uniformly in time but arrive at the photodetector randomly in time. Therefore, both the power P_s of the optical signal and the number $\mathcal{S}$ of photons received in a given time interval Δt fluctuate randomly around their respective average values of P_s and $\overline{\mathcal{S}}$. The random fluctuations of the photon numbers are characterized by Poisson statistics. In any given time interval Δt, the probability of receiving $\mathcal{S}$ photons is given by the Poisson probability distribution:

$$p(\mathcal{S}) = \frac{\overline{\mathcal{S}}^{\mathcal{S}} \mathrm{e}^{-\overline{\mathcal{S}}}}{\mathcal{S}!}. \tag{11.28}$$

The mean square noise in photon number fluctuations can then be calculated as

$$\overline{\mathcal{S}_\mathrm{n}^2} = \sigma_{\mathcal{S}}^2 = \sum_{\mathcal{S}} p(\mathcal{S})\left(\mathcal{S} - \overline{\mathcal{S}}\right)^2 = \overline{\mathcal{S}}. \tag{11.29}$$

This photon contribution to the noise of a photodetector is independent of the physical properties of the photodetector because it is external to the photodetector. It is the ultimate lower limit of the noise in an optical detection system. It sets the fundamental limit on the detectivity of a photodetector.

The photons received by a photodetector are converted to photoelectrons or electron–hole pairs, depending on the type of photodetector, through the photoelectric effect. With a quantum efficiency of η_e, which has a value between 0 and 1, the number of photoelectrons generated is only a fraction of that of the photons received by the photodetector. Because a given photon can only generate either one or no electron, but not a fraction of an electron, the photoelectric process is clearly quantum mechanical and probabilistic. The shot noise associated with this process has to be considered if the quantum efficiency is less than unity. This effect is fully accounted for by considering the statistics of the number $\mathcal{N}$ of charge carriers that are generated in the photodetector with a quantum efficiency η_e. The random fluctuations of the photogenerated charge carriers are also characterized by the Poisson probability distribution:

$$p(\mathcal{N}) = \frac{\overline{\mathcal{N}}^{\mathcal{N}} \mathrm{e}^{-\overline{\mathcal{N}}}}{\mathcal{N}!}, \tag{11.30}$$

where $\mathcal{N} = \eta_\mathrm{e}\mathcal{S}$ as given in (11.3). We find, through a procedure similar to that used in (11.29), that the mean square noise in the number of photogenerated carriers is

$$\overline{\mathcal{N}_\mathrm{n}^2} = \sigma_{\mathcal{N}}^2 = \overline{\mathcal{N}}. \tag{11.31}$$

Because $\mathcal{N} < \mathcal{S}$ when $\eta_\mathrm{e} < 1$, the noise is actually reduced by an imperfect quantum efficiency. This result seems odd. However, what really counts in a detection system is not the noise alone, but the SNR. While the noise is reduced by an imperfect quantum efficiency of $\eta_\mathrm{e} < 1$, the signal is reduced even more. Consequently, a photodetector that has a poorer quantum efficiency has a lower SNR.

We consider here a photodetector that has no internal gain, such that $i_\mathrm{s} = i_\mathrm{ph}$. Using (11.4) and (11.31), we find the shot current noise in the photodetector:

$$\overline{i_{\mathrm{n,sh}}^2} = 4e^2B^2\overline{\mathcal{N}_\mathrm{n}^2} = 4e^2B^2\overline{\mathcal{N}} = 2eB\overline{i_\mathrm{s}}. \tag{11.32}$$

We then find the mean square current fluctuations for the shot noise in a photodetector that receives an optical power of P_s from an input optical signal: