Chapter 5

à Question 1

```
In[1]:= Needs["DiscreteMath`RSolve`"]
        Needs["Graphics`Graphics3D`"]
        Needs["Graphics`PlotField3D`"]
```

The two equations are:

$$x_{t+1} = -8 - x_t + y_t$$
$$y_{t+1} = 4 - 0.3\,x_t + 0.9\,y_t$$

with difference equations:

$$\Delta x_t = -8 - 2\,x_t + y_t$$
$$\Delta y_t = 4 - 0.3\,x_t - 0.1\,y_t$$

```
In[4]:= Solve[{-8 - 2 xstar + ystar == 0, 4 - 0.3 xstar - 0.1 ystar == 0}, {xstar, ystar}]

        General::spell1 :
         Possible spelling error: new symbol name "xstar" is similar to existing symbol "ystar".

Out[4]= {{xstar → 6.4, ystar → 20.8}}
```

```
In[5]:= matrixA = ( -1    1
                    -0.3  0.9 )

Out[5]= {{-1, 1}, {-0.3, 0.9}}

In[6]:= Eigenvalues[matrixA]

Out[6]= {-0.826209, 0.726209}

In[7]:= Eigenvectors[matrixA]

Out[7]= {{-0.985232, -0.171225}, {-0.501268, -0.865292}}
```

It will be noted that although the eigenvalues are those given in the text, it appears that the eigenvectors are different. But this is not the case. Consider the eigenvector $\boldsymbol{v}^r$ associated with the eignenvalue $r = 0.726209$. This is given in the programme by,

$$\boldsymbol{v}^r = \begin{pmatrix} -0.501268 \\ -0.865292 \end{pmatrix}$$

But in the text we arbitrarily set $v_2 = 1$. We therefore must divide each element of the vector by -0.865292. Since,

```
In[8]:= -0.501268 / -0.865292

Out[8]= 0.579305
```

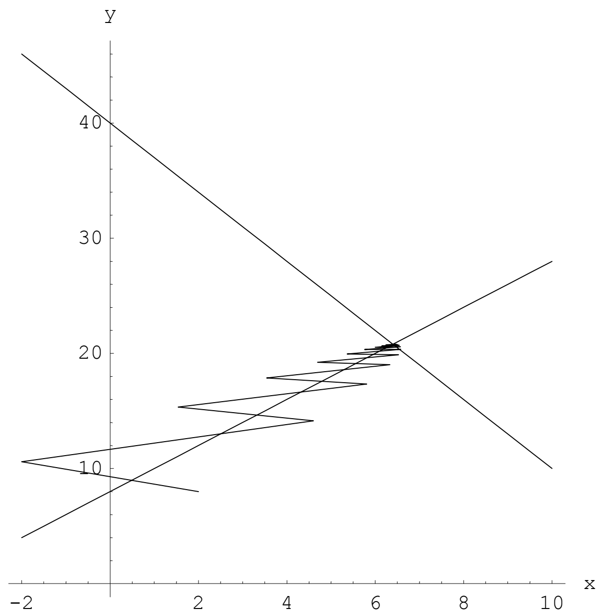
then the eigenvector is,

$$\boldsymbol{v}^r = \begin{pmatrix} 0.579305 \\ 1 \end{pmatrix}$$

which is the eigenvector derived in the text.

```
In[9]:= V = Transpose[Eigenvectors[matrixA]]

Out[9]= {{-0.985232, -0.501268}, {-0.171225, -0.865292}}

In[10]:= V1 = Inverse[V]

Out[10]= {{-1.12862, 0.653812}, {0.223331, -1.28506}}

In[11]:= u0 = {-4.4, -12.8}

Out[11]= {-4.4, -12.8}

In[12]:= matrixD = ( (-0.8262087348)^t        0
                            0          0.7262087348^t )

        General::spell1 :
         Possible spelling error: new symbol name "matrixD" is similar to existing symbol "matrixA".

Out[12]= {{(-0.826209)^t, 0}, {0, 0.726209^t}}

In[13]:= u = V.matrixD.V1.u0

Out[13]= {-4.4 (1.11195 (-0.826209)^t - 0.111949 0.726209^t) -
            12.8 (-0.644157 (-0.826209)^t + 0.644157 0.726209^t),
          -4.4 (0.193247 (-0.826209)^t - 0.193247 0.726209^t) -
            12.8 (-0.111949 (-0.826209)^t + 1.11195 0.726209^t)}

In[14]:= u1 = u[[1]]

Out[14]= -4.4 (1.11195 (-0.826209)^t - 0.111949 0.726209^t) -
          12.8 (-0.644157 (-0.826209)^t + 0.644157 0.726209^t)

In[15]:= u2 = u[[2]]

Out[15]= -4.4 (0.193247 (-0.826209)^t - 0.193247 0.726209^t) -
          12.8 (-0.111949 (-0.826209)^t + 1.11195 0.726209^t)

In[16]:= xt = 6.4 + u1

Out[16]= 6.4 - 4.4 (1.11195 (-0.826209)^t - 0.111949 0.726209^t) -
          12.8 (-0.644157 (-0.826209)^t + 0.644157 0.726209^t)

In[17]:= yt = 20.8 + u2

Out[17]= 20.8 - 4.4 (0.193247 (-0.826209)^t - 0.193247 0.726209^t) -
          12.8 (-0.111949 (-0.826209)^t + 1.11195 0.726209^t)

In[18]:= points = Table[{xt, yt}, {t, 0, 20}];

In[19]:= traj = ListPlot[points, PlotJoined -> True, DisplayFunction -> Identity];

In[20]:= isoclines = Plot[{8 + 2 x, 40 - 3 x}, {x, -2, 10}, DisplayFunction -> Identity];
```

```
In[21]:= Show[{traj, isoclines}, AspectRatio -> 1,
           AxesLabel -> {"x", "y"}, DisplayFunction -> $DisplayFunction];
```



This figure clearly shows the trajectory oscillating across the phase line, but converging on the fixed point, which confirms the stability established in Section 5.3.

à Question 2

Example 5.10 is given by the following equations

$x_{t+1} = -0.85078\, x_t - y_t$

$y_{t+1} = x_t + 2.35078\, y_t$

which has a fixed point at the origin. Subtracting x_t from both sides of the first equation and y_t from both sides of the second equation we obtain,

$\Delta x_t = x_{t+1} - x_t = -1.85078\, x_t - y_t$

$\Delta y_t = y_{t+1} - y_t = x_t + 1.35078\, y_t$

The two phase lines are found by setting $\Delta x_t = 0$ and $\Delta y_t = 0$ respectively, i.e.,

```
In[22]:= Solve[-1.85078 x - y == 0, y]

Out[22]= {{y → -1.85078 x}}

In[23]:= Solve[x + 1.35078 y == 0, y]

Out[23]= {{y → -0.740313 x}}
```

It should be noted that both phase lines have a negative slope.

The vector forces can be established by noting:

If $\Delta x_t > 0$ then $y_t < -1.85078\, x_t$, i.e. below $\Delta x_t = 0$, x is rising

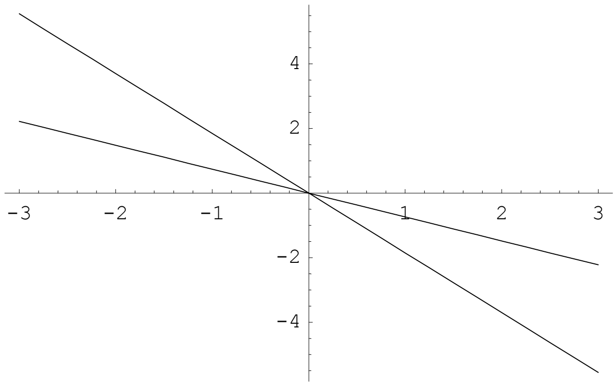
If $\Delta x_t < 0$ then $y_t > -1.85078\, x_t$, i.e. above $\Delta x_t = 0$, x is falling

and

If $\Delta y_t > 0$ then $y_t > -0.740313\, x_t$, i.e. above $\Delta x_t = 0$, y is rising

If $\Delta y_t < 0$ then $y_t < -0.740313\, x_t$, i.e. below $\Delta x_t = 0$, y is falling

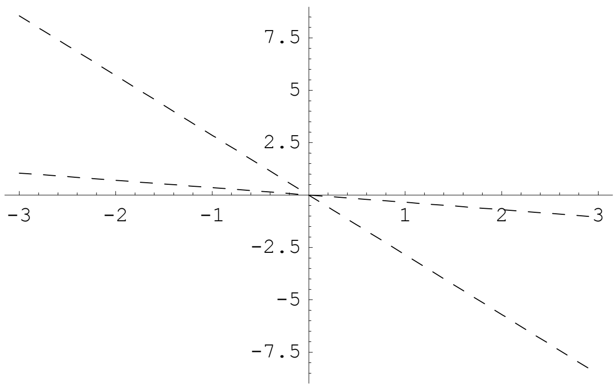
```
In[24]:= isoclines = Plot[{-1.85078 x, -0.740313 x}, {x, -3, 3}];
```



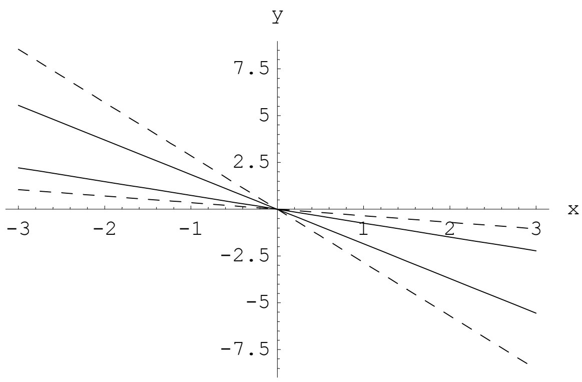

```
In[25]:= matrixA = ( -0.85078    -1
                         1     2.35078 )

Out[25]= {{-0.85078, -1}, {1, 2.35078}}

In[26]:= Eigenvalues[matrixA]

Out[26]= {2., -0.499999}

In[27]:= Eigenvectors[matrixA]

Out[27]= {{0.331007, -0.943628}, {-0.943628, 0.331007}}
```

To find the saddle-path equations

```
In[28]:= Solve[0.331007 x == -0.943628 y, y]

Out[28]= {{y → -0.350781 x}}

In[29]:= Solve[-0.943628 x == 0.331007 y, y]

Out[29]= {{y → -2.85078 x}}

In[30]:= saddles =
           Plot[{-0.350781 x, -2.85078 x}, {x, -3, 3}, PlotStyle -> Dashing[{0.02}] ];
```



```
In[31]:= Show[isoclines, saddles, AxesLabel -> {"x", "y"}];
```



à Question 3

The two equations of Example 5.4 are:

$$x_{t+1} = -8 - x_t + y_t$$
$$y_{t+1} = 4 - 0.3\,x_t + 0.9\,y_t$$

and although the question refers to setting up the problem on a spreadsheet, we can also use *Mathematica* to show a table of values. We do this by solving the system of equations for each of the initial conditions and set the results out as a table.

```
In[32]:= Clear[x, y]
```

For (3,10)

```
In[33]:= sol1 = RSolve[{x[t + 1] == -8 - x[t] + y[t],
            y[t + 1] == 4 - 0.3 x[t] + 0.9 y[t], x[0] == 3, y[0] == 10}, {x[t], y[t]}, t]

Out[33]= {{x[t] → -3. (-1. If[t == 0, 1, 0] +
                 If[t ≥ 1, -0.24078 (-0.826209)^t + 1.24078 0.726209^t - 2. 1.^t, 0]) +
              3.7 (-1. If[t == 1, 1, 0] + If[t ≥ 2, 0.291428 (-0.826209)^t +
                   1.70857 0.726209^t - 2. 1.^t, 0]) - 3.9 (-1. If[t == 2, 1, 0] +
                 If[t ≥ 3, -0.352729 (-0.826209)^t + 2.35273 0.726209^t - 2. 1.^t, 0]),
          y[t] → -1.66667 (-10. (0.144468 (-0.826209)^t - 0.744468 0.726209^t + 1.2 1.^t) +
                If[t ≥ 1, 1.11348 (-0.826209)^t - 0.633476 0.726209^t - 0.48 1.^t, 0])}}
```

```
In[34]:= path1 = Table[{t, sol1[[1, 1, 2]], sol1[[1, 2, 2]]}, {t, 0, 20}] ;
         TableForm[path1, TableHeadings -> {{ }, {"t", "x[t]", "y[t]"}}]
```

Out[34]//TableForm=

t	x[t]	y[t]
0	3.	10.
1	-1.	12.1
2	5.1	15.19
3	2.09	16.141
4	6.051	17.8999
5	3.8489	18.2946
6	6.44571	19.3105
7	4.86477	19.4457
8	6.58095	20.0417
9	5.46077	20.0633
10	6.60249	20.4187
11	5.81621	20.3961
12	6.57987	20.6116
13	6.03174	20.5765
14	6.54475	20.7093
15	6.16457	20.675
16	6.51039	20.7581
17	6.2477	20.7292
18	6.48147	20.7819
19	6.30047	20.7593
20	6.45883	20.7932

```
In[35]:= Clear[x, y]
```

For (3,30)

```
In[36]:= sol2 = RSolve[{x[t + 1] == -8 - x[t] + y[t],
           y[t + 1] == 4 - 0.3 x[t] + 0.9 y[t], x[0] == 3, y[0] == 30}, {x[t], y[t]}, t]
```

```
Out[36]= {{x[t] → -3. (-1. If[t == 0, 1, 0] +
              If[t ≥ 1, -0.24078 (-0.826209)^t + 1.24078 0.726209^t - 2. 1.^t, 0]) -
           16.3 (-1. If[t == 1, 1, 0] + If[t ≥ 2, 0.291428 (-0.826209)^t +
                1.70857 0.726209^t - 2. 1.^t, 0]) + 16.1 (-1. If[t == 2, 1, 0] +
              If[t ≥ 3, -0.352729 (-0.826209)^t + 2.35273 0.726209^t - 2. 1.^t, 0]),
         y[t] → -1.66667 (-30. (0.144468 (-0.826209)^t - 0.744468 0.726209^t + 1.2 1.^t) +
             If[t ≥ 1, 5.34622 (-0.826209)^t - 28.8662 0.726209^t + 23.52 1.^t, 0])}}
```

```
In[37]:= path2 = Table[{t, sol2[[1, 1, 2]], sol2[[1, 2, 2]]}, {t, 0, 20}] ;
         TableForm[path2, TableHeadings -> {{ }, {"t", "x[t]", "y[t]"}}]
```

Out[37]//TableForm=

t	x[t]	y[t]
0	3.	30.
1	19.	30.1
2	3.1	25.39
3	14.29	25.921
4	3.631	23.0419
5	11.4109	23.6484
6	4.23751	21.8603
7	9.62279	22.403
8	4.78023	21.2759
9	8.49565	21.7142
10	5.21857	20.9941
11	7.77553	21.3291
12	5.55359	20.8636
13	7.30996	21.1111
14	5.80116	20.807
15	7.00586	20.986
16	5.98011	20.7856
17	6.80551	20.913
18	6.10751	20.7801
19	6.67255	20.8698
20	6.19725	20.7811

```
In[38]:= Clear[x, y]
```

For (10,10)

```
In[39]:= sol3 = RSolve[{x[t + 1] == -8 - x[t] + y[t],
            y[t + 1] == 4 - 0.3 x[t] + 0.9 y[t], x[0] == 10, y[0] == 10}, {x[t], y[t]}, t]
```

```
Out[39]= {{x[t] → -10. (-1. If[t == 0, 1, 0] +
                If[t ≥ 1, -0.24078 (-0.826209)^t + 1.24078 0.726209^t - 2. 1.^t, 0]) +
             17. (-1. If[t == 1, 1, 0] + If[t ≥ 2, 0.291428 (-0.826209)^t +
                  1.70857 0.726209^t - 2. 1.^t, 0]) - 10.2 (-1. If[t == 2, 1, 0] +
                If[t ≥ 3, -0.352729 (-0.826209)^t + 2.35273 0.726209^t - 2. 1.^t, 0]),
           y[t] → -1.66667 (-10. (0.144468 (-0.826209)^t - 0.744468 0.726209^t + 1.2 1.^t) +
               If[t ≥ 1, 0.301839 (-0.826209)^t + 0.178161 0.726209^t - 0.48 1.^t, 0])}}
```

```
In[40]:= path3 = Table[{t, sol3[[1, 1, 2]], sol3[[1, 2, 2]]}, {t, 0, 20}] ;
         TableForm[path3, TableHeadings -> {{ }, {"t", "x[t]", "y[t]"}}]
```

Out[40]//TableForm=

t	x[t]	y[t]
0	10.	10.
1	-8.	10.
2	10.	15.4
3	-2.6	14.86
4	9.46	18.154
5	0.694	17.5006
6	8.8066	19.5423
7	2.73574	18.9461
8	8.21039	20.2308
9	4.02041	19.7446
10	7.72419	20.564
11	4.83982	20.1904
12	7.35053	20.7194
13	5.36884	20.4423
14	7.07344	20.7874
15	5.71396	20.5866
16	6.87267	20.8138
17	5.94111	20.6706
18	6.72949	20.8212
19	6.09172	20.7202
20	6.62852	20.8207

```
In[41]:= Clear[x, y]
```

For (10,30)

```
In[42]:= sol4 = RSolve[{x[t + 1] == -8 - x[t] + y[t],
           y[t + 1] == 4 - 0.3 x[t] + 0.9 y[t], x[0] == 10, y[0] == 30}, {x[t], y[t]}, t]
```

Out[42]= $\{\{x[t] \to -10.\ (-1.\ \mathrm{If}[t == 0, 1, 0] + \mathrm{If}[t \geq 1, -0.24078\ (-0.826209)^{t} + 1.24078\ 0.726209^{t} - 2.\ 1.^{t}, 0]) - 3.\ (-1.\ \mathrm{If}[t == 1, 1, 0] + \mathrm{If}[t \geq 2, 0.291428\ (-0.826209)^{t} + 1.70857\ 0.726209^{t} - 2.\ 1.^{t}, 0]) + 9.8\ (-1.\ \mathrm{If}[t == 2, 1, 0] + \mathrm{If}[t \geq 3, -0.352729\ (-0.826209)^{t} + 2.35273\ 0.726209^{t} - 2.\ 1.^{t}, 0]),$

$y[t] \to -1.66667\ (0.200544\ (-0.826209)^{t} - 5.72054\ 0.726209^{t} - 12.48\ 1.^{t})\}\}$

```
In[43]:= path4 = Table[{t, sol4[[1, 1, 2]], sol4[[1, 2, 2]]}, {t, 0, 20}] ;
         TableForm[path4, TableHeadings -> {{ }, {"t", "x[t]", "y[t]"}}]
```

Out[43]//TableForm=

t	x[t]	y[t]
0	10.	30.
1	12.	28.
2	8.	25.6
3	9.6	24.64
4	7.04	23.296
5	8.256	22.8544
6	6.5984	22.0922
7	7.49376	21.9034
8	6.40966	21.465
9	7.05529	21.3956
10	6.34027	21.1394
11	6.79915	21.1234
12	6.32425	20.9713
13	6.64706	20.9769
14	6.32984	20.8851
15	6.55525	20.8976
16	6.34238	20.8413
17	6.49891	20.8545
18	6.35554	20.8193
19	6.46379	20.8307
20	6.36694	20.8085

à Question 4

The system for example 5.5 is

$$x_t = x_{t-1} + 2\, y_{t-1} + z_{t-1}$$
$$y_t = -x_{t-1} + y_{t-1}$$
$$z_t = 3\, x_{t-1} - 6\, y_{t-1} - z_{t-1}$$

(a) We can first use *Mathematica* to check the solution in the text for this set of equations for the initial conditions $x(0) = 3$, $y(0) = -4$ and $z(0) = 3$.

```
In[44]:= Clear[x, y, z, t]
```

```
In[45]:= sol = RSolve[{x[t] == x[t - 1] + 2 y[t - 1] + z[t - 1], y[t] == -x[t - 1] + y[t - 1],
                z[t] == 3 x[t - 1] - 6 y[t - 1] - z[t - 1], x[0] == 3, y[0] == -4, z[0] == 3},
           {x[t], y[t], z[t]}, t]
```

Out[45]= $\{\{x[t] \to 6\,(-1)^t + 2^{1+t} - 5\,\text{If}[t == 0, 1, 0],\ y[t] \to 3\,(-1)^t - 2^{1+t} - 5\,\text{If}[t == 0, 1, 0],\ z[t] \to -18\,(-1)^t + 3\,2^{1+t} + 15\,\text{If}[t == 0, 1, 0]\}\}$

Note that this is a fuller answer than in the text. Here we have a qualification for the initial condition that - 5 must be added to $x(t)$ to obtain the correct answer. Similarly for $y(t)$ and $z(t)$ -5 and +15 must be added respectively for the initial condition to be correct.

```
In[46]:= Clear[x, y, z]
```

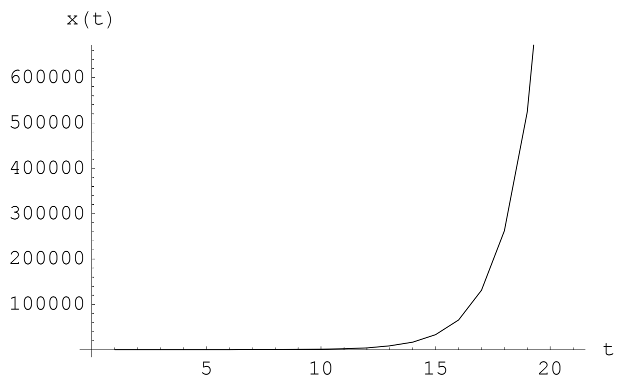
(b)

```
In[47]:= path = Table[{t, sol[[1, 1, 2]], sol[[1, 2, 2]], sol[[1, 3, 2]]}, {t, 0, 20}] ;
         TableForm[path, TableHeadings -> {{ }, {"t", "x[t]", "y[t]", "z[t]"}}]

General::spell1 :
 Possible spelling error: new symbol name "path" is similar to existing symbol "Path".

Out[47]//TableForm=
```

