Different quantities in Maxwell's equations have different transformation properties. An understanding of these properties is important and leads to a fundamental appreciation of the difference between the characteristics of the electric and magnetic fields, which is the origin of the difference between the electric and magnetic symmetry properties of materials. It also helps in understanding many basic characteristics of the electro-optic, magneto-optic, and nonlinear optical properties of materials to be addressed in later chapters.

The electric field vectors, $\boldsymbol{E}$ and $\boldsymbol{D}$, have the same transformation properties as those of $\boldsymbol{P}$, while the transformation properties of the magnetic field vectors, $\boldsymbol{H}$ and $\boldsymbol{B}$, are the same as those of $\boldsymbol{M}$. The origin of the electric properties of a material is the charge-density distribution, $\rho(\mathbf{r}, t)$, at the atomic level in the material, whereas that of the magnetic properties stems from the current-density distribution, $\boldsymbol{J}(\mathbf{r}, t)$. The transformation properties of the scalar quantity ρ are such that *the sign of ρ remains unchanged under the transformation of either space inversion or time reversal*. In contrast, $\boldsymbol{J}$ is a *polar vector* because it is charge density times velocity, $\rho\mathbf{v}$, where velocity, $\mathbf{v}$, is a polar vector. Thus, *the vector $\boldsymbol{J}$ changes sign under the transformation of either space inversion or time reversal*. It changes sign under space inversion because a polar vector changes sign under space inversion, and it changes sign under time reversal because $\mathbf{v}$ is the first time derivative of $\mathbf{r}$. The electric polarization $\boldsymbol{P}$ is a polar vector because it is the volume average of the electric dipole moment density defined by $\rho(\mathbf{r}, t)\mathbf{r}$, and the product of a scalar quantity ρ and a polar vector $\mathbf{r}$ is a polar vector. In contrast, magnetization $\boldsymbol{M}$ is an *axial vector* because it is the volume average of the magnetic dipole moment density defined by $\mathbf{r} \times \boldsymbol{J}(\mathbf{r}, t)$, and the cross product of two polar vectors, $\mathbf{r}$ and $\boldsymbol{J}$, is an axial vector. Therefore, we find the following transformation properties.

1. **Electric fields.** The electric field vectors, $\boldsymbol{P}$, $\boldsymbol{E}$, and $\boldsymbol{D}$, change sign under space inversion but not under time reversal.
2. **Magnetic fields.** The magnetic field vectors, $\boldsymbol{M}$, $\boldsymbol{H}$, and $\boldsymbol{B}$, change sign under time reversal but not under space inversion.

With these transformation properties understood, the invariance of Maxwell's equations and the continuity equation under the transformation of space inversion or time reversal or both can be easily verified.

Response of medium

Polarization and magnetization in a medium are generated, respectively, by the response of the medium to the electric and magnetic fields. Therefore, $\boldsymbol{P}(\mathbf{r}, t)$ depends on $\boldsymbol{E}(\mathbf{r}, t)$, while $\boldsymbol{M}(\mathbf{r}, t)$ depends on $\boldsymbol{B}(\mathbf{r}, t)$. *At optical frequencies, the magnetization vanishes*, $\boldsymbol{M} = 0$. Consequently, for optical fields, the following relation is always true:

$$\boldsymbol{B}(\mathbf{r}, t) = \mu_0 \boldsymbol{H}(\mathbf{r}, t). \tag{1.14}$$