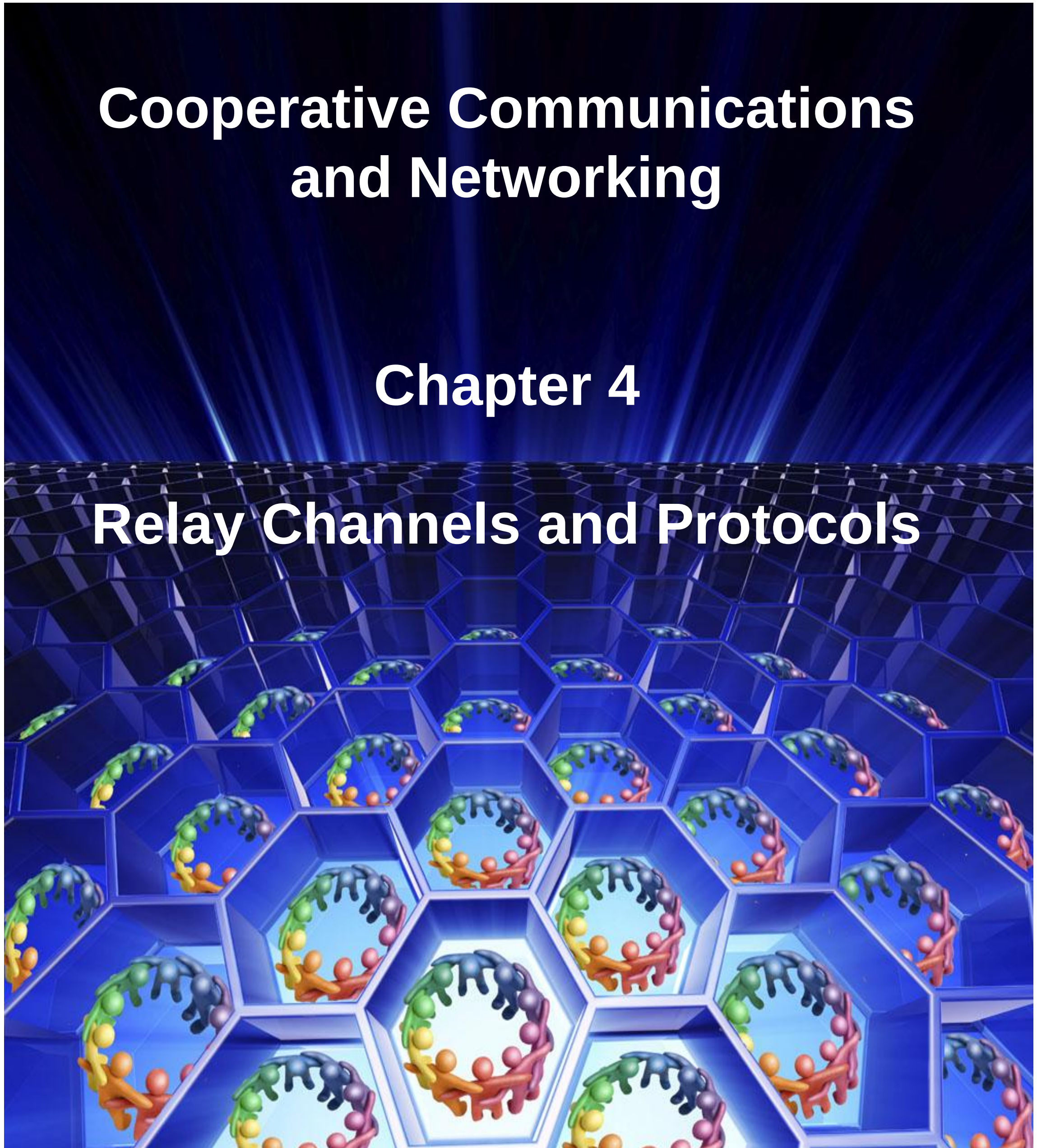

Cooperative Communications
and Networking
Chapter 4
Relay Channels and Protocols

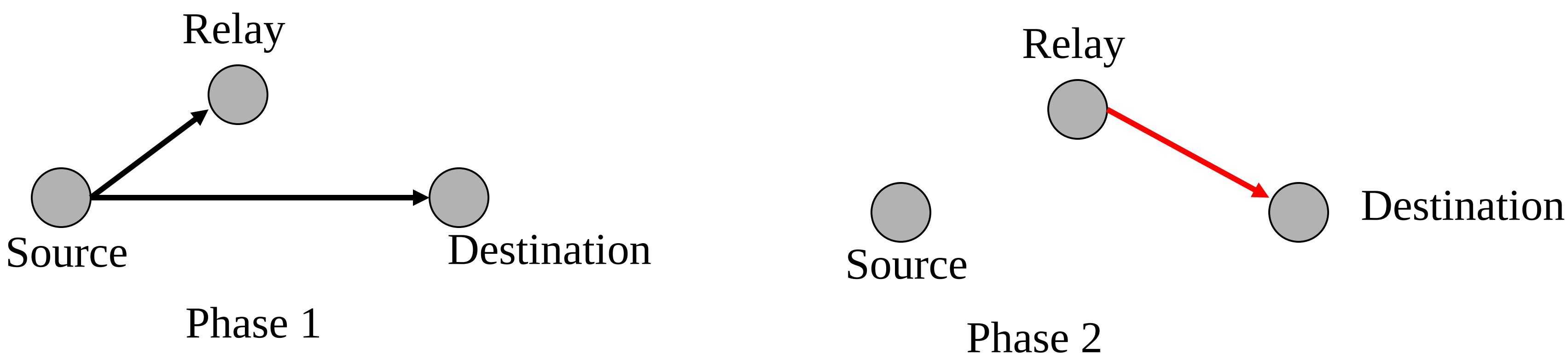
Outline

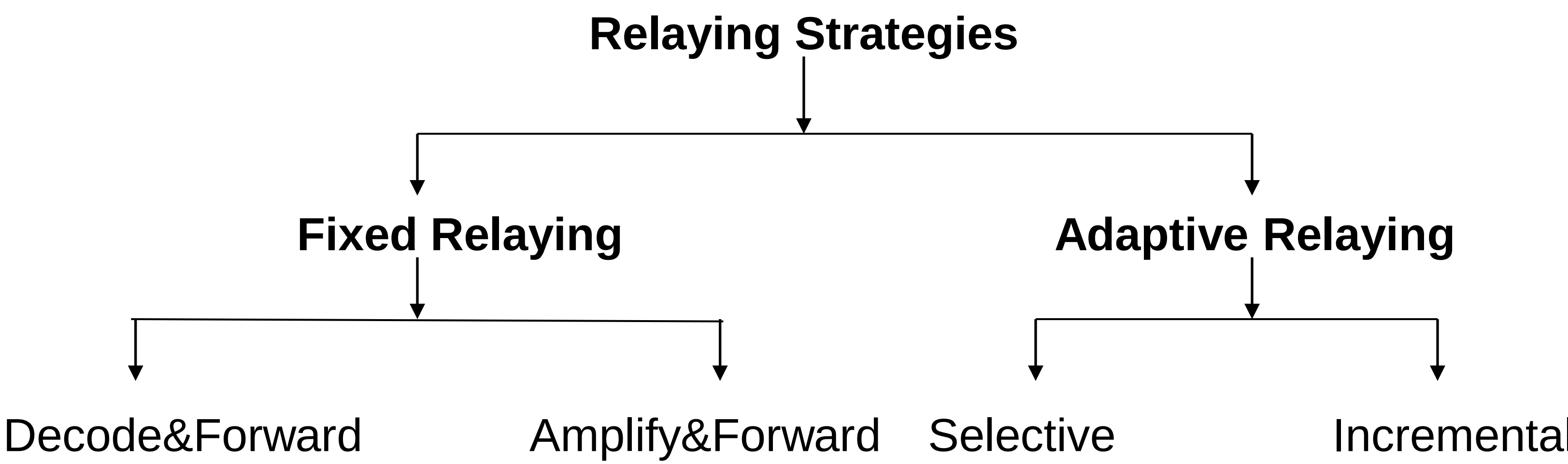
- Cooperative communications: a paradigm shift
- Definitions and system model
- Relaying strategies
  - Amplify & forward
  - Selection decode & forward
  - Incremental relaying
- Hierarchical cooperation

A Paradigm Shift

- Future generations of wireless systems promise very high spectral efficiencies
- MIMO is a key enabler for achieving higher data rates
- The number of antennas on any device is limited by the device size
- Cooperation offers a new communication paradigm in which the number of antennas virtually increases with the number of terminals
- Such a virtual antenna system can offer significant capacity gains

Relaying Strategies





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Definitions and System Model

- A channel is:
  - Fast fading if each packet encounters several channel realizations
  - Slow fading if each packet encounters one channel realization
- Quasi-static Rayleigh fading channel is assumed
- Channel state information (CSI) at receiver
- Gaussian channel input, unless otherwise stated
- Half-duplex devices

Definitions

- The multiplexing gain

$$r = \lim_{SNR\to\infty} \frac{R(SNR)}{\log(SNR)}$$

- The diversity gain

$$d(r) = -\lim_{SNR\to\infty} \frac{\log p_{out}(r \log SNR)}{\log SNR}$$

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Definitions
(Diversity and Multiplexing Gains)

- The spatial multiplexing gain of a $n_t \times n_r$ MIMO scales like

$$\min(n_t, n_r)\log(SNR) \longrightarrow$$ For 0 div gain

- The diversity gain of $n_t \times n_r$ MIMO is

