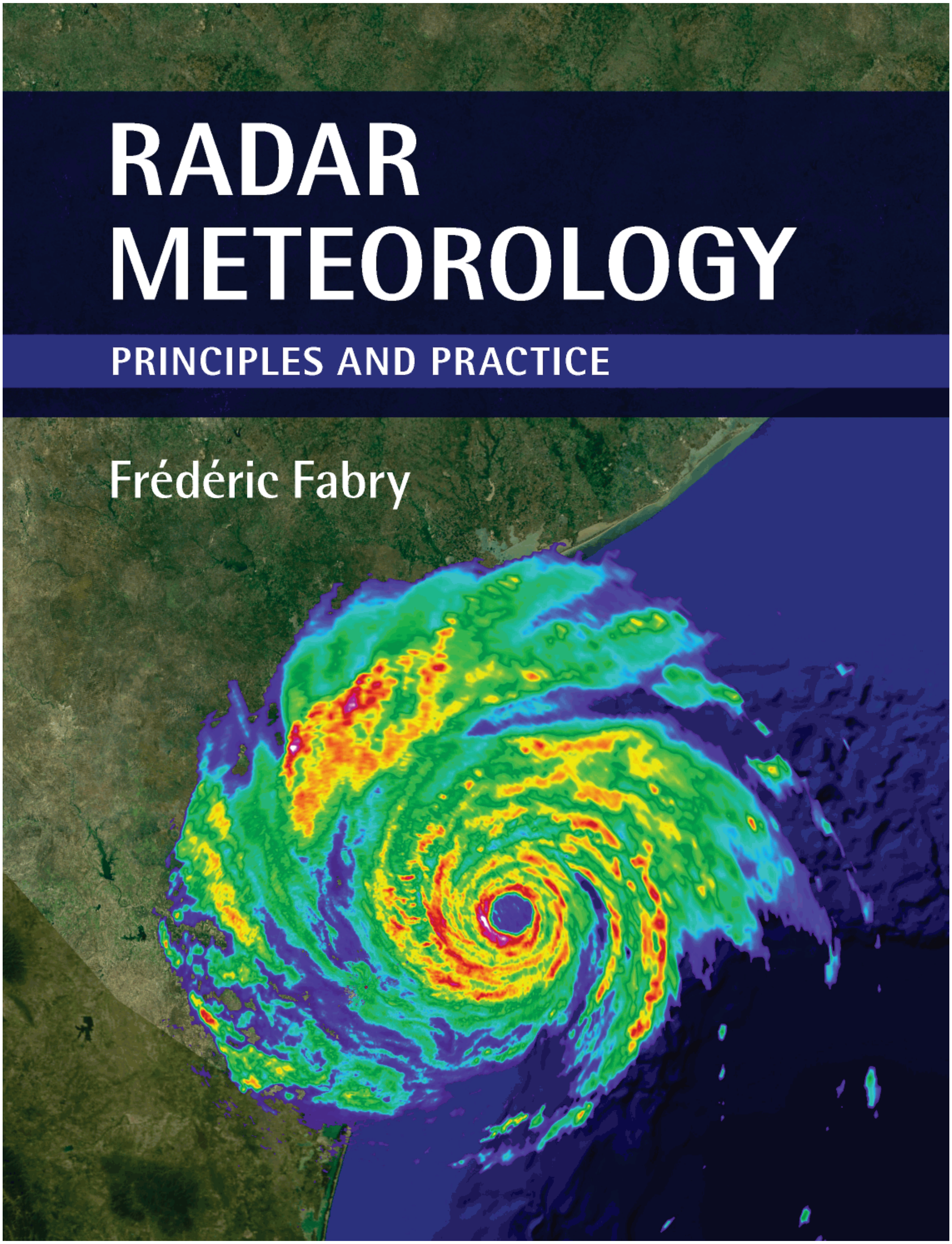


PROBLEM SETS

Version 1; July 2015

2 Fundamentals of weather radar measurements

2.1 What are the names of the four scattering regimes we have covered in this chapter, and under what conditions do they apply? In addition, provide an example of a target that would qualify for each of those scattering regimes for a Ka-band radar (λ=8.6 mm).

2.2 Consider the following potential targets: a mosquito-size insect, a large bird, a condensation nuclei, a raindrop, a cloud droplet, and a hailstone. For an S-band radar, which of these targets are Rayleigh scatterers? And for a W-band radar?

2.3 Provide three reasons why two radars of similar beam width, resolution, and sensitivity, one at 3-mm wavelength and one at 6-m wavelength, observe different echo patterns.

2.4 You are provided with funding to design your own radar for your research. You can choose between three common radar frequencies (2.8, 9.4, or 35 GHz corresponding to S-, X-, or Ka-band). Given that all other parameters like transmit power and antenna beam width are the same for all frequencies, which of the three frequencies would you choose for the following research topics (Shortly explain/justify your answer. Note: More than one frequency band might be a correct choice for some of the questions):

a) observing thin cirrus clouds?

b) observing severe thunderstorms?

c) observing widespread precipitation?

d) observing high Doppler velocities? [requires knowledge from Chapter 5]

2.5 What do we mean by super-refraction? How does it affect radar data quality?

2.6 Consider the radar from São Paulo on the lower-right of Fig. 2.6 (D_a = 3.6 m with a 10.4 cm wavelength) and the Middle and Upper Atmosphere radar on Fig. 2.5 (D_a = 103 m with a 6.45 m wavelength). Contrast their expected angular beam width.

2.7 The first elevation of a typical weather surveillance radar is 0.5º.

a) Assuming that the 4/3 Earth radius approximation holds, at what altitude will the center of the radar beam be at 240 km? 480 km? (radius of the Earth a_e is approximately 6371 km)

b) What would be the altitude of the radar beam at 240 km in the absence of beam bending (if $dn/dz = 0$)?

2.8 Consider a cloud with 10^6 droplets of 10 μm diameter per liter and rain with 1 millimeter-size drop per liter. Compute the backscattering cross-section of cloud and rain at visible (0.5 μm) and radar (5 cm) wavelengths. Compare and contrast. Use $n(\lambda) = 1.333$ at visible wavelengths and $n(\lambda) = 8.6 + 1.7\,i$ at radar wavelengths.

2.9 Use Fig. 2.13 to demonstrate graphically that even though the pulse length is $c\tau/n$ long, the radar signal coming back at the receiver at any specific instant is from a region with a depth in range of only $c\tau/(2n)$. What is then the typical range resolution of a radar with a 1 μs transmit pulse?

2.10 It can be shown that, if the effect of the curvature of the Earth is neglected, elevation angles must be spaced such that $\tan(\theta)$ increases geometrically to have the same height resolution at far range than at near range at a given altitude. Use this result to determine the elevation angles to use in a 14 angle volume scan extending up to 20°, assuming a starting angle of 0.5°. Contrast your result with the WSR-88D VCP-12 volume scan pattern shown in Fig. 2.17. Where are the main differences?

3 Radar reflectivity and products

3.1 You are tasked to explore the possibility of designing a weather surveillance radar ten times more sensitive than the ones currently used. Use the Radar Equation to explain what you would try to improve, and reflect on the consequences to explain what obstacles you would face or how such a choice would affect data quality.

3.2 Besides the PPI and CAPPI, name and define three products made by radar product generation software and used to display radar data.

3.3 Consider the vertical section of reflectivity in Fig. 3.5.

a) Explain why this product is particularly useful to help aviation.

b) What is the gray inverted triangle, and what causes it?

3.4 The VIL product is commonly used to detect cells that contain hail. Explain why, given what the VIL tries to compute and given the structure of hail storms.

3.5 In addition to rainfall, radars can detect clear air echoes. These can either be insects or layers of turbulence. Typically, insect echoes average 0 dBZ (but can be stronger in some areas or times of the year). The reflectivity of a turbulent layer depends on the structure parameter of turbulence C_n, and can be computed as $Z = 1.35 \cdot 10^{15} C_n^2 \lambda^{11/3}$, all in MKS units . Typical values of C_n^2 are of about 10^{-15} for average turbulence and 10^{-13} in strong turbulence. Compute a) the reflectivity of normal and strong turbulence for the radar wavelength used by weather surveillance radars in your country (S- or C-band), b) the wavelength at which echoes associated with normal and strong turbulence become comparable to those caused by insects, and c) the wavelength at which echoes associated with normal and strong turbulence become comparable to those caused by a stratiform rainfall of 1 mm/h.

3.6 A radar attempts to measure precipitation at 120 km range with a beam of width 1º whose elevation angle for the centre of the beam is 0.5º above the horizon. Throughout the area, the vertical profile of reflectivity is as follows: 25 dBZ from 0 to 2 km height (rain); 30 dBZ from 2 to 2.5 km height (bright band); 20 dBZ from 2.5 to 6 km height (snow); no echo above.

a) Compute the reflectivity measured by the radar. Assume that the beam has uniform sensitivity (or "gain") over the 1º beam width and is completely blind to echoes outside of that beam (or has zero gain outside of the 1º beam).

b) If rainfall rate is estimated from that reflectivity instead of the one at the surface (25 dBZ), what is the ratio between the radar-measured rainfall aloft and the observed surface rainfall?

3.7 Drop-size distributions can often be approximated by (3.8). Assuming that the fall speed of a raindrop is $w_r(D) = 4\, D^{.5}$ ($w_r(D)$ in m s^{-1}, D in mm), and using the fact that, in the interval $0 \rightarrow \infty$, $\int \exp(-ax)\, x^b\, dx = \Gamma(b+1) / a^{b+1}$ (where $\Gamma(x)$ is the Gamma function), compute:

a) The equivalent reflectivity factor (in dBZ) at 2 mm/h and 50 mm/h (do not use Table 3.2 or (3.9));

b) The size of drops that contribute the most to the reflectivity and to the rainfall rate (they are different) at these two rainfall rates.

Gamma function properties and some useful values: $(\forall n \in \mathbb{N}), \Gamma(n) = (n-1)!$; $(\forall x > 1), \Gamma(x) = (x-1)\Gamma(x-1)$; $\Gamma(0.3) \approx 3$; $\Gamma(0.5) = \sqrt{\pi}$; $\Gamma(1) = 1$.

3.8 Show that, even though a snowflake reflects 4 times less than a raindrop of the same mass, the reflectivity of snow and the reflectivity of rain are very similar for the same precipitation rate. Use the $w_r(D)$ relationship of the previous problem for rain and $w_s(D) = 0.8\ D^{0.3}$ for snow and consider two very different rain rates such as 0.2 mm/h and 10 mm/h as examples. Hint: Use (3.8) for the drop size distribution in rain and assume that each raindrop originated from one snowflake of the same mass.

3.9 a) What is the reflectivity factor of wet spherical hail of the size of a small coin (2 cm diameter) if its concentration is one per cubic meter? What rainfall intensity would give the same reflectivity factor assuming a Marshall-Palmer drop size distribution? Consider that wet hail, even if the water layer covering the hail is very thin, has the same dielectric constant as liquid water.

b) Is such hail in the Rayleigh region at X-band, C-band or S-band? How is the radar received power increased or decreased by this for each wavelength?

c) How different is the reflectivity of hail if it is dry?

3.10 In supplement e03.3 are nine average drop size distributions for different reflectivity intervals from 5 years of disdrometer data (data courtesy of GyuWon Lee). The data is in 11 columns of text, each column being separated by commas, and can be read as an Excel spreadsheet or as data by many computer languages. Use the data to:

a) Plot the nine drop size distributions;

b) Compute the reflectivity (in mm^6/m^3 and in dBZ) for each drop size distribution. Hint: When integrating, be aware of the interval in diameter dD over which integration step is performed;

c) Compute the rainfall rate (in mm/h) for each distribution;

d) Establish a power law relationship to obtain the best estimate of rainfall rate given reflectivity.

4 Reflectivity patterns

4.1 How would you recognize the presence of hail in a storm detected by radar? Use only clues from reflectivity imagery.

4.2 The bright-band is an important feature found in many radar observations of precipitation and seen in different scan products:

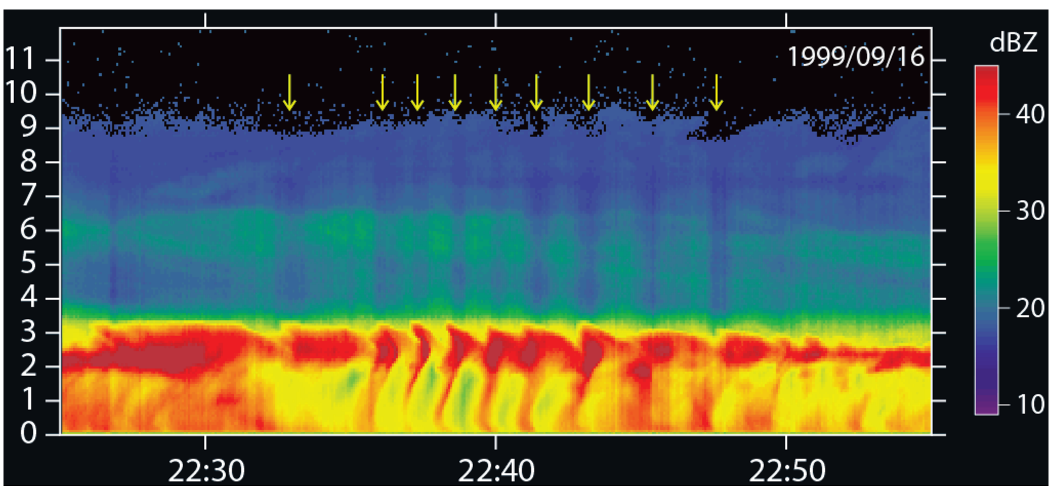
a. Explain what microphysical processes cause the feature we usually refer to as the “bright band” in radar observations.

b. Based on your explanations above, sketch a vertical profile of radar reflectivity factor and Doppler velocity as observed by a vertically pointing radar.

c. Sketch how the melting layer appears in a PPI image of radar reflectivity factor from a large widespread rainfall event.

d. What effect has the melting layer on the derived rainfall rates (from PPI) if not properly corrected?

4.3 If bright bands occur as a result of melting snowflakes, and the precipitation from the top of thunderstorms is in the form of ice, why is no clear bright band observed in thunderstorms? Note: Answering this question requires a good understanding of precipitating microphysics in strong vertical motions.

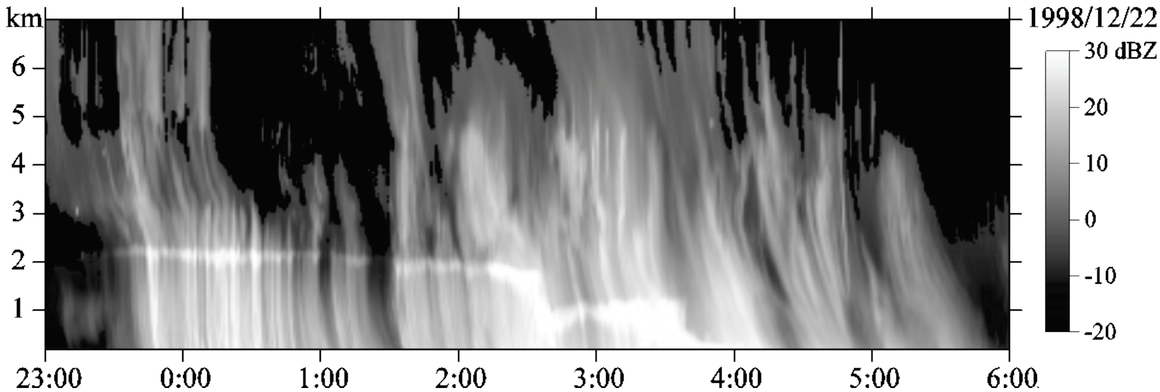


4.4 The image above was obtained by the McGill vertically pointing radar operating at X-band.

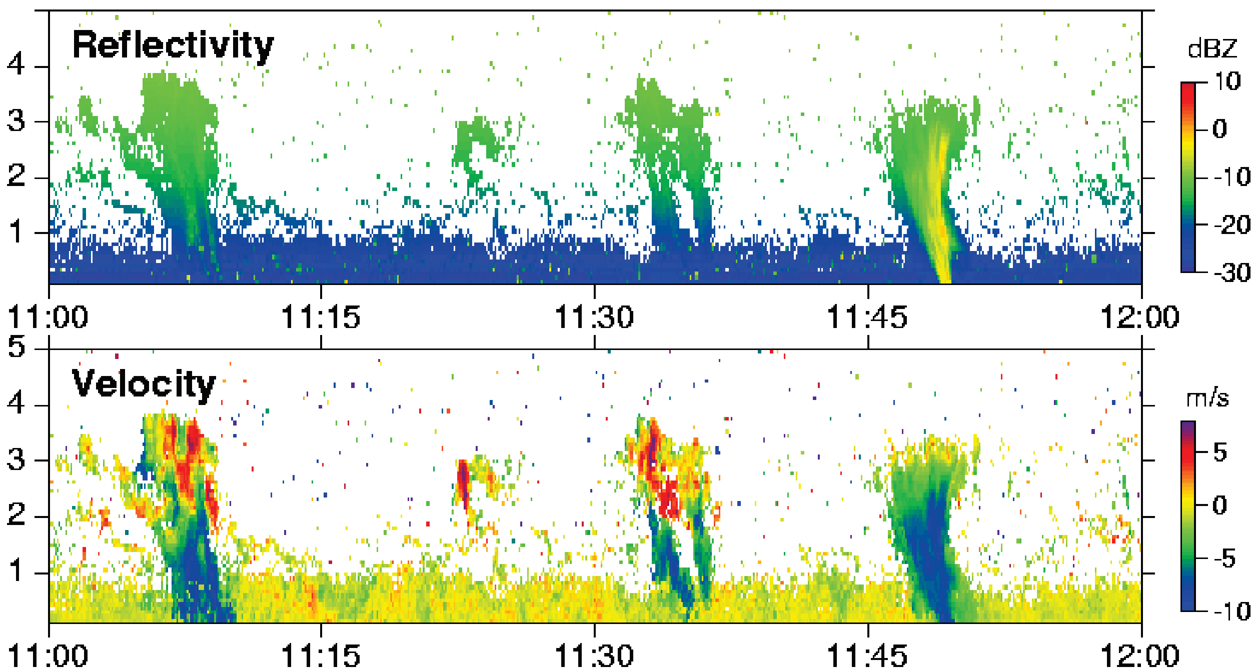
a) What type of precipitation process produces this reflectivity pattern? Mention both dynamical (convective, stratiform…) and microphysical (precipitation formation process) aspects. Justify your answer.

b) Based on the reflectivity information, estimate the rainfall rate at 22:30.

c) Arrows indicate times when the signal above 3 km appears weaker. Speculate on what can cause this. Reminder that this data was collected by a X-band radar.



4.5 Using the radar image above collected in Montreal, Canada, by the McGill vertically pointing radar, determine the type of precipitation observed at ground level for the following time periods: a) 23:00 - 23:30 (yes, echoes reach the surface); b) 23:30 - 03:20; c) 03:20 - 04:00; d) after 04:00. Note that over each period considered, the type of precipitation remains the same. Justify your answer.

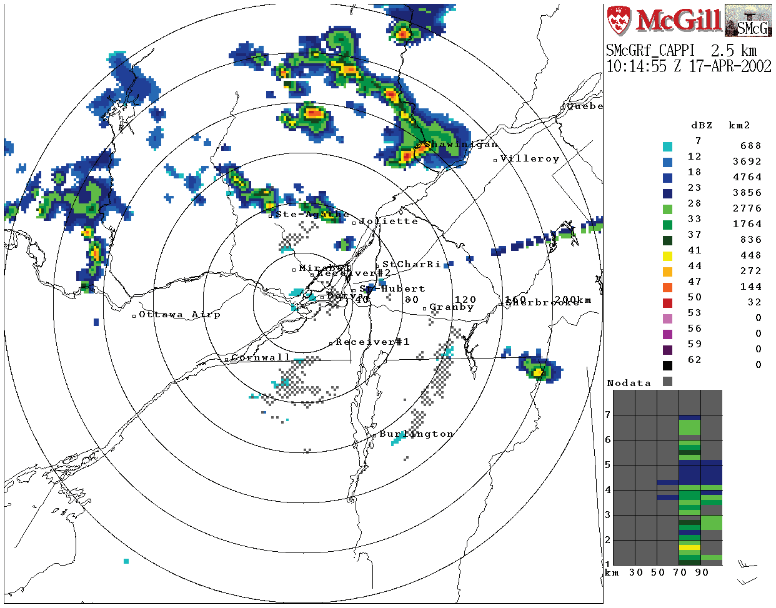


4.6 The image above shows radar reflectivity factor and vertical Doppler velocity (blue colors or negative values are towards the radar) from a vertically pointing radar.

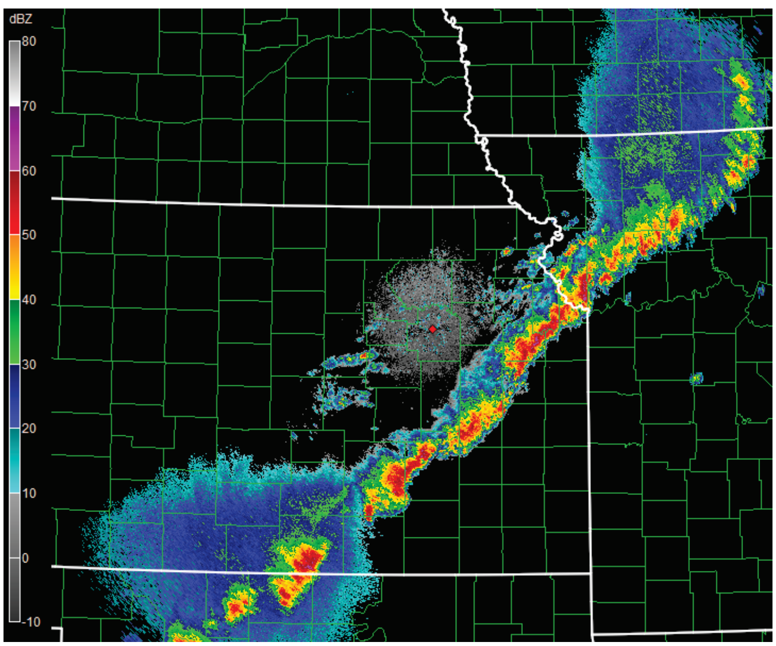
a) Interpret the images and identify all meteorological and non-meteorological targets.

b) What does the combined view of reflectivity and Doppler velocity tell you about the stages of the different convective elements?

c) What can you say about precipitation (e.g. when, intensity, duration, relation to convective system)?

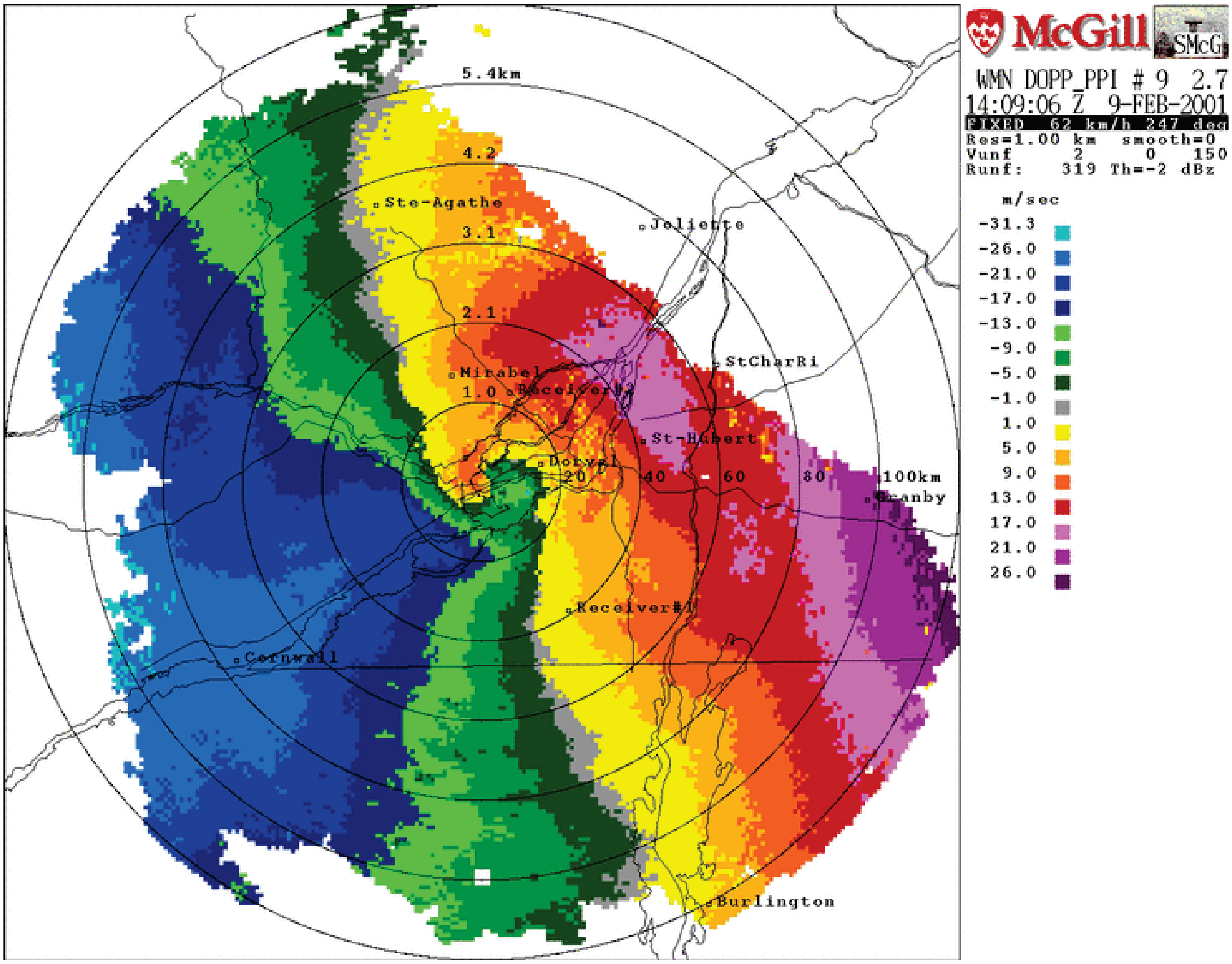


4.7 The image on the left shows weak echoes along a thin line that goes from the radar towards the east-north-east at far ranges (and at low elevation angles). This line appears when the radar points towards the rising sun. Can you explain why this occurs?



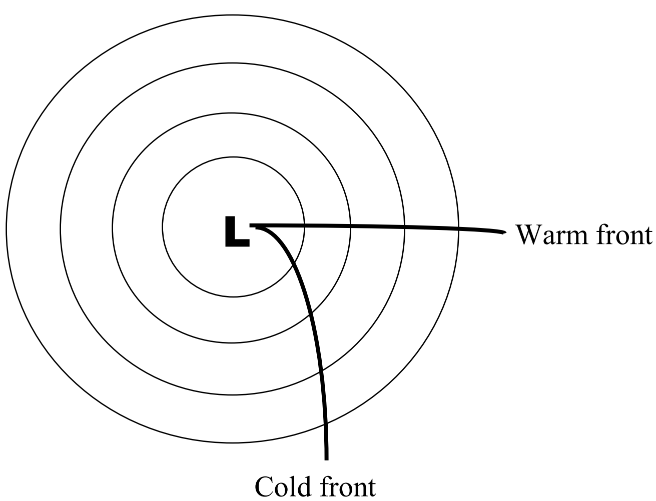
4.8 What types of echoes are observed on this particular image? Contrast the green-to-red areas from the blue-cyan areas to the NE and SW and from the light gray area around the radar on low-level PPI collected on a late afternoon in summer (all echoes show movement). Justify your answer.

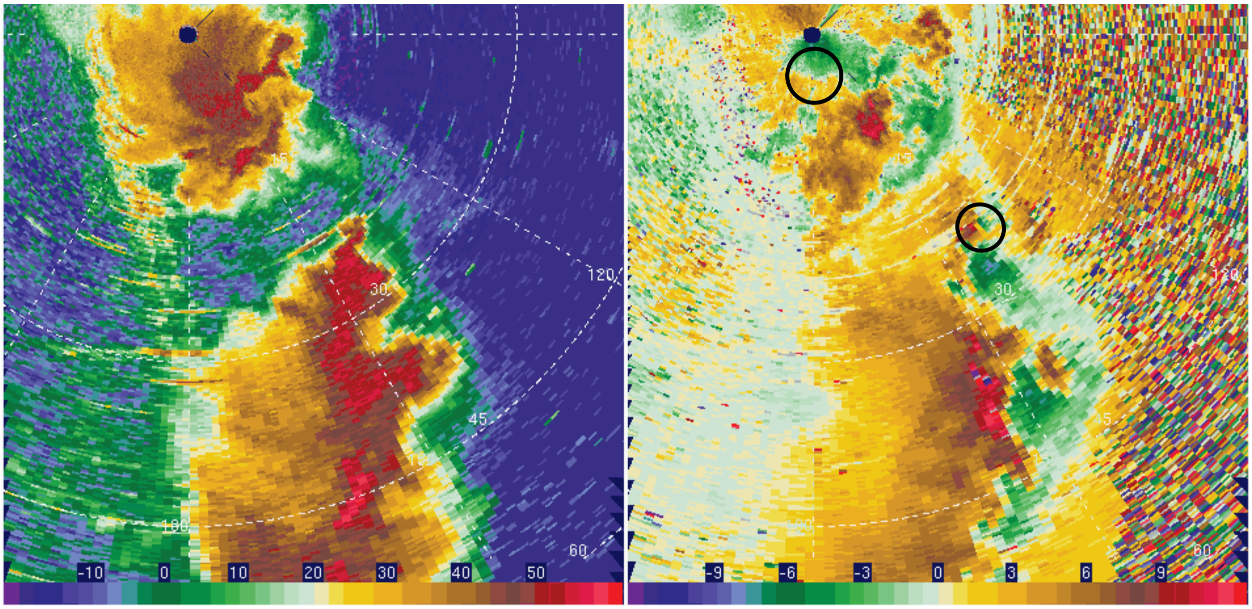
5 Doppler velocity information



5.1 a) Use the Doppler pattern shown above to retrieve the wind near the surface and at the altitude of each range ring (1, 2.1, 3.1, 4.2, and 5.4 km).

b) Consider the simplistic low pressure illustrated below. Where would you expect that this radar image was collected with respect to the center of the low pressure system? Close? Far? In which direction? Explain/justify your answer based on the wind pattern retrieved and your knowledge of the structure of a low pressure system.

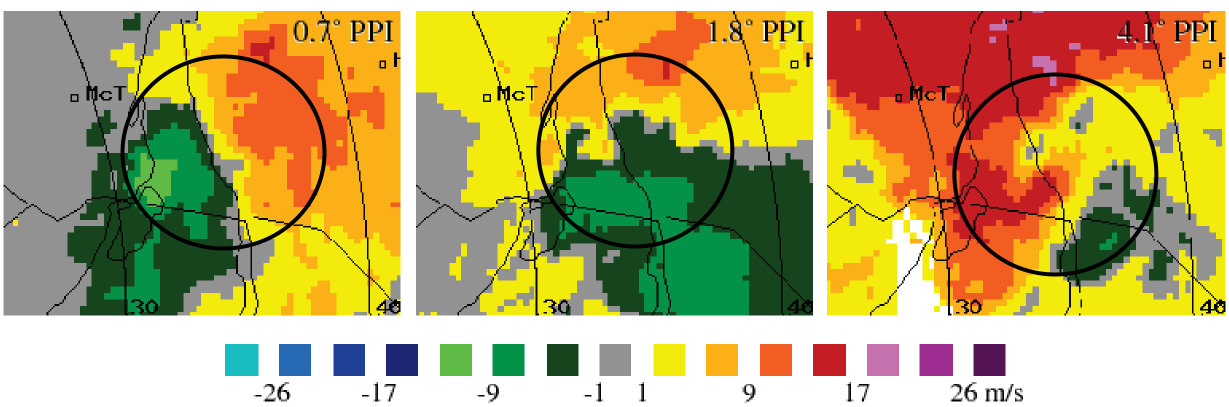




5.2. The Figure above shows a 30 km by 30 km portion of a low PPI, the radar being in the dark blue disk near the upper-left corner. The image on the left is for reflectivity, while the image on the right is Doppler velocity.

a) What kind of weather is being observed on these images? Justify your answer.

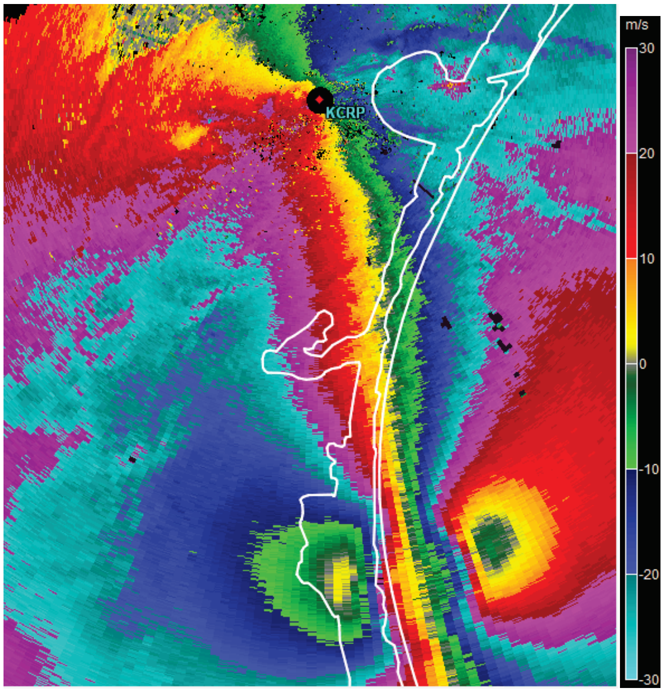
b) What are the signatures in each of the circled areas on the right image? What causes them meteorologically speaking?



5.3 The figure above shows how the local Doppler pattern associated with a convective storm evolved with height. What are the Doppler signatures at each of the three heights typical of? Use that to describe the air circulation as a function of height for that region.

5.4 a) Define and explain velocity aliasing.

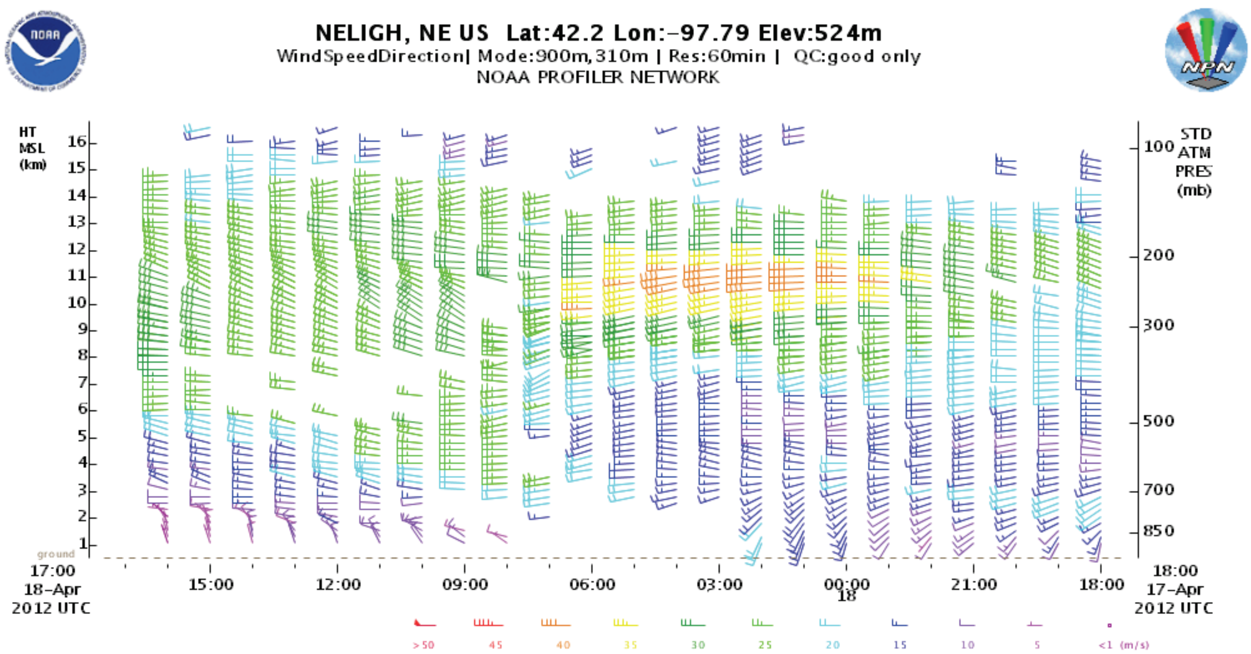
b) Consider an S-band radar (λ = 10.5 cm) and a C-band radar (λ = 5.5 cm) with a maximum unambiguous range of 125 km. Beyond what velocity is aliasing expected for both radars?



5.5 On the left is a Doppler PPI image from the KCRP radar with a Nyquist velocity of 29 m/s.

a) What is the direction and the wind speed at the radar site near the surface?

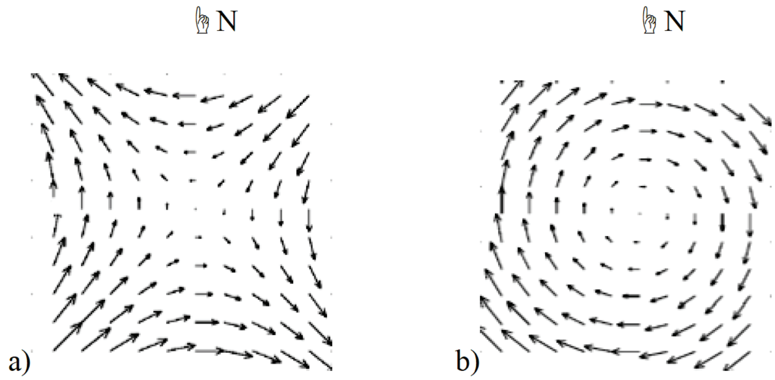
b) Where are the strongest winds on this image, and in what direction and at what speed do they blow?



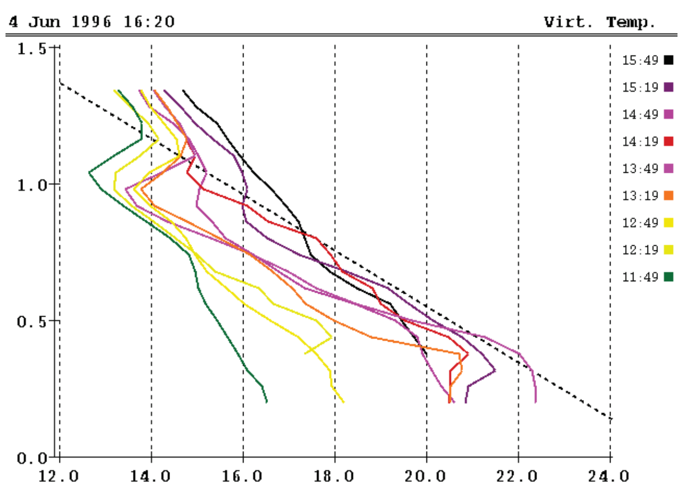
5.6 Above is the time series of winds measured by a wind profiler.

a) How does a wind profiler obtain its wind estimates? And what are its targets?

b) If you had a Doppler velocity image from a scanning radar made on a 2° PPI (up to 120 km range, corresponding to a maximum altitude of 4 km) at 15UTC, and targets were present everywhere, how would the Doppler image look like? Sketch a round display, the zero velocity line, and where you would see cold colors and warm colors.



5.7 The vectors in the diagrams a) and b) below represent hypothetical air-flow circulations within a larger scale pattern. What kind of circulations are these? Make sketches of the radial velocity as seen by a radar located North, North-East, East and South of each of the circulations in the two diagrams. How would the resulting Doppler signatures be interpreted in each case? Do you find ambiguities? How would you resolve these ambiguities?



5.8 The sequence of temperature profiles in the figure was obtained by the RASS system on the roof of the McGill meteorology building in Montreal, Canada (times are in UTC, 12:00 UTC = 07:00 local). The dashed line is the dry adiabatic lapse rate. Comment these profiles and give plausible reason for their time evolution.

5.9 a) A C-band radar (λ = 5.5 cm) is transmitting with a PRF such that the maximum unambiguous range is 130km. Find what pulse repetition frequency is being used.

b) There is precipitation at 20 km and at 150 km from the radar. The intensity of precipitation is the same at the two ranges. Is the measured intensity at 20 km affected? If yes, what is the error in the measurement of reflectivity in %? Based on this exercise, comment on the circumstances when range folding is most problematic.

c) Hydrometeors are moving at 0 m/s at near range (20 km) and at 40 m/s toward the radar at 150 km. What Doppler velocity does the radar measure at 20 km range? State your assumptions and explain your reasoning.

5.10 A Doppler spectrum measured in light rain by a vertically pointing radar can be approximated by a Gamma function such that $S(w_r) = 5w_r^6 \exp(-10w_r)$, where $S(w_r)$ expressed the contribution to reflectivity of targets with fall speed w_r and is expressed in (mm^6/m^3)/(m/s). For drizzle or very light rain, the fall velocity as a function of drop diameter can be approximated by w_r [m/sec] = 4 D [mm]. Derive the expression for the drop size distribution that produced the observed Doppler spectrum assuming negligible vertical air motion at the height of the measurement. Can you estimate the rain rate?

5.11 The following equation forms the basis for the one-harmonic VAD processing if winds only vary with the height z:

$$v_{DOP}\left[z(r),\phi\right]=\left[w(z)-w_f\right]\sin\theta'+\left[u(z)\cos\theta'\right]\sin\phi+\left[v(z)\cos\theta'\right]\cos\phi\,. \qquad (5.6)$$

By fitting a function $v_{\mathrm{DOP}}(r)= a_0+ a_1\sin\phi + b_1\cos\phi$ through Doppler data, and knowing the elevation θ and the azimuth ϕ, we can get from a_1 and b_1 both $u(z)$ and $v(z)$, the east-west and the north-south components of the wind as a function of height, assuming that winds do not vary horizontally. From the a_0 constant, we can estimate the average fall speed of hydrometeors w_{f} with respect to still air, combined with the air vertical velocity $w(z)$.

Let us now assume that, instead of a constant wind, we have a linear wind such that $u=u_{\mathrm{o}}+(\mathrm{d}u/\mathrm{d}x)x+(\mathrm{d}u/\mathrm{d}y)y$, $v=v_{\mathrm{o}}+(\mathrm{d}v/\mathrm{d}x)x+(\mathrm{d}v/\mathrm{d}y)y$, with x and y being respectively the east-west and north-south distance from the radar.

a) Show that under this scenario, and if the elevation θ is 0°, one can use the two-harmonic VAD to fit this linear wind, i.e., $v_{\mathrm{DOP}}(r)= a_0+ a_1\sin\phi + b_1\cos\phi + a_2\sin(2\phi) + b_2\cos(2\phi)$ where $a_0=[(\mathrm{d}u/\mathrm{d}x)+(\mathrm{d}v/\mathrm{d}y)]r/2$, $a_1=u_{\mathrm{o}}$, $b_1=v_{\mathrm{o}}$, $a_2=[(\mathrm{d}u/\mathrm{d}y)-(\mathrm{d}v/\mathrm{d}x)]r/2$, $b_2=[(\mathrm{d}v/\mathrm{d}y)-(\mathrm{d}u/\mathrm{d}x)]r/2$, and r is range. Hint: this derivation requires doing a few trigonometric transformations using formulas possibly forgotten since high school or the first years of college.

b) What do the five retrieved parameters a_0, a_1, b_1, a_2, and b_2 correspond to physically or meteorologically speaking?