t	x[t]	y[t]	z[t]
0	3	-4	3
1	-2	-7	30
2	14	-5	6
3	10	-19	66
4	38	-29	78
5	58	-67	210
6	134	-125	366
7	250	-259	786
8	518	-509	1518
9	1018	-1027	3090
10	2054	-2045	6126
11	4090	-4099	12306
12	8198	-8189	24558
13	16378	-16387	49170
14	32774	-32765	98286
15	65530	-65539	196626
16	131078	-131069	393198
17	262138	-262147	786450
18	524294	-524285	1572846
19	1048570	-1048579	3145746
20	2097158	-2097149	6291438

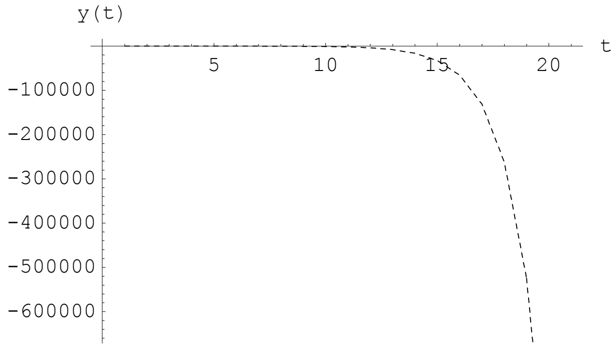
which is explosive. Plotting each against time we have,

```
In[48]:= listx = Table[sol[[1, 1, 2]], {t, 0, 20}];

In[49]:= listy = Table[sol[[1, 2, 2]], {t, 0, 20}];

General::spell1 :
 Possible spelling error: new symbol name "listy" is similar to existing symbol "listx".

In[50]:= listz = Table[sol[[1, 3, 2]], {t, 0, 20}];

General::spell :
 Possible spelling error: new symbol name "listz" is similar to existing symbols {listx, listy}.

In[51]:= linex = ListPlot[listx, PlotJoined -> True, AxesLabel -> {"t", "x(t)"}];
```



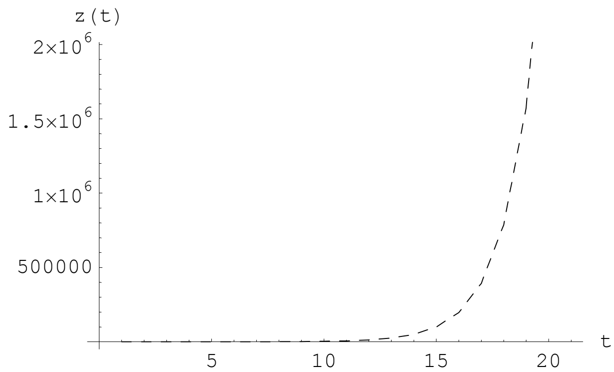
```
In[52]:= liney = ListPlot[listy, PlotJoined -> True,
           AxesLabel -> {"t", "y(t)"}, PlotStyle -> Dashing[{0.01}]];
```

```
General::spell1 :
 Possible spelling error: new symbol name "liney" is similar to existing symbol "linex".
```

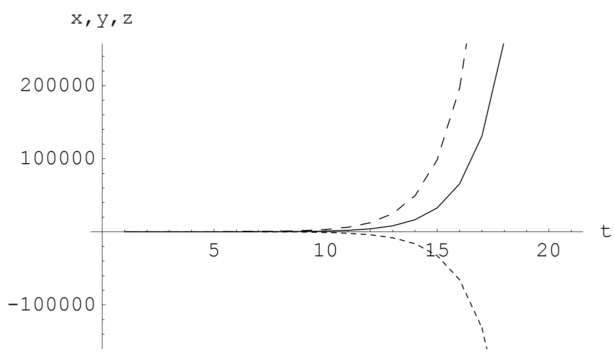



```
In[53]:= linez = ListPlot[listz, PlotJoined -> True,
           AxesLabel -> {"t", "z(t)"}, PlotStyle -> Dashing[{0.02}]];
```

```
General::spell :
 Possible spelling error: new symbol name "linez" is similar to existing symbols {linex, liney}.
```