$$SNR^{-n_t n_r} \longrightarrow$$ For 0 multiplexing gain

There is a Tradeoff

Relevant Performance Metrics

- Outage probability
  - Probability of not meeting specified transmission rate R due to poor channel realizations

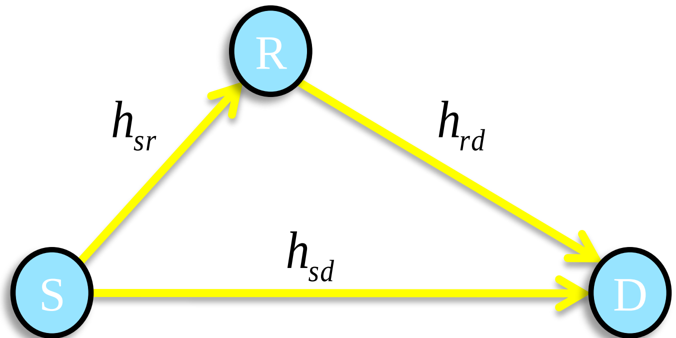
$$P_{out} = \Pr\left[I(x,y) < R\right]$$

  - $I(x,y)$ is the maximum mutual information between input and output signals
- Diversity gain
  - Reduction in outage probability due to SNR improvement

$$\nabla = -\lim_{\gamma \to \infty} \frac{\log P_{out}}{\log \gamma}$$

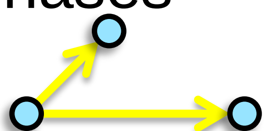
γ : signal-to-noise ratio

Relay Model



$$P_s = P_r = P$$

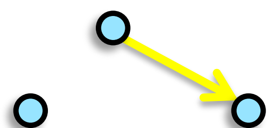
- Single relay
  - Assume equal transmission powers
- Two phases



Phase I

$$y_{sd} = \sqrt{P} h_{sd} x + n_{sd}$$

$$y_{sr} = \sqrt{P} h_{sr} x + n_{sr}$$



Phase II

$$y_{rd} = h_{rd} q(y_{sr}) + n_{rd}$$

$$x \sim \mathrm{C.G.}(0,1)$$

$$n_{sd}, n_{sr}, n_{rd} \sim \mathrm{C.G.}(0, N_0)$$

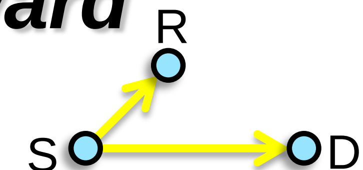
- i.i.d. 0-mean complex Gaussian signals
  - input x, noise terms n_{sd}, n_{sr}, n_{rd}
- Rayleigh fading channels
  - Nodes know channel coefficients

$$h_{sd} \sim \mathrm{R}(\sigma_{sd})$$

$$h_{sr} \sim \mathrm{R}(\sigma_{sr})$$

$$h_{rd} \sim \mathrm{R}(\sigma_{rd})$$

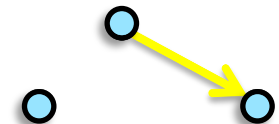
Fixed Relaying: Amplify and Forward



Phase I

- Amplify y_{sr} and forward to destination

$$
\begin{aligned}
y_{rd} &= h_{rd} q(y_{sr}) + n_{rd} \\
&= h_{rd}\left[\frac{\sqrt{P}}{\sqrt{P|h_{sr}|^2 + N_0}}\left(\sqrt{P}h_{sr}x + n_{sr}\right)\right] + n_{rd} \\
&= \left(\frac{\sqrt{P}}{\sqrt{P|h_{sr}|^2 + N_0}}\sqrt{P}h_{rd}h_{sr}\right)x + \left(\frac{\sqrt{P}}{\sqrt{P|h_{sr}|^2 + N_0}}h_{rd}n_{sr} + n_{rd}\right)
\end{aligned}
$$


Phase II

- Both y_{sd} and y_{rd} are fused with maximum ratio combining at destination to maximize SNR

$$
y = \left[\frac{\sqrt{P}h_{sd}^*}{N_0}\right]y_{sd} + \left[\frac{\frac{\sqrt{P}}{\sqrt{P|h_{sr}|^2 + N_0}}\sqrt{P}h_{rd}^*h_{sr}^*}{\left(\frac{P|h_{rd}|^2}{P|h_{sr}|^2 + N_0} + 1\right)N_0}\right]y_{rd}
$$

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Fixed Relaying: AF

- Instantaneous SNR at destination

$$\gamma = \gamma_{sd} + \gamma_{rd} = \frac{P|h_{sd}|^2}{N_0} + \frac{1}{N_0}\frac{P^2|h_{rd}|^2|h_{sr}|^2}{P|h_{sr}|^2 + P|h_{rd}|^2 + N_0}$$

- Mutual information
  - Source and relay each take half of resource

$$I_{AF} = \frac{1}{2}\log(1+\gamma_{sd}+\gamma_{rd}) = \frac{1}{2}\log\left(1+\Gamma|h_{sd}|^2 + f(\Gamma|h_{sr}|^2, \Gamma|h_{rd}|^2)\right)$$

$$\Gamma = P/N_0$$

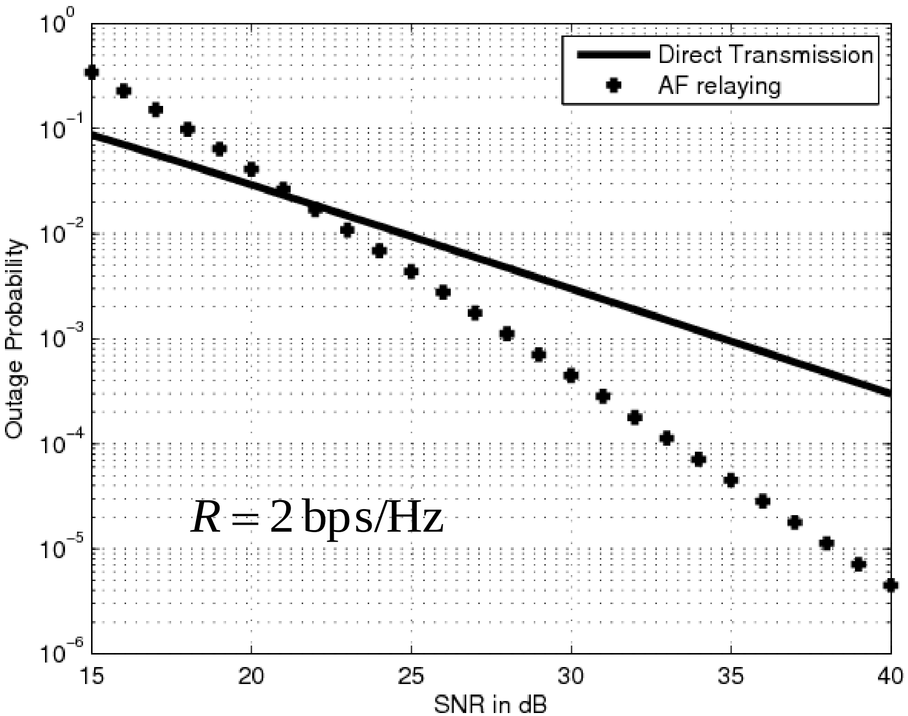
$$f(a,b) = \frac{ab}{a+b+1}$$

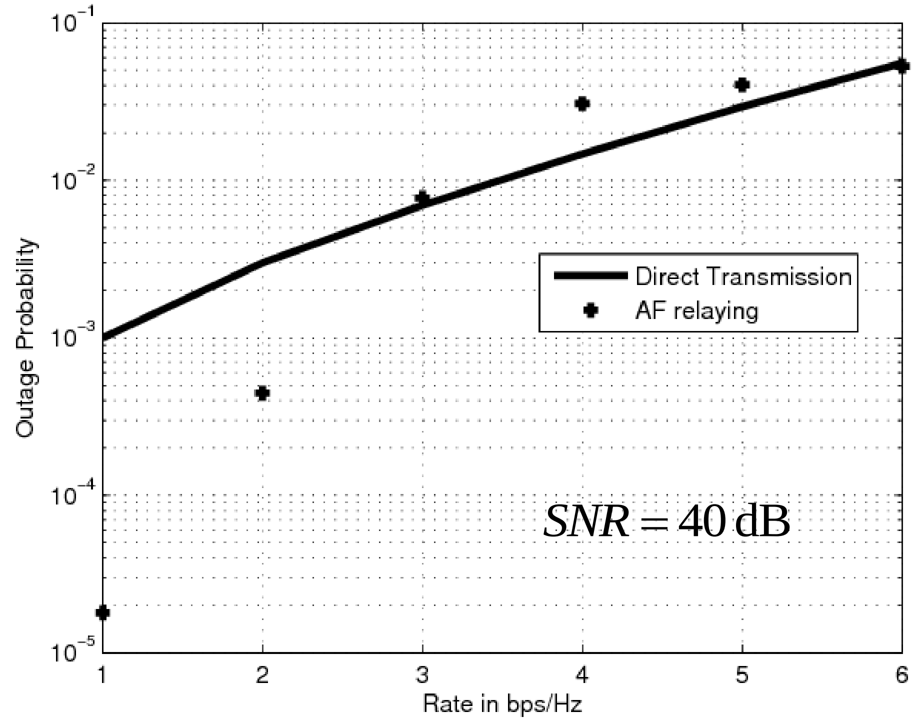
- Outage probability

$$P_{out} = \Pr[I_{AF} < R] \approx \left(\frac{\sigma_{sr}^2 + \sigma_{rd}^2}{2\sigma_{sd}^2(\sigma_{rd}^2\sigma_{sr}^2)}\right)\left(\frac{2^{2R}-1}{\Gamma}\right)^2$$

$$\nabla = 2$$

Fixed Relaying: AF





- Easy implementation
- Low spectral efficiency

$\sigma_{sd}^2 = 1$

$\sigma_{sr}^2 = 0.5 \qquad N_0 = 1$

$\sigma_{rd}^2 = 0.5$

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Fixed Relaying: Decode and Forward

