A First Course in Random Matrix Theory: Erratum

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page 77

$$\sqrt{V'(s)^2 - 4\Pi(s)} \approx 2C(s-\lambda_+)^\theta, \tag{5.82}$$

$$\Phi(\lambda_{\max}) = \frac{2C}{\theta+1}(\lambda_{\max}-\lambda_+)^{\theta+1} \qquad \lambda_{\max}-\lambda_+ \ll 1. \tag{5.83}$$

page 236

$$\begin{aligned}
\mathbf{M}_{12}\mathbf{G}_{22}(z)\mathbf{M}_{21} &= \sum_{k,\ell=1}^{T} \frac{1}{T^2} f(y_k)f(y_\ell)[\mathbf{a}_k]_1[\mathbf{a}_\ell]_1 \sum_{i,j>1}^{N} [\mathbf{a}_k]_i[\mathbf{G}_{22}(z)]_{ij}[\mathbf{a}_\ell]_j \\
&= \sum_{k=1}^{T} \frac{1}{T^2} f^2(y_k)([\mathbf{a}_k]_1)^2 \sum_{i,j>1}^{N} [\mathbf{a}_k]_i[\mathbf{G}_{22}(z)]_{ij}[\mathbf{a}_k]_j \\
&\overset{T\to\infty}{\to} qc_2h(z)
\end{aligned} \tag{14.66}$$

where $q = N/T$ and

$$c_2h(z) = \tau\left(\frac{\mathbf{HDH}^T}{T}\mathbf{G}_{22}(z)\right) \quad , \quad [\mathbf{H}]_{ik} = [\mathbf{a}_k]_i \quad i>1, \quad \mathbf{D}_{k\ell} = f^2(y_k)([\mathbf{a}_k]_1)^2\delta_{k\ell}. \tag{14.67}$$

page 237

For completeness we show here how to compute the function $h(z)$.

$$c_2h(z) = \frac{1}{N}\operatorname{Tr}\left(\frac{\mathbf{HDH}^T}{T}\left(z\mathbf{1} - \frac{\mathbf{HD}_0\mathbf{H}^T}{T}\right)^{-1}\right) = \frac{1}{N}\operatorname{Tr}\left(\mathbf{DF}(z)\right), \tag{14.73}$$

where $[\mathbf{D}_0]_{k\ell} = f(y_k)\delta_{k\ell}$ and

$$\begin{aligned}
\mathbf{F}(z) &= \frac{1}{T}\mathbf{H}^T\left(z\mathbf{1} - \frac{\mathbf{HD}_0\mathbf{H}^T}{T}\right)^{-1}\mathbf{H} \\
&= \sum_{n=0}^{\infty}\frac{1}{z^{n+1}}\frac{1}{T}\mathbf{H}^T\left(\frac{\mathbf{HD}_0\mathbf{H}^T}{T}\right)^n\mathbf{H} \\
&= \mathbf{D}_0^{-1/2}\left[\tilde{\mathbf{M}}\left(z\mathbf{1}-\tilde{\mathbf{M}}\right)^{-1}\right]\mathbf{D}_0^{-1/2}.
\end{aligned} \tag{14.74}$$

The term in brackets is the T-matrix of $\tilde{\mathbf{M}} := q\mathbf{D}_0^{1/2}\mathbf{HH}^T\mathbf{D}_0^{1/2}/N$ a $T\times T$ colored Wishart with $\tilde{q} = 1/q$ and true covariance $q\mathbf{D}_0$. In the large N limit we have the following subordination relation

$$\mathbb{E}\left[\mathbf{F}(z)\right] = \mathbf{D}_0^{-1/2}\left[q\mathbf{D}_0\left(\frac{z}{1+\tilde{q}\mathfrak{t}_{\tilde{\mathbf{M}}}(z)}\mathbf{1} - q\mathbf{D}_0\right)^{-1}\right]\mathbf{D}_0^{-1/2}. \tag{14.75}$$

Using the fact that $\tilde{q}\mathfrak{t}_{\tilde{\mathbf{M}}}(z) = \mathfrak{t}_{\mathbf{M}}(z)$, we find for the function $h(z)$ in the large N limit

$$\begin{aligned}
c_2h(z) &= \frac{1}{N}\operatorname{Tr}\left(\mathbf{D}\left(Z(z)\mathbf{1} - q\mathbf{D}_0\right)^{-1}\right) \\
&= \frac{1}{q}\int_{-\infty}^{\infty}\frac{\mathrm{d}a}{\sqrt{2\pi}}\mathrm{e}^{-a^2/2}\frac{a^2f^2(a^2)}{Z(z)-f(a^2)},
\end{aligned} \tag{14.76}$$

where $Z(z) = z/(q + q\mathbf{t_M}(z))$. The t-transform $\mathbf{t_M}(z)$ satisfies the following subordination relation

$$\begin{aligned} \mathbf{t_M}(z) &= \frac{1}{q}\mathbf{t_{D_0}}(Z(z)) \\ &= \frac{1}{q}\int_{-\infty}^{\infty} \frac{\mathrm{d}a}{\sqrt{2\pi}} \mathrm{e}^{-a^2/2} \frac{f(a^2)}{Z(z) - f(a^2)}. \end{aligned} \tag{14.77}$$

page 247

$$x_{\pm} = 2p + 1 \pm 2\sqrt{p(p+1)}. \tag{15.29}$$

page 281

18.1.1 General Framework

Imagine we have an unobservable variable x that we would like to infer the observation of a related variable y.