6 The added value of dual polarization

6.1 Many meteorological services have been or are planning to spend considerable resources to upgrade radars to have dual-polarization capability. Why? What added value do such types of radars bring?

6.2 How does fuzzy logic function? Include in your answer a quick description of the basic elements of a fuzzy logic system. Why is this approach so useful for automatic target identification?

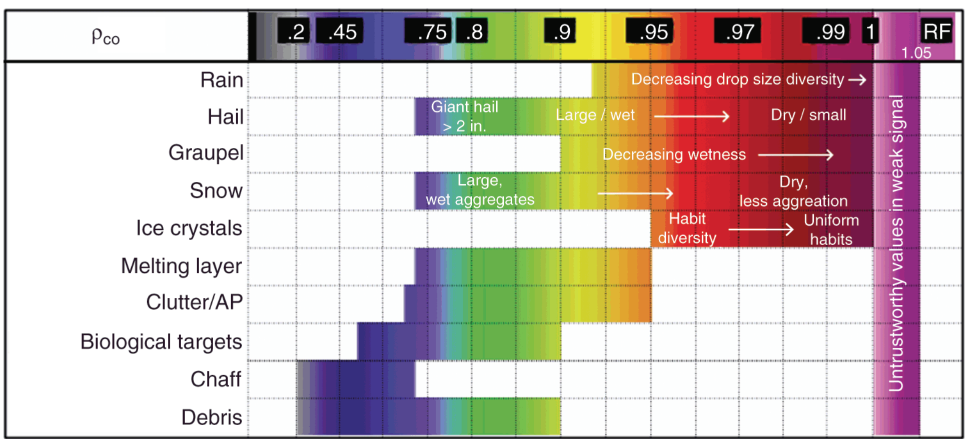
6.3 The ratio of the shortest to the longest axis of a raindrop (see Fig. 6.2) can be approximated by

$$r = 0.9951 + 0.02510D - 0.03644D^2 + 0.005030D^3 - 0.0002492D^4,$$

where D is the equivolume diameter of the drop in mm. In that context, and using Fig. 6.3, what value of Z_{dr} would one expect to find for a light widespread rainfall rate of 1 mm/h and a downpour of 100 mm/h? Hint: Recall exercise 3.7 on how to get the size of drops that contribute the most to the reflectivity.

6.4 A radar makes a RHI through convective cold rain. How do you expect Z_{dr}, and ρ_{co} to vary with height? Explain your answer.

6.5 A radar makes a RHI through heavy rain at very close range (1 km), temperatures being above 15°C everywhere within that range. How do you expect Z_{dr}, and ρ_{co} to vary with elevation angle? Justify your answer.



6.6 Above is Fig. 6.9, charting typical values of ρ_{co} for different target types. Explain why:

a) Meteorological targets in general should have higher ρ_{co} than non-meteorological targets;

b) Rain with wider size distributions should have lower ρ_{co} than rain with more uniform drop sizes;

c) Wet hail and wet graupel should have lower ρ_{co} than their dry counterparts.

6.7 The Air Force is experimenting with a release of a new type of chaff shaped like rings of different size but identical axis ratio (diameter 10 times larger than the thickness of the ring), and all small enough to be Rayleigh scatterers. They are designed to fall with their longest axis falling horizontally. How do you expect Z_{dr}, LDR, and ρ_{co} for such targets to compare with that of familiar targets like rain, snow, insects, or traditional chaff made of fibers at low elevation angles? Explain your reasoning.

6.8 What would happen to dual-polarization measurements if one accidently plugged the signal coming from horizontal channel into the digitizer input of the vertical channel and the signal coming from the vertical channel into the digitizer input of the horizontal channel? Consider Z, v_{DOP}, Z_{dr}, ρ_{co}, and ψ_{dp}.

7 Convective storm surveillance

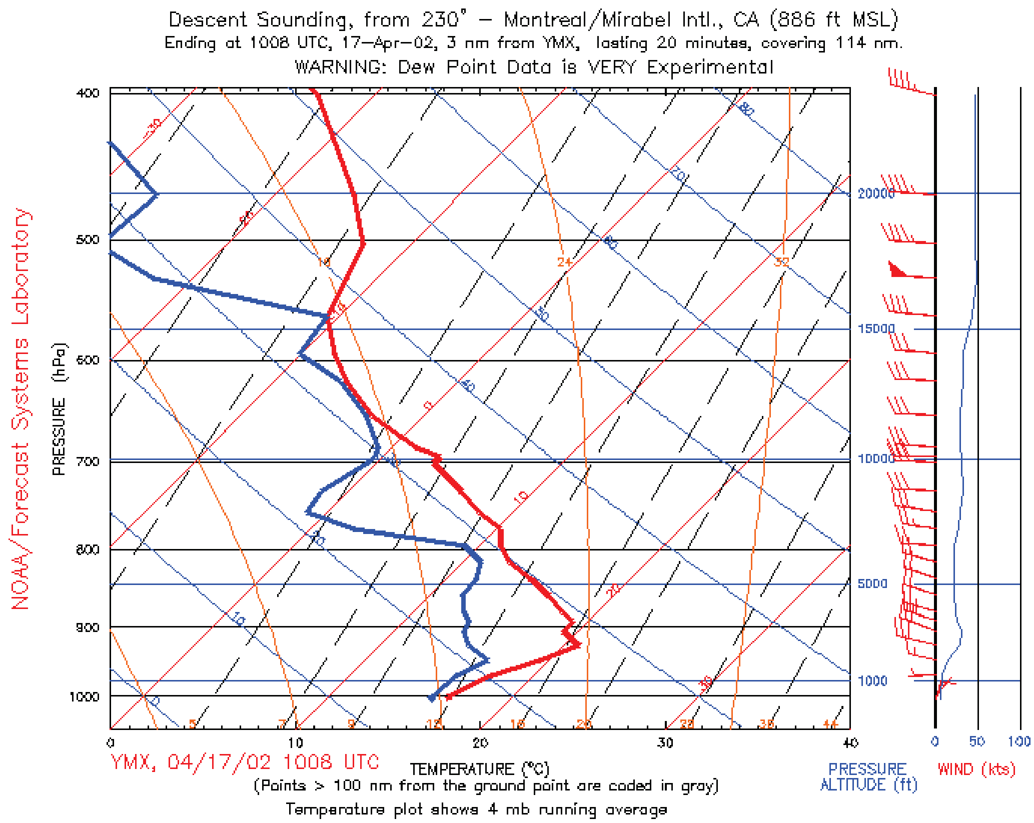
7.1 If a potentially severe storm is within the radar coverage, what signatures do you look for on radar pictures to determine its severity?

7.2 A long time ago, the Montreal weather office issued the following severe weather watch bulletin:

```
SEVERE WEATHER BULLETIN
ISSUED BY ENVIRONMENT CANADA
AT 08:45 AM EDT WEDNESDAY 17 APRIL 2002.

SEVERE THUNDERSTORM WATCH FOR MONTREAL, LAURENTIANS, EASTERN
TOWNSHIPS…

WEATHER CONDITIONS FOR THESE REGIONS ARE FAVOURABLE TO THE
DEVELOPMENT OF SEVERE THUNDERSTORMS. SOME OF THEM COULD PRODUCE
LARGE HAILSTONES - HIGH WINDS - HEAVY RAIN AND INTENSE
LIGHTNING.

THIS SEVERE WATCH IS IN EFFECT FROM 08:45 AM TO 05:30 PM EDT.
```

a) Use the information from the sounding below to justify why the weather office decided to broadcast this severe weather watch;