- Decode, re-encode, then forward signal
  - Re-encoded signal might be incorrect

- Mutual information limited by the weakest link between the source-relay and the combined source-destination and relay-destination.

$$I_{DF} = \frac{1}{2}\min\left\{ \log\left(1+\Gamma|h_{sr}|^2\right), \log\left(1+\Gamma|h_{sd}|^2+\Gamma|h_{rd}|^2\right) \right\}$$

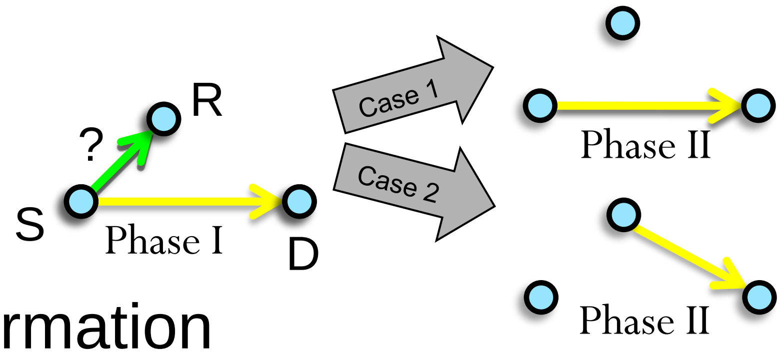
- Outage probability

$$\nabla = 1$$

$$P_{out} = \Pr\left[I_{DF} < R\right] \approx \frac{1}{\sigma_{sr}^2}\frac{2^{2R}-1}{\Gamma}$$

Adaptive Relaying: Selective DF

- DF if source-relay link exceeds SNR threshold



- Mutual information

$$I_{SDF} = \begin{cases} \frac{1}{2}\log\left(1+2\Gamma|h_{sd}|^2\right), & |h_{sr}|^2 < \frac{2^{2R}-1}{\Gamma} \\ \frac{1}{2}\log\left(1+\Gamma|h_{sd}|^2+\Gamma|h_{rd}|^2\right), & |h_{sr}|^2 \geq \frac{2^{2R}-1}{\Gamma} \end{cases}$$

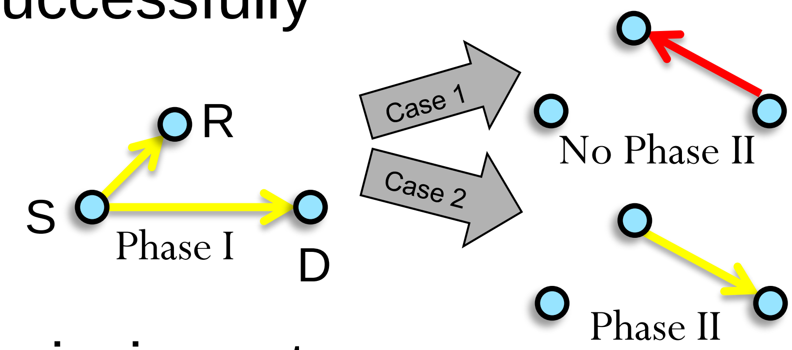
- Outage probability

$$P_{out} = \Pr\left[I_{SDF} < R\right] \approx \left(\frac{\sigma_{sr}^2+\sigma_{rd}^2}{2\sigma_{sd}^2\left(\sigma_{rd}^2\sigma_{sr}^2\right)}\right)\left(\frac{2^{2R}-1}{\Gamma}\right)^2$$

$\nabla = 2$

Adaptive Relaying: Incremental

- Feedback from the destination if source signal is transmitted successfully



- Overall transmission rate

$$\overline{R} = R \Pr\left[|h_{sd}|^2 \geq \frac{2^R - 1}{\Gamma}\right] + \frac{R}{2} \Pr\left[|h_{sd}|^2 < \frac{2^R - 1}{\Gamma}\right] \geq \frac{R}{2}$$

- Outage probability

$$\Pr[I_{IR} < R] = \Pr[I_{Direct} < R] \Pr\left[I_{AF} < \frac{R}{2} \mid I_{Direct} < R\right]$$

$$\approx \left(\frac{\sigma_{sr}^2 + \sigma_{rd}^2}{\sigma_{sd}^2 \left(\sigma_{rd}^2 \sigma_{sr}^2\right)}\right)\left(\frac{2^{\overline{R}} - 1}{\Gamma}\right)^2$$

