Essentials of Micro and Nanofluidics with Applications to the Biological and Chemical Sciences

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Errata/corrections/clarifying notes

Note: this list is organized by page number. All equation numbers remain the same.

1. page 4 Add "Contributed by Professor Minami Yoda" to the bottom in the box
2. page 4 last paragraph on the page line 3 the italics should be "lab-on-a-chip" replaces "lab on a chip"; also line 1 page 5.
3. page 26: equation 1.18, "K" should be italicised consistent with Chapter 7. See equations 7.41 and 7.42
4. page 31 first sentence second full paragraph "...that incorporates van der Waals forces and electrostatic forces." Delete the words "the attractive' and "the repulsive".
5. page 43 top of the page "$Watt = \frac{1Joule}{sec}$"; eliminate space after first t.
6. page 43, midpage "The *mole fraction*, $X_A = \frac{c_A}{c}$" Eliminate the space after "fraction".
7. page 49 equation 2.23 should read $\vec{F} = -\nabla \mu_A$
8. page 72: problem 2.3, line 6: "c=1M" replaces "T=1M"
9. page 72: problem 2.4, Second sentence should read "Assuming that the sodium ion..."

10. page 83 caption of Figure 3.8, last sentence should read: The x denotes the crossing points and $a = 2$, $b = 1$.

11. page 89 first line below equation 3.59 replace "is equivalent to" with "is proportional to"

12. page 89 equation 3.60 should read

$$\rho \frac{D\vec{V}}{Dt} = \frac{\vec{F}}{\mathcal{V}} = \frac{\vec{F}_{Surface} + \vec{F}_{Body}}{\mathcal{V}}$$

13. page 89 equation 3.63 should read

$$F_{x,net} = (\tau_{xx,front} - \tau_{xx,back})dydz + (\tau_{yx,right} - \tau_{yx,left})dxdz + (\tau_{zx,top} - \tau_{zx,bottom})dxdy$$

14. page 94 second full paragraph, end of first full sentence "$M_O = 16Da$"

15. page 99 equation 3.111 should read

$$\frac{Dc_A}{Dt} = D_{AB}\nabla^2 c_A - \frac{D_{AB} z_A F}{RT}\left(\frac{\partial c_A E_x}{\partial x} + \frac{\partial c_A E_y}{\partial y} + \frac{\partial c_A E_z}{\partial z}\right)$$

16. page 103 equation 3.127 $\frac{D\vec{b}\bullet\vec{r}}{Dt}$ replaces $\vec{g}\bullet\vec{r}$

17. page 104 equation 3.134 should read

$$\vec{V}\bullet\nabla\bullet\vec{\tau}_{ij} = \rho\left(\vec{V}\bullet\frac{D\vec{V}}{Dt} + \vec{V}\bullet\vec{b}\right)$$

18. page 114 equation 3.171 $\frac{\partial c_s}{\partial t}$ replaces $\frac{\partial c_A}{\partial t}$

19. page 117 Figure 3.20, change "a" to "d"

20. page 117 second line from bottom, should read $D(\rho, \mu, d, U)$.

21. page 119 equation (3.193), first term on the right side should read "-$\epsilon_1 \frac{\partial p}{\partial x}$".

22. page 120 equation (3.199), the pressure can be scaled using either h or L. To be consistent with equation (3.93) equation (3.199) should read $p = \frac{p^*}{\frac{\mu U}{h}}$ unless $\epsilon_1 = 1$ where $h = L$ where either h or L can be used.

23. page 135 last sentence should read "using these..." Omit "the".

