 type C_Set is new Set and Abstract_Sets.Set
 with null record;
end;
package body Container_Sets is
 function Empty return C_Set is
 begin
 return (Empty_Set with null record);
 end Empty;

 function Unit(E: Element) return C_Set is
 begin
 return R: C_Set do
 R.Insert(E);
 end return;
 end;

 function Union(S, T: C_Set) return C_Set is
 begin
 return S or T;
 end Union;

 function Intersection(S, T: C_Set) return C_Set is
 begin
 return S and T;
 end Intersection;

 procedure Take(From: in out C_Set;
 E: out Element) is
 begin
 E := From.First_Element;
 From.Delete_First;
 end Take;
end Container_Sets;

```

Using multiple inheritance makes the body a bit shorter. Note that Union and Intersection are not simply inherited; this is because although C\_Set does inherit Union and Intersection from Set nevertheless they get overridden by the new ones we

## Answers to exercises 51

are trying to declare. But luckily the renamings **and** and **or** are also inherited and so we can use them instead. We could equally have used a renaming as body thus

```

function Union(S, T: C_Set) return C_Set
renames "or";

```

Maybe the wrapper solution is easier to understand.

*Exercise 24.10*

```

1 function Most(The_Index: Text_Map)
 return String is
 Max: Count_Type := 0;
 L: Count_Type;
 Best_One: Indexes.Cursor;

 begin
 for C in The_Index.Iterate loop
 L := Indexes.Element(C).Length;
 if L > Max then
 Best_One := C;
 Max := L;
 end if;
 end loop;
 return Indexes.Element(Best_One);
 end Most;

```

## 52 Answers to exercises

### Answers 25

#### Exercise 25.4

```

1 with Ada.Unchecked_Deallocation; use Ada;
 package body Queues is
 procedure Free is
 new Unchecked_Deallocation(Cell, Cell_Ptr);
 ...
 procedure Remove(Q: in out Queue; X: out Item) is
 Old_First: Cell_Ptr := Q.First;
 begin
 if Q.Count = 0 then
 raise Empty;
 end if;
 X := Q.First.Data;
 Q.First := Q.First.Next;
 Q.Count := Q.Count - 1;
 if Q.Count = 0 then
 Q.Last := null;
 end if;
 Free(Old_First);
 end Remove;
 ...
 end Queues;

```

Note that we assign **null** to Q.Last if the last item is removed. Otherwise it would continue to refer to the deallocated cell – of course, this should do no harm since it will never be used again but it seems wise not to tempt fate.

### Answers 27

#### Exercise 27.1

- 1 (a) dynamic – Integer  
 (b) static – Integer  
 (c) static – *root\_integer*