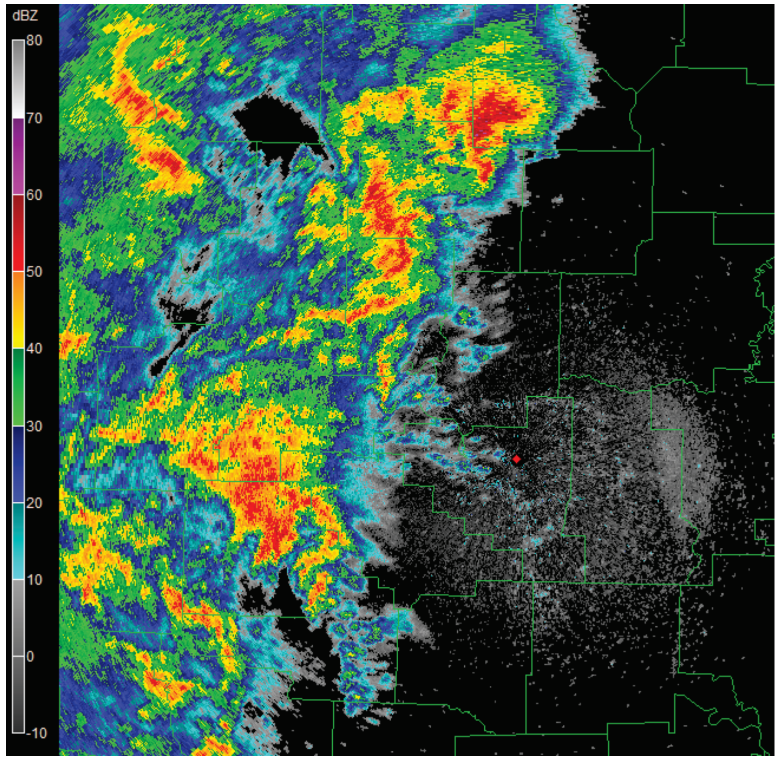
b) Given the wind profile, what type of severe weather thunderstorm and/or phenomena do you <u>not</u> expect? Justify your answer.



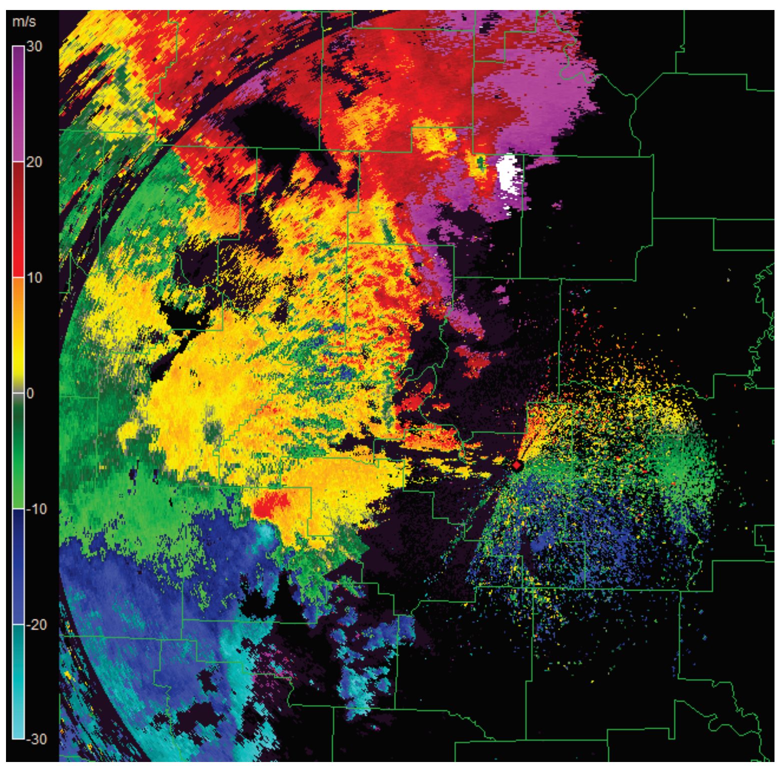
Temperature (red) and dew point (blue) sounding obtained by in-situ sensors on a commercial airplane landing at Montreal-Mirabel airport near 06:00EDT. Altitude conversion tip: 10000 ft = 3.048 km.

7.3 A large storm system is sweeping the eastern half of the North American continent. The two images on the right show the 0.5° PPI of reflectivity (top) and Doppler velocity (bottom) from the Little Rock (Arkansas) radar at 22:00 UTC (17:00 Local Daily Time), the image measuring roughly 300*300 km.

What severe weather watches or warnings would you broadcast for each of the three intense storms closest to the radar (one to the west, one to the north-west, and one to the north)?

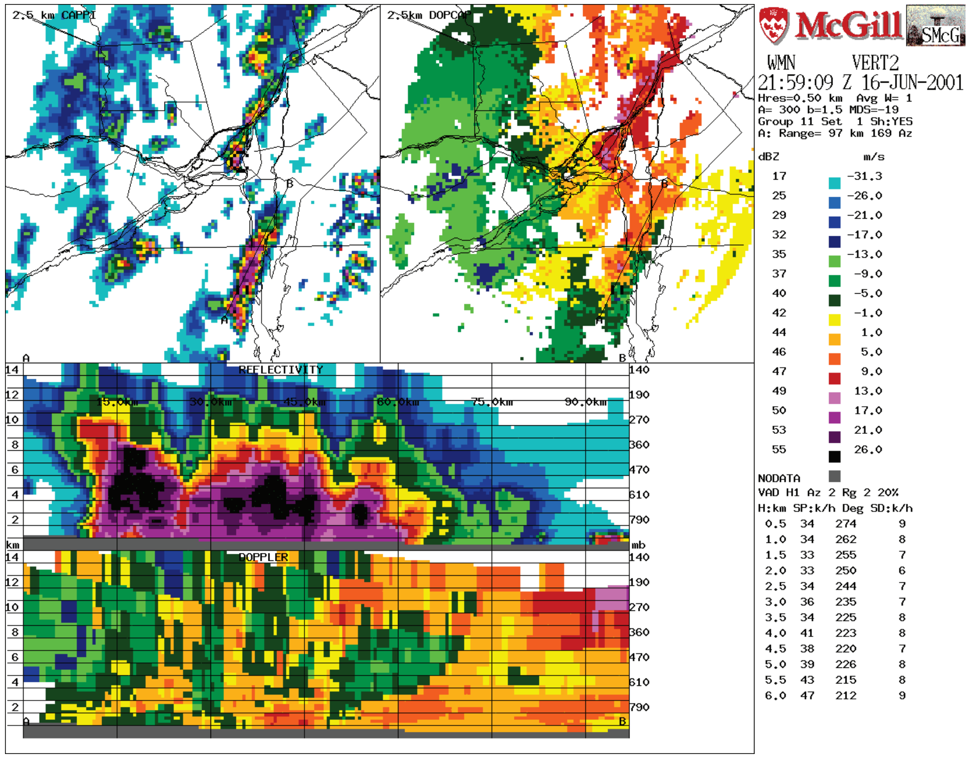


Note: On the Doppler image, the deep purple color, such as immediately west of the radar illustrated by a red lozenge indicates areas where no information was obtained on velocity (bad data). White colors represent velocities greater than +30 m/s.



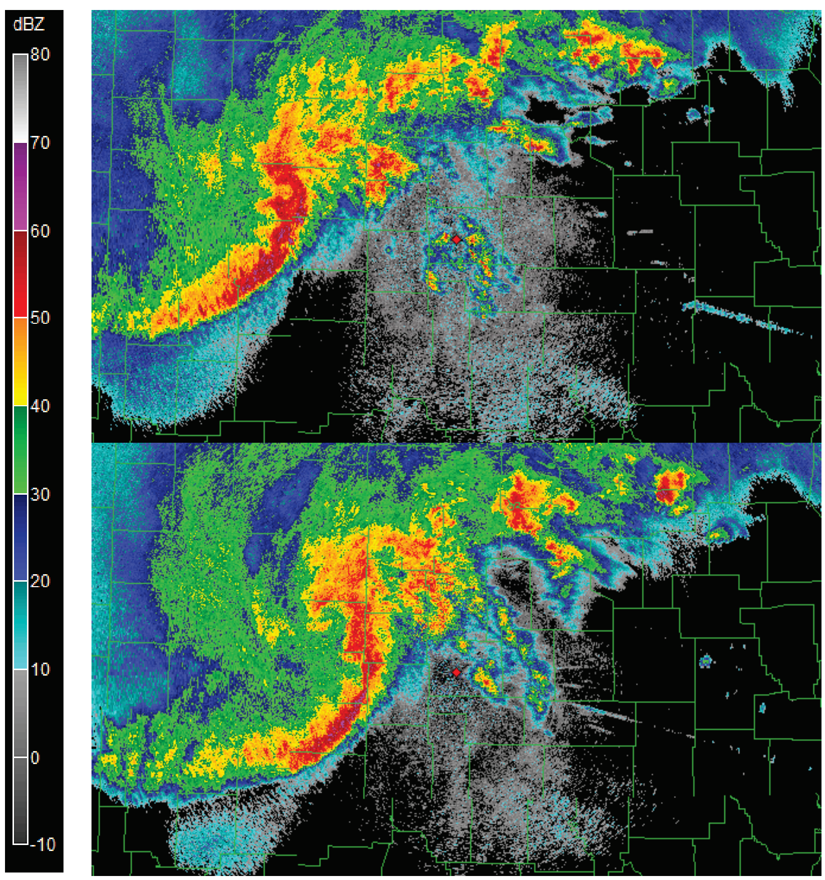
7.4 A vertical cross-section from SSW-to-NNE was done on the lines of cells SSE of the radar.