$$\nabla = 2$$

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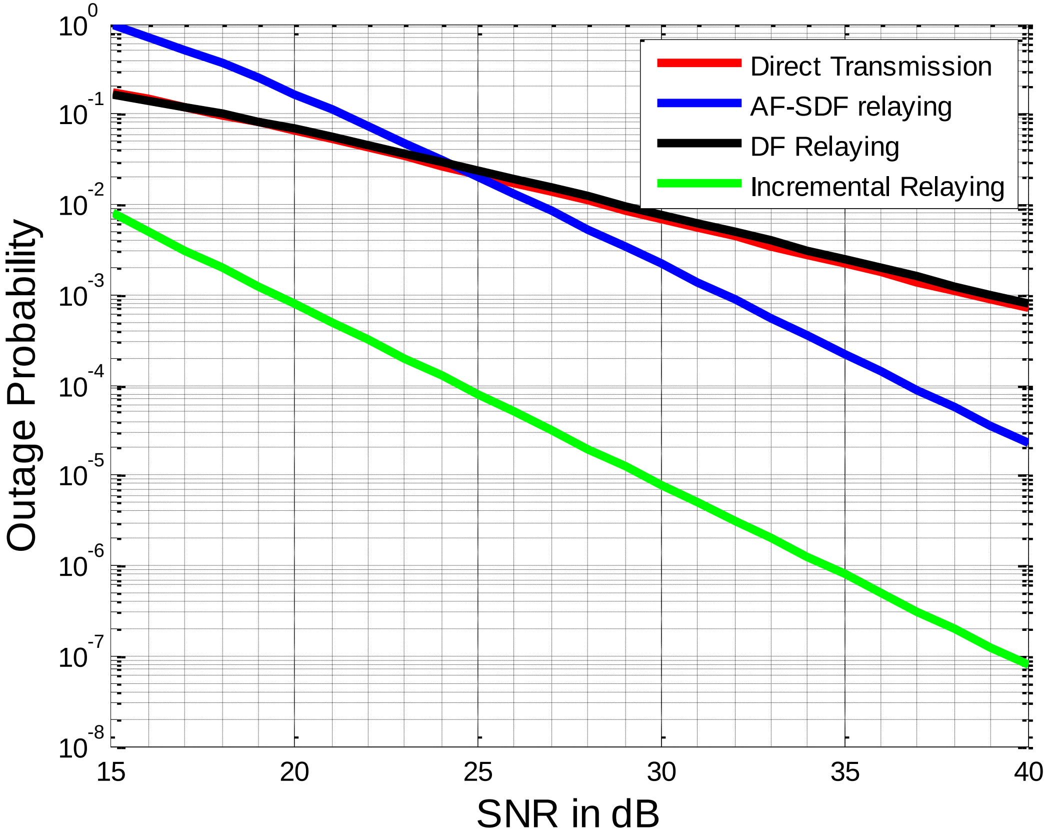
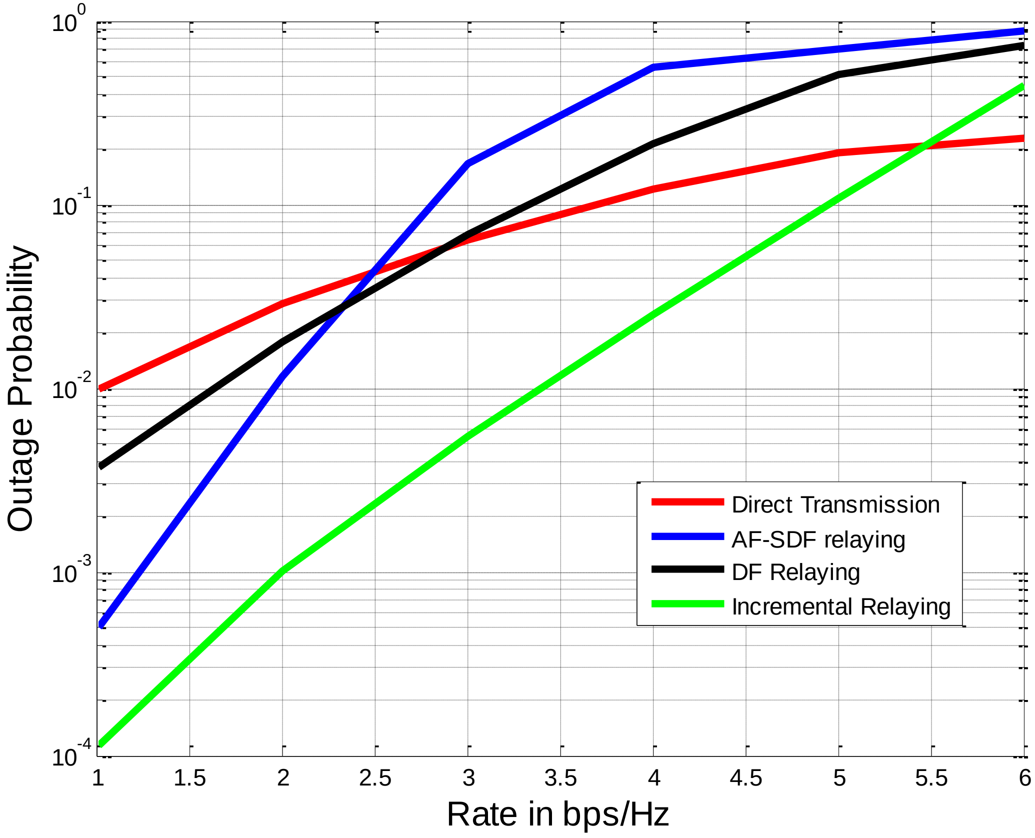
Outage Versus SNR for fixed Rate



Outage Versus Rate for fixed SNR



Hierarchical Cooperation

- Motivation: n nodes within communication range want to communicate at rate $R(n)$ per node
  - Each node is assumed to have a specific and fixed receiver
- Problem: Long-range transmission using large power causes too much interference
  - Aggregate throughput $T(n) = nR(n)$ degrades as n grows

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Hierarchical Cooperation

- What is the maximum aggregate throughput that can be achieved in a wireless network?
- In a n-node network, nearest neighbor multihop transmission has a network capacity that is upper bounded as [Gupta/Kumar, 00]

$$T(n) \leq O(\sqrt{n})$$

- Throughput per node drops as the density of the nodes increases as

$$R(n) = \frac{T(n)}{n} \leq O(\frac{1}{\sqrt{n}})$$

- What can cooperation help?

Hierarchical Cooperation

- ***Hierarchical cooperation*** can achieve linear scaling!

$$nR(n) \approx O(n)$$

- This means: As the network density increases, the throughput/node does not degrade!

- Network throughput is bounded above by

$$nR(n) \leq O(n \log n)$$

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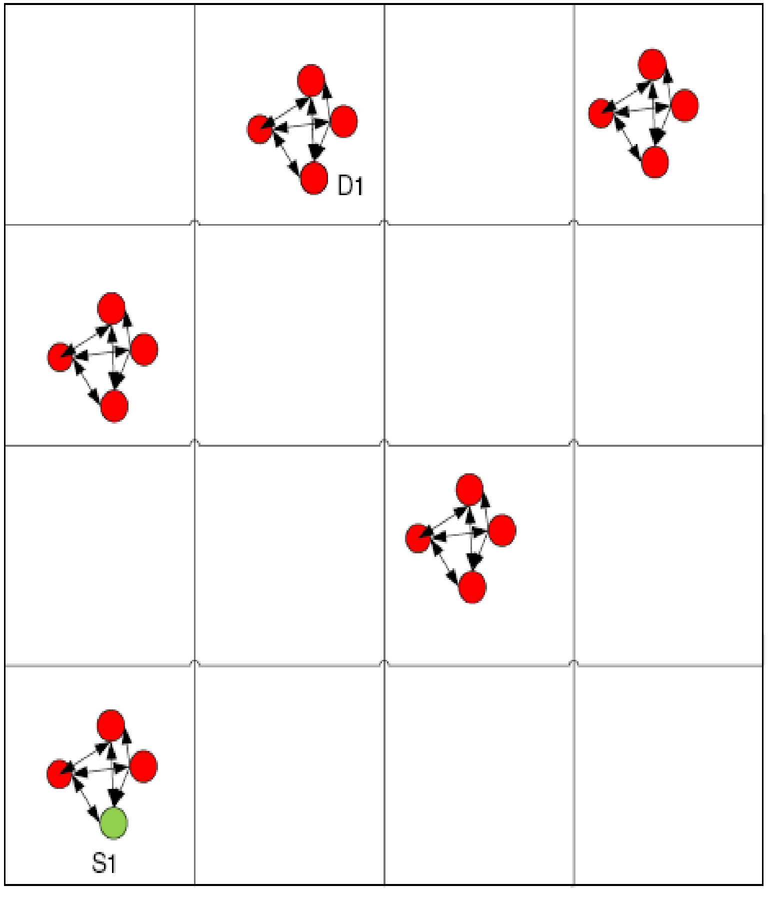
Hierarchical Cooperation (2)