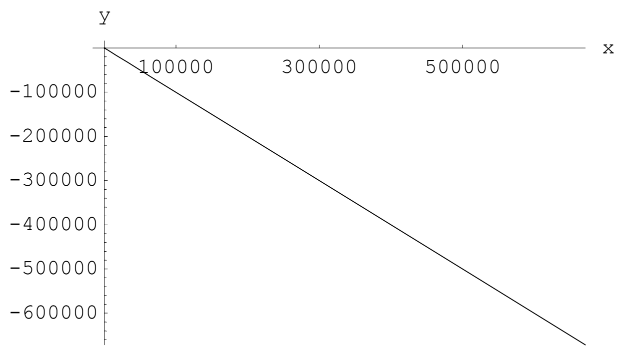

```
In[54]:= Show[linex, liney, linez, AxesLabel -> {"t", "x,y,z"}];
```



(c)

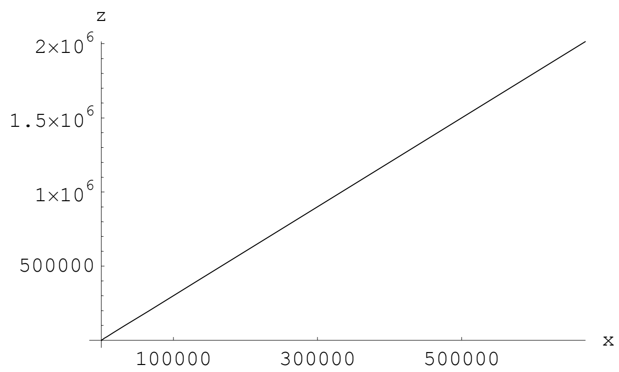
```
In[55]:= pointsxy = Table[{sol[[1, 1, 2]], sol[[1, 2, 2]]}, {t, 0, 20}];
```

```
In[56]:= ListPlot[pointsxy, PlotJoined -> True,
           Ticks -> {{100000, 300000, 500000}, Automatic}, AxesLabel -> {"x", "y"}];
```



(d)

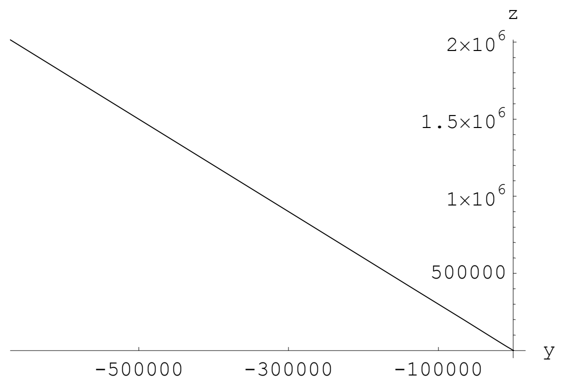
```
In[57]:= pointsxz = Table[{sol[[1, 1, 2]], sol[[1, 3, 2]]}, {t, 0, 20}];

         General::spell1 :
          Possible spelling error: new symbol name "pointsxz" is similar to existing symbol "pointsxy".

In[58]:= ListPlot[pointsxz, PlotJoined -> True,
           Ticks -> {{100000, 300000, 500000}, Automatic}, AxesLabel -> {"x", "z"}];
```



(e)

```
In[59]:= pointsyz = Table[{sol[[1, 2, 2]], sol[[1, 3, 2]]}, {t, 0, 20}];

         General::spell1 :
          Possible spelling error: new symbol name "pointsyz" is similar to existing symbol "pointsxz".
```

```
In[60]:= ListPlot[pointsyz, PlotJoined -> True,
           Ticks -> {{-100000, -300000, -500000}, Automatic}, AxesLabel -> {"y", "z"}];
```