24. page 136 exercise 3.8 " Show by direct calculation that in cartesian coordinates and in two-dimensions, the vorticity satisfies" replaces "Show by direct calculation that the vorticity satisfies"

25. page 136 exercise 3.9 should read "and at $y = 0$" replaces "and $t = 0$ at $y = 0$"

26. page 138 exercise 3.14 Equations 1 and 3 (unnumbered) are inconsistent and should read

$$\frac{\partial X_A}{\partial t} - \frac{\partial^2 X_A}{\partial y^2} + \frac{\beta}{\epsilon^2} z_A X_A \sum_{i=1}^{n} z_i X_i - z_A \frac{\partial \phi}{\partial y} \frac{\partial X_A}{\partial y} = 0$$

$$\alpha \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \frac{\beta E_x(t)}{\epsilon^2} \sum z_i X_i$$

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\beta}{\epsilon^2} \sum z_i X_i = 0$$

27. page 138 Exercise 3.14, $E_x(t)$ should be removed from the third equation of exercise 3.14.

28. page 138 exercise 3.16 $u(0) = 0$ not $u(0) = 1$

29. page 160 equation 4.106 $K(Re_w)$ replaces $\frac{K}{Re_w}$

30. page 165 just after equation (4.121) should be "where here $m = \frac{h_o - h_i}{h_i}$"

31. page 166 just after equation (4.124) "...the flow rate $Q = 8.3 \times 10^{-8} \frac{m^3}{sec}$ and for $\frac{h_o}{h_i} = 0.5$, $\frac{h_m}{h_i} = 2/3$; that is ..."

32. page 178 exercise 4.12 "solution of the form $\psi = f(r) sin\theta$" replaces "solution of the form $\psi = U r sin\theta$"

33. page 178 exercise 4.12 Ar^3 replaces Ar^2 in the equation defining the form of ψ.

34. page 185 end of the third complete paragraph: "..., concentration profiles will most often not be fully developed in channels of interest in microfluidics, unless the channels are very long, for Reynolds number of order one and greater. However, if the Reynolds number is extremely small as is most often the case in nanofluidics, the concentration profile can be fully developed." replaces "..., concentration profiles are rarely considered to be fully developed in channels of interest in microfluidics and nanofluidics."

35. page 186, equation (5.26) should read

$$\frac{\partial \theta}{\partial r} = -\frac{\frac{\partial T}{\partial r}|_{r=a}}{T_s - T_m} \neq f(x)$$

36. page 186 after equation (5.26), delete "which means that the heat flux across the tube is constant."

37. page 189 equation (5.37) c should be c_p.

38. page 196, equation 5.76 should read

$$\frac{\partial}{\partial t}|_z = \frac{\partial}{\partial t}|_{\bar{z}} + \frac{\partial \bar{z}}{\partial t}_z \frac{\partial}{\partial \bar{z}}|_{\bar{z}} = \frac{\partial}{\partial t}|_{\bar{z}} - \langle w \rangle \frac{\partial}{\partial \bar{z}}|_{\bar{z}}$$

39. page 203, just below equation (5.111), should read $B = \frac{L}{a+L}c_0$.

40. page 208 exercise 5.2 "...will be fully developed..." replaces "...will not be fully developed..."

41. page 208 exercise 5.2 add to the end of the statement "Take the diffusion coefficient to be $D_A = 10^{-9}\frac{m^2}{sec}$."

42. page 210, missing a reference, generated a "?": it should be

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\cite{guell1}

@article{guell1,
author="D. C. Guell and R. G. Cox and H. Brenner",
title="Taylor Dispersion in Conduits of Large Aspect Ratio",
journal="Chem. Eng. Comm.",
volume="58",
year="1987",
pages="231-244",
}
```

43. page 210 exercise 5.10 $\bar{z} = z - \langle w \rangle t$ replaces $\eta = z - \langle w \rangle t$

44. page 210, exercise 5.13

$$c_A(r,t) = 0 \ at \ r = a \ for \ t > 0$$

The first zero is missing.

45. page 210 In exercise 5.13, the equation should read

$$\frac{\partial c_A}{\partial t} = D_A \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial c_A}{\partial r}\right]$$

46. page 228, problem 6.6 the first boundary condition needs a negative sign:

$$\frac{d\phi}{dy} = -\frac{\sigma}{\epsilon_e} \ at \ y = 0$$

47. page 244, line under equation (7.56), $X_R < Xx_D$ should be $X_R < X_D$

48. page 254 equation (7.79) $\frac{F}{\epsilon_e}$ replaces $\frac{e}{\epsilon_e}$

49. page 264 equation (7.132), should read

$$\frac{\epsilon_e RT}{zF\lambda}(b^{z/2} - b^{-z/2}) = \frac{N_s F}{N_A}\left(\frac{K_1}{K_1 + b^z K_{Sym}[Sym^+] + b[H^+]}\right)$$

50. page 279, exercise 7.1 $-5mV$ instead of $-5V$

51. page 279 exercise 7.2 The expression for the surface charge density should read

$$\sigma = -\frac{N_s F}{N_A}\frac{K_1}{K_1 + [H^+]_s + K_M[M^+]}$$

52. page 279 exercise 7.2 K_{Na} in the second to last line and add at the end of the statement specifying pH,"$pH = 7$, and $[Na] = 10mM$ in the bulk. "

53. page 279 exercise 7.4 "...is known to be $\sigma = -.02\,\frac{C}{m^2}$." replaces "...is known to be $\sigma = -20\frac{C}{m^2}$."

54. page 279 exercise 7.6 "...is known to be $\sigma = -.02\,\frac{C}{m^2}$, and at $y = h$, the surface charge density is $\sigma = -.015\frac{C}{m^2}$." replaces "... is known to be $\sigma = -20mV$ and at $y = h$ the surface charge density is $\sigma = 15mV$. "

55. page 280 exercise 7.9 "defined as when the potential $\mid \phi \mid < 0.01$?" replaces "defined as when the potential $\phi < 0.01$?"

56. page 280 first equation should read

$$\frac{F}{A} = \mp\frac{1}{2}\epsilon_e\zeta^2\kappa^2 \;\; at \;\; y = 0, h$$

and the second equation should read

$$\frac{F}{A} = p(h) - p(0) = \frac{1}{2}\epsilon_e\zeta^2\kappa^2 tanh^2\kappa h \;\; for \;\; \kappa h = O(1)$$

57. page 295, Figure 8.7 caption should read "Illustration of a hormone binding to a protein. Image courtesy of the National Library of Medicine."

58. page 302 first full paragraph, line 5 "order of $1\mu sec$" replaces "order of $1msec$"

59. page 318 "In Figure 9.8" replaces "In Figure 9.9"

60. page 318 Figure 9.9 is not discussed so after "...for the parameters of Figure 9.8." Add "The electroosmotic velocity profiles in this Section are typical of those channels that make up the nanopore membrane sketched on Figure 9.9, although such membranes may contain thousands of channels. "

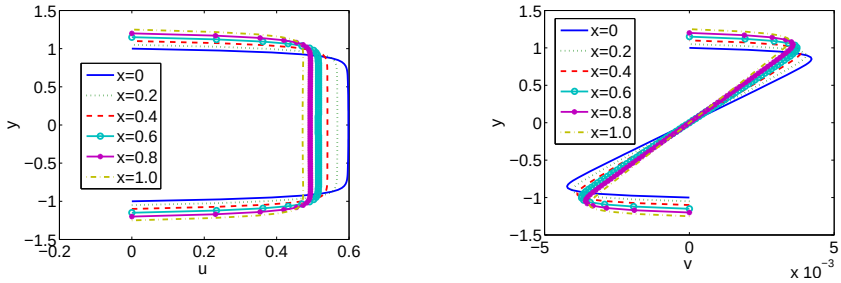
61. page 319 equation (9.44) $m_A z_A X_A E_x$ replaces $m_A F z_A X_A E_x$

62. page 325 last paragraph second line should read "for which $\frac{h_i}{L}Re_h << 1$ in the ..."

63. page 326, $+e^{-(\frac{h+y}{\epsilon})}$ replaces $-e^{-(\frac{h+y}{\epsilon})}$ in equations 9.68,9.69,9.70.

64. page 3.27 first line under equation 9.75: "distribution is comprised of...."

65. page 327 Figure 9.15 caption and figures should be replaced. Figure environment should read



(a) streamwise velocity (b) transverse velocity

Figure 9.15 The dimensionless streamwise and transverse velocity of EOF in a nano-diffuser. The height of the diffuser is $20\,nm$ at the inlet and $25\,nm$ at the outlet and the length of the diffuser is $600\,nm$ and the EDLs are thin compared to the diffuser. The imposed electric field is $8000V/m$ and the ζ-potential is $-15mV$. The pressure is zero both at the inlet and the outlet. The concentration of each electrolyte component in the upstream reservoir is $0.1M$.

66. Page 327, paragraph starting with "Results for the ..." should read
Results for the electro-osmotic flow in a nano-diffuser are shown in Figure 9.15. In this example, the height of the diffuser is $20\,nm$ at the inlet and $25\,nm$ at the outlet and the length of the diffuser is $600\,nm$; the parameter $\epsilon \sim 0.05 \ll 1$. Each electrolyte concentration is $0.1\,M$ in the reservoir and the ζ-potential walls is $-15mV$ which is typical of PMMA (Kirby 2004 and Hasselbrink, 2004c). Note that the EDLs are thin compared to the height of the diffuser and as required by continuity the velocity decreases in the streamwise direction.

67. page 328, paragraph beginning with "Figure 9.16 shows..." should read
Figure 9.16 shows the electroosmotic flow in a nano-nozzle. The height of the nano-nozzle at the inlet is $20\,nm$ and the length of the nozzle is $600\,nm$; the height at the outlet is $15\,nm$. Here $\epsilon = 0.15$ and the EDLs are overlapped as shown in the streamwise velocity profile in Figure 9.16. The streamwise velocity is nearly parabolic and the transverse velocity is small compared to the streamwise velocity.

68. Page 329, figure 9.16 needs to be replaced by

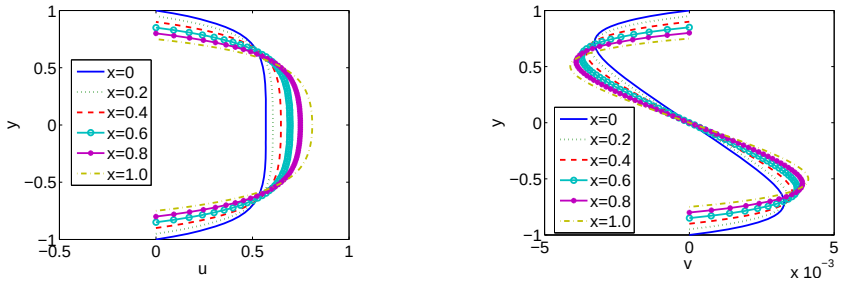
69. page 335, just below equation 9.101 $\xi = \frac{\epsilon_e - \epsilon_e^{'}}{2\epsilon_e + \epsilon_e^{'}}$ replaces $\xi = \frac{\epsilon_e - \epsilon_e^{'}}{2\epsilon_e - \epsilon_e^{'}}$

70. page 335, equation 9.103 $\nabla \bullet \vec{u}$ replaces $\nabla \cdot \vec{u}$

71. page 337, Figure 9.20 Omit "Here $y = \frac{F\zeta}{RT}$ and E is the electric field strength." from the caption.

72. page 337, Figure 9.20 add to caption: "Here $\alpha = \frac{6\pi\mu\phi_0}{\epsilon_e}$. "

73. page 337, Figure 9.20, vertical axis label should read "$\alpha\mu_{\,e}$" where the current label is.



(a) streamwise velocity (b) spanwise velocity

Figure 9.16 The dimensionless streamwise and transverse velocity of EOF in a nano-nozzle. The height of the nozzle is $20\,nm$ at the inlet and $15\,nm$ at the outlet and the length of the nozzle is $600\,nm$; $\epsilon = 0.15$ and the EDLs are overlapped. The imposed electric field is assumed to be $8000\,V/m$ and the ζ-potential of the walls is $-15\,mV$. The concentration of each electrolyte solution in the upstream reservoir is $0.01M$.