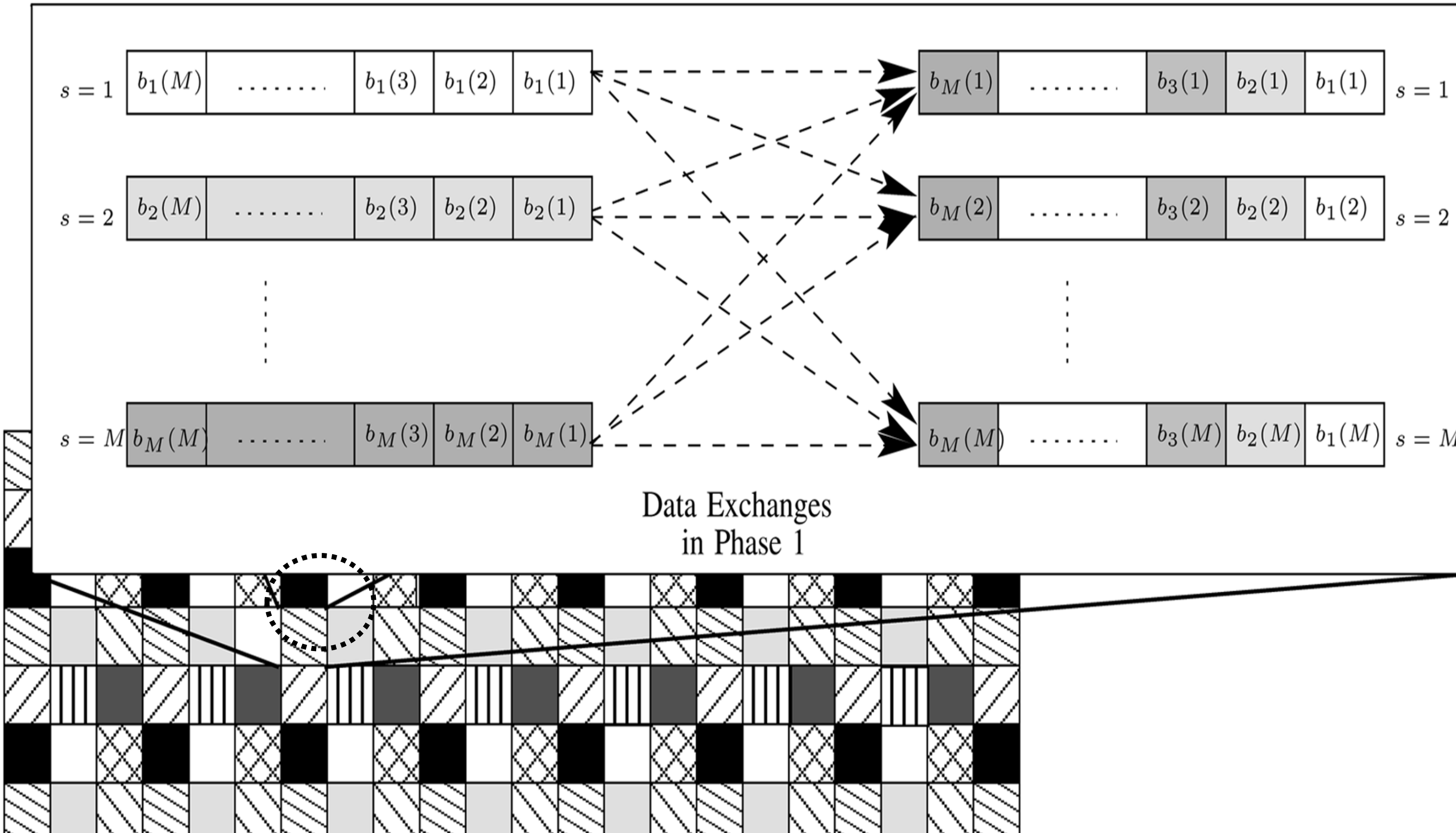
- Hierarchical cooperation scheme can achieve linear scaling of network capacity in ad hoc networks if it leverages [Özgür/Lévêque/Tse, 07]
  - Clustering (spatial reuse)
  - Long-range MIMO transmission across clusters (high spatial multiplexing gain)
- Hierarchical cooperation scheme recursively applies 3-phase protocol each iteration, which are
  - Phase I: Setting up transmit cooperation
  - Phase II: MIMO transmission
  - Phase III: Cooperate to decode



Phase 1:
Setting Up Transmit Cooperation

- The network is divided into clusters each with M nodes
- Clusters operate in parallel following the 9-TDMA scheme
- Each node as a source divides a block of length LM bits into M sub-blocks, and disseminates these sub-blocks among the nodes inside its clusters

Phase 1:
Setting Up Transmit Cooperation



Clusters are divided into 9 groups
(corresponding to 9 different patterns shown)