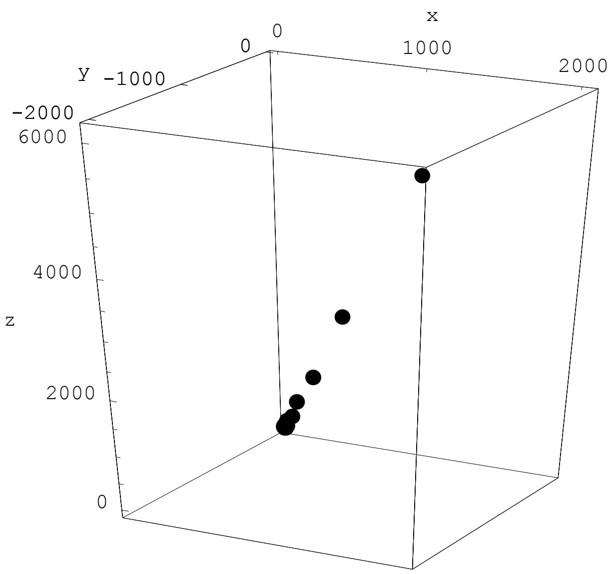


à Question 5

We already have the values for x[t],y[t] and z[t] in Question 4. These are specified as sol[[1,1,2]] for x, sol[[1,2,2]] for y and sol[[1,3,2]] for z. However, to plot such a set of points in 3-dimensional space, we need to utilise the **ScatterPlot3D** command which is available in the Graphics3D package. This package must first be loaded. Since the figures become large (both negatively and positively) we plot only the first 10 points.

```
In[61]:= points = Table[{sol[[1, 1, 2]], sol[[1, 2, 2]], sol[[1, 3, 2]]}, {t, 0, 10}];
```

```
In[62]:= ScatterPlot3D[points, AxesLabel -> {x, y, z},
           Ticks -> {{0, 1000, 2000}, {0, -1000, -2000}, Automatic},
           PlotStyle -> PointSize[0.03], AspectRatio -> 1];
```



à Question 6

Example 5.5 can be written first as a set of difference equations:

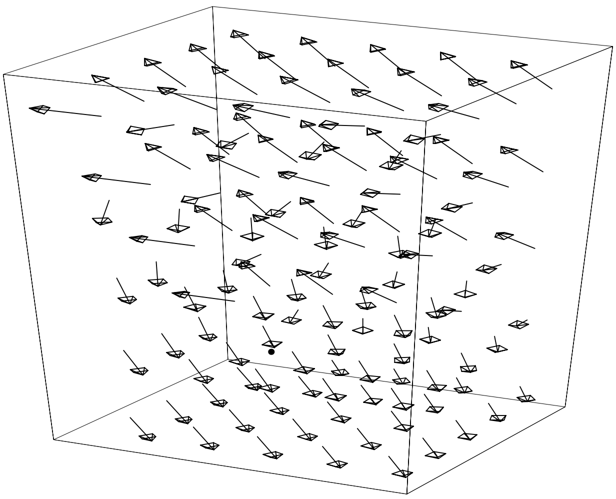
$\Delta x_t = 2\, y_t + z_t$

$\Delta y_t = -x_t$

$\Delta z_t = 3\, x_t - 6\, y_t - 2\, z_t$

We first need to load the **PlotField3D** command. The 3-dimensional direction field can then be obtained using the following instructions.

```
In[63]:= PlotVectorField3D[{2 y + z, -x, 3 x - 6 y - 2 z},
           {x, 0, 2000}, {y, 0, -2000}, {z, 0, 6000}, PlotPoints -> 5,
          VectorHeads -> True, ScaleFunction -> (1 &), AspectRatio -> 0.8];
```



But this does not appear to give us much additional insight.

à Question 7

The matrix for each system is respectively:

$$\mathbf{A1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \mathbf{A2} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \qquad \mathbf{A3} = \begin{pmatrix} 3 & -4 \\ 1 & -2 \end{pmatrix}$$

If r and s are the eigenvalues of the system, then the diagonal matrix, **D**, is given by:

$$\mathbf{D} = \begin{pmatrix} r & 0 \\ 0 & s \end{pmatrix}$$

■ (i) System 1

```
In[64]:= matrixA1 = ( 0  1
                     -1  0 )

Out[64]= {{0, 1}, {-1, 0}}
```

(a)

```
In[65]:= Eigenvalues[matrixA1]

Out[65]= {-i, i}
```

(b)

```
In[66]:= Eigenvectors[matrixA1]

Out[66]= {{i, 1}, {-i, 1}}
```

(c)

The diagonal matrix for this first system is

$$\mathbf{D1} = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

■ (ii) System 2

```
In[67]:= matrixA2 = ( -2   1
                       1  -2 )

Out[67]= {{-2, 1}, {1, -2}}
```

(a)

```
In[68]:= Eigenvalues[matrixA2]

Out[68]= {-3, -1}
```

(b)

```
In[69]:= Eigenvectors[matrixA2]

Out[69]= {{-1, 1}, {1, 1}}
```

(c)

The diagonal matrix for this first system is

$$\mathbf{D2} = \begin{pmatrix} -3 & 0 \\ 0 & -1 \end{pmatrix}$$

■ (iii) System 3

```
In[70]:= matrixA3 = ( 3  -4
                      1  -2 )

Out[70]= {{3, -4}, {1, -2}}
```

(a)

```
In[71]:= Eigenvalues[matrixA3]

Out[71]= {-1, 2}
```

(b)

```
In[72]:= Eigenvectors[matrixA3]

Out[72]= {{1, 1}, {4, 1}}
```

(c)

The diagonal matrix for this first system is

$$\mathbf{D3} = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$$