74. page 337, Figure 9.20, horizontal axis label should read "$\frac{\zeta F}{RT}$" where the current label is.

75. page 345 exercise 9.5 just below the equation, should be "where $x = e^{\psi_o} = e^{u_o}$, the outer..."

76. page 346 exercise 9.6 second to last line: "electro-osmotic" replaces "elecro-osmotic".

77. page 346 exercise 9.6 Add to the end "Assume the imposed electric field is $E_x = 5000\frac{V}{m}$."

78. page 346 exercise 9.11 Delete "Compare this value with the value obtained from an electromigration calculation of $1mm/sec$."

79. page 346 exercise 9.11 Add to the end "Assume the imposed electric field is $E_x = 5000\frac{V}{m}$."

80. page 346 exercise 9.12 Add to the end "Assume the imposed electric field is $E_x = 5000\frac{V}{m}$."

81. page 346, exercise 9.12, line 3, "one species of colloidal" should be "one species particle"

82. page 347 exercise 9.13 $2rdr$ replaces rdr in the denominator of the equation at the top of the page.

83. page 347, exercise 9.17, the first γ equation should read

$$\gamma = \frac{\left(\frac{K}{D_A} - 1\right)_{EOF}}{\left(\frac{K}{D_A} - 1\right)_{PR}}$$

84. page 374 equation (10.66) should read

$$f_i'' \cong \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2} + O(h^2)$$

85. page 396 Definition of Thomas Algorithm, the following equations replace the four unnumbered equations following the sentence beginning with "Different schemes..." The equations for α_1 and and γ_1 remain. After that:

$$\alpha_i = \frac{c_i}{a_i - b_i\alpha_{i-1}} \ , \gamma_i = \frac{d_i - b_i\gamma_{i-1}}{a_i - b_i\alpha_{i-1}} \ for \ i = 2, ..., N-1$$

with the back substitution step as

$$x_n = \gamma_n \ , x_i = \gamma_i - c_i x_{i+1} \ for \ i = N-1, ..., 2$$

replaces

$$\alpha_i = \frac{c_i}{a_1 - b_i c_{i-1}}, \gamma_i = \frac{d_i - b_i d_{i-1}}{a_i - b_i c_{i-1}} \ for \ i = 2, ..., N-1$$

with the back substitution step as

$$x_n = \frac{\gamma_n}{\alpha_n}, x_i = \frac{(\gamma_i - \alpha_i x_{i+1})}{\alpha_i} \ for \ i = N-1, ..., 2$$

86. page 408 Just under equation (10.188) "suppose we have used the elimination algorithm discussed in section 10.9.3" replaces "suppose we have used the Thomas algorithm"

87. page 444, exercise 10.18, add "Assume $\zeta = -3$." at the end of the exercise.

88. page 445 In exercise 10.23 "yvst" should be "y as a function of t"

89. page 445 In exercise 10.25 "For a square channel" should read "For a rectangular channel"

90. page 463 equation 11.55 should read

$$\frac{d^2x_i}{dt^2} = \frac{x_i^{k+1} - 2x_i^k + x_i^{k-1}}{\Delta t^2} + O(\Delta t^2)$$

91. page 466, second full paragraph, line 6 "particles" replaces "partiles"

92. page 506 After the sentence beginning with "The question is," next sentence, "In the outer solution..." replaces "In the inner solution..."

93. page 519 Day reference "Erkenntnis" replaces "Erkenntis"

94. page 532 3rd reference authors should read: "Xuan, X., Xu,B., Sinton, D. and Li, D."