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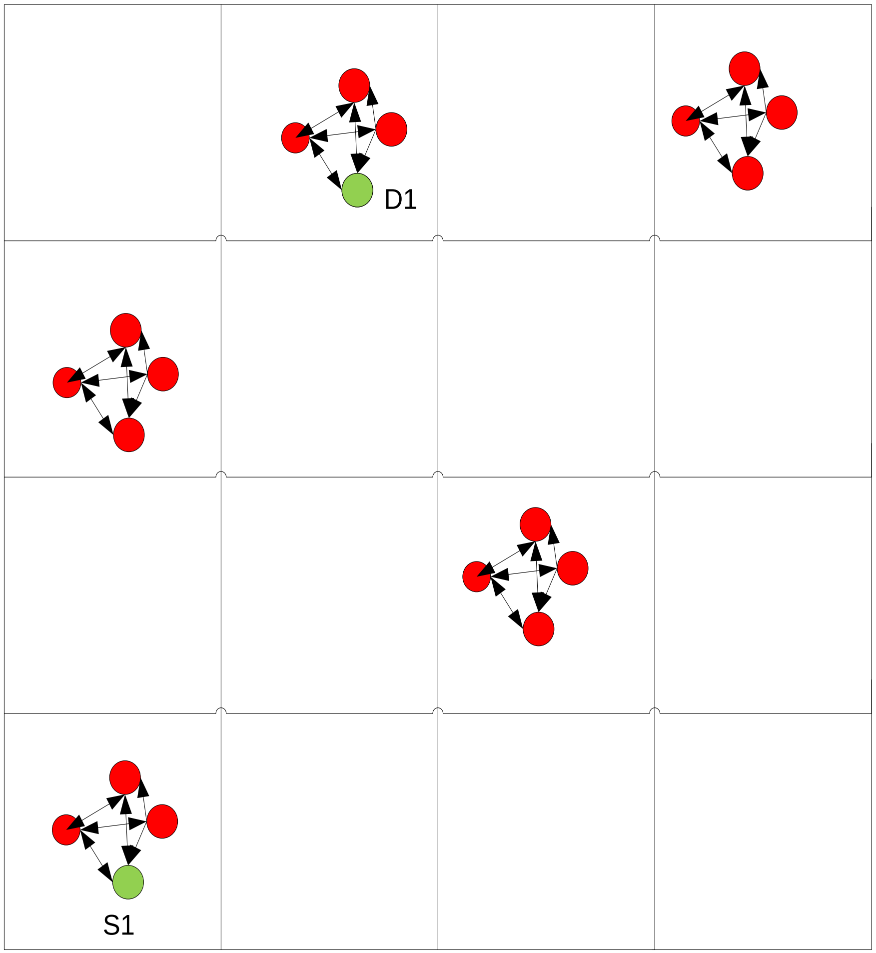
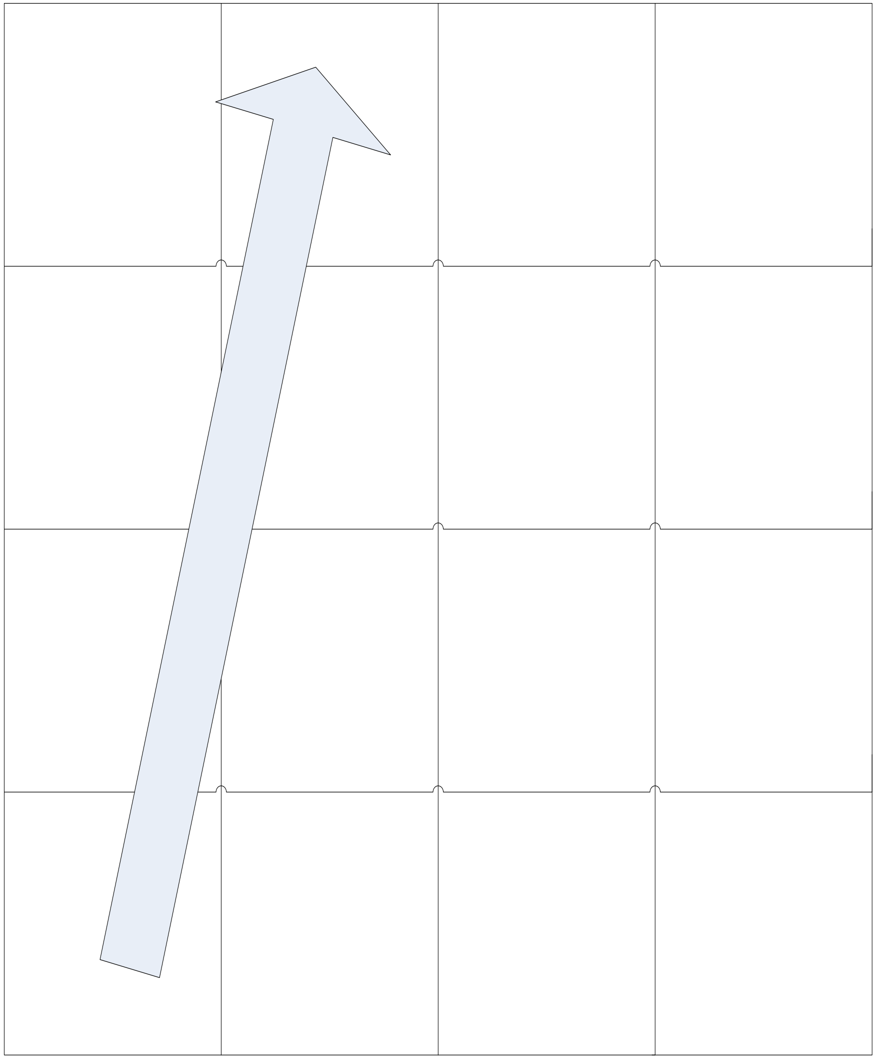
Phase 1: Local transmit cooperation in clusters



Phase 2: MIMO long range transmission between clusters



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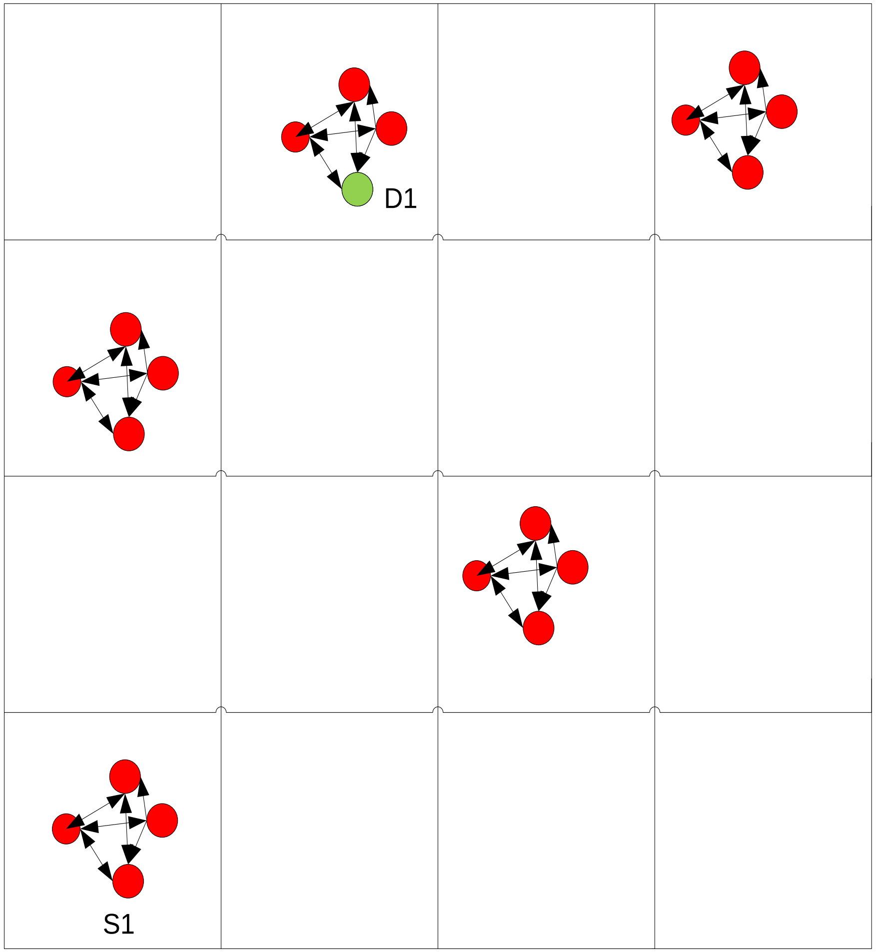
Phase 3: Local receive cooperation in clusters



Why Does it Work?

- Via clustering one can achieve spatial reuse
- This is because clusters that are well separated can work in parallel
- The nodes operating in each cluster can lower their power accordingly to limit the interference to other clusters working in parallel
- Via long-range MIMO we can achieve high spatial multiplexing gain

Analysis: Does It Work?

- Key Lemma: If there exists a scheme that can achieve an aggregate throughput of

$$T(n) \geq K_1 n^b, \qquad 0 \leq b < 1$$

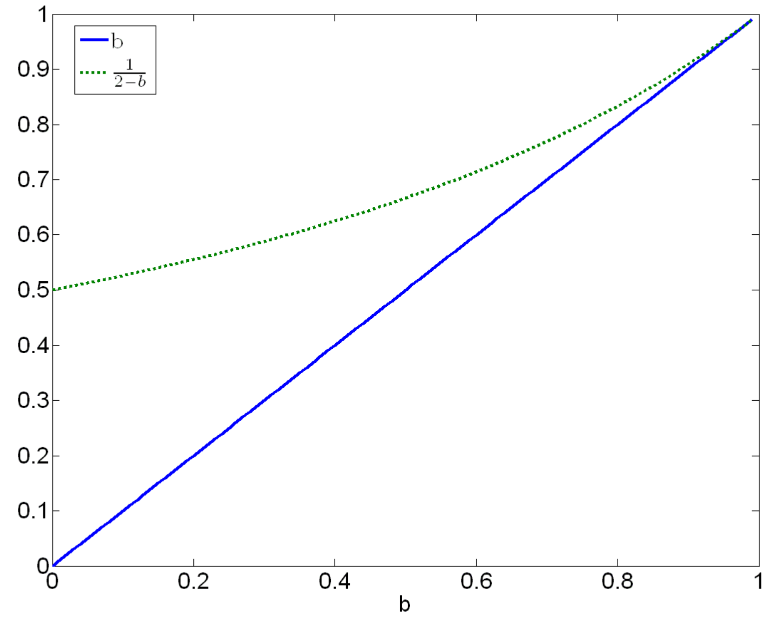
with upper-bounded average power, then one can construct another scheme that achieves

$$T(n) \geq K_2 n^{\frac{1}{2-b}}$$

with the same average power

Analysis: Does It Work?

- $$n^{\frac{1}{2-b}} \geq n^b, \quad 0 \leq b < 1$$



- Performance is improved, particularly for small b

Proof Sketch of Key Lemma: Analysis of Phase 1

- Each node in a cluster has LM bits to transmit
- Each node scales its power proportional to the cluster size
  - Interference at each node is bounded
  - Aggregate throughput of $K_1 M^b$ can be achieved
- Total time slots to complete Phase 1 is therefore

$$T_1 = \underbrace{2M \times \frac{LM}{K_1 M^b}}_{\substack{\text{required time slots for} \\ \text{total traffic} \,\square\, M \times LM}} \times \underbrace{9}_{\because\ \text{9-TDMA Scheme}} = \frac{18L}{K_1} M^{2-b}$$

(independent of n due to parallel processing)

Analysis of Phase 2

- Long-range MIMO transmission takes place
- An MxM multiple antenna MIMO system can transmit O(M) bits in 1 time slot
  - Essential step to achieve high capacity scaling
- Time slots required to complete all MIMO transmission is proportional to n

$$T_2 = 2Cn$$

Analysis of Phase 3

- Each node delivers the observation it receives from Phase 2 to the destination node
- Similar to Phase 1, clusters work in parallel using 9-TDMA, and the required number of time slots is given by

$$T_3 = \frac{18CQ}{K_1} M^{2-b}$$

Combining Phase 1, 2, and 3

- Total number of time slots is given by

$$T_t = T_1 + T_2 + T_3 = \frac{18L}{K_1} M^{2-b} + 2Cn + \frac{18CQ}{K_1} M^{2-b}$$

- Aggregate throughput will be thus as given in the lemma

$$T(n) = \frac{nML}{T_t}$$

$$= \frac{nML}{\frac{18L}{K_1} M^{2-b} + 2Cn + \frac{18CQ}{K_1} M^{2-b}}$$

$$\geq K_2 n^{\frac{1}{2-b}}, \text{ for some } K_2 \text{ if } M = n^{\frac{1}{2-b}} \text{ is chosen}$$

Tradeoff between cooperation overhead (T1+T3) and MIMO performance

Achieving Linear Capacity Scaling

- Idea: recursively apply the 3-phase protocol in Phase 1 and 3 to achieve higher throughput (hence "Hierarchical")
  - Recursion starts with smaller areas and grows until including the whole network
- Theorem: For any $\varepsilon > 0$, there exists a constant K_ε such that, with high probability, an aggregate throughput of $T(n) \geq K_\varepsilon n^{1-\varepsilon}$ is achievable
- Eventually, we get the linear capacity scaling! This means that cooperative communication can help to make the network scalable.

Achieving Linear Capacity Scaling

- Hierarchical Cooperation