Based on the information available on this image, and using the Lemon technique, would you characterize any of these cells as severe? Justify your answer.



7.5 Consider these two low-level PPIs thirty minutes apart over an area measuring roughly 400*250 km.

Based on the information available on these two images, what warnings would you broadcast and why?



7.6 Study the climatology of occurrence of convection as a function of time of day (slide 11 of supplement e07.2). Find two unexpected signatures beyond the ones listed in the textbook and in that supplement, and describe your findings. If you can explain them using meteorological reasoning or as artifacts of radar measurements, do so; if you cannot, explain why. [This question is very open-ended by design]

8 Monitoring widespread systems

8.1 On the right are a set of images from the McGill radar:

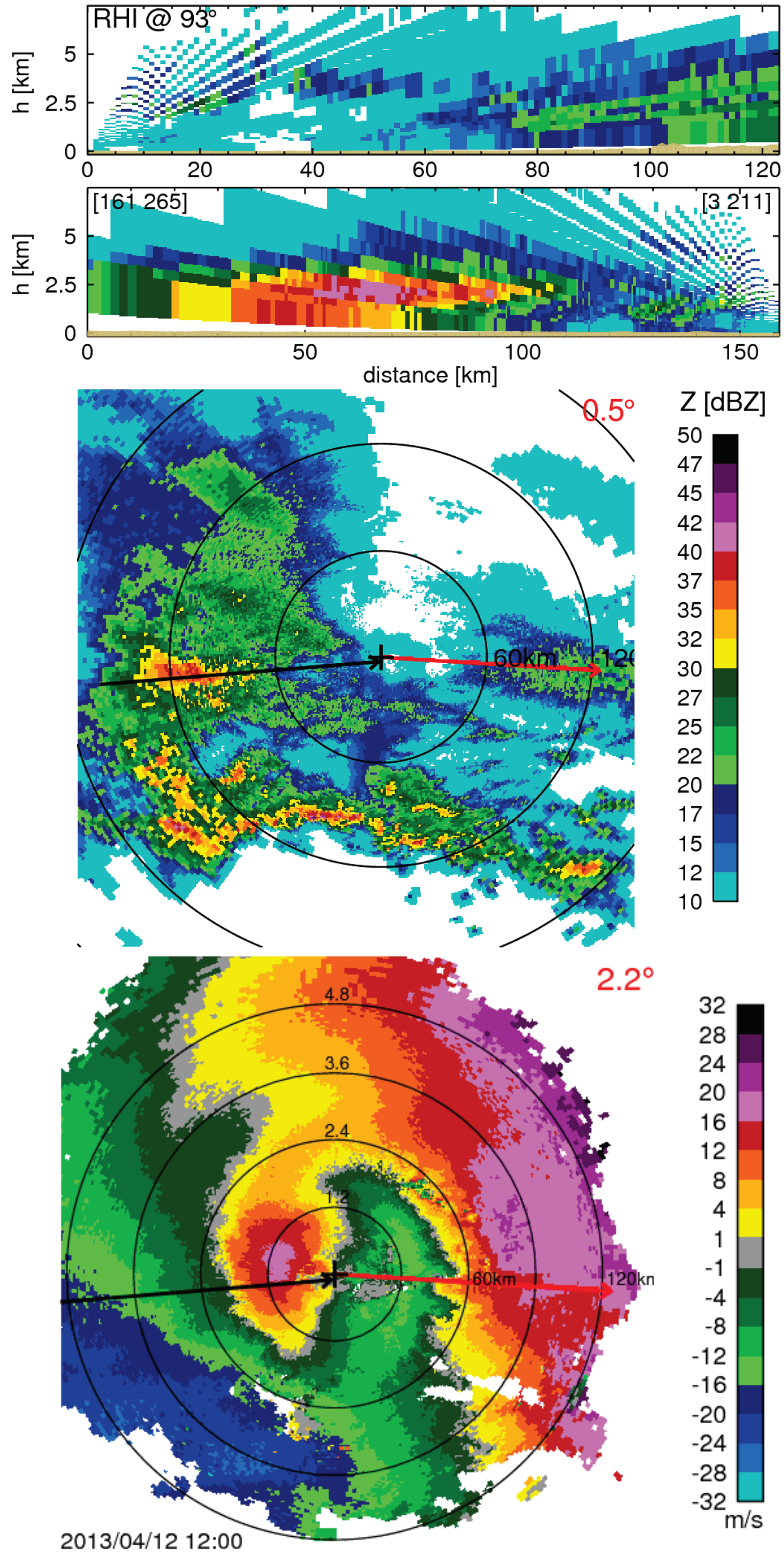
- Two vertical cross-sections of reflectivity, the first one being east of the radar and the second one being west of the radar;

- A 0.5° PPI of reflectivity factor (dBZ); and,

- A 2.2° PPI of Doppler velocity.

What follows is the model-predicted sounding for the radar site with, from left to right: height above ground (km), temperature (°C), wind direction (°), and wind speed (m/s).

Height	Temp.	Dir	Speed
.16	-2.8	74	9
.36	-4.3	80	11
.56	-5.3	93	13
.99	-5.9	103	16
1.43	-7.0	100	15
1.91	-3.8	127	9
2.42	-1.0	202	10
2.97	-1.8	224	16
3.56	-3.0	234	19
4.19	-7.0	238	21
4.86	-10.9	248	22



a) Based on the radar data, what type of precipitation process (microphysics and dynamics) is responsible for the heavier precipitation observed west of the radar? Justify your answer.

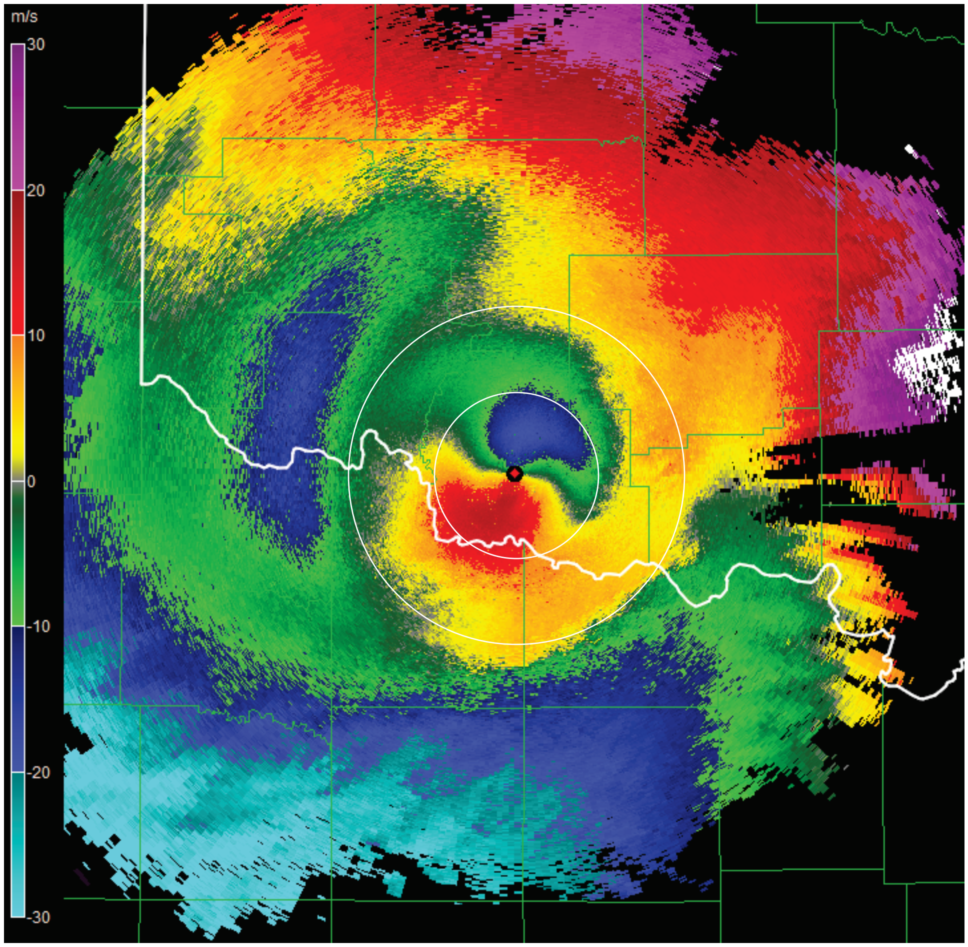
b) Validate or find discrepancies between the radar information and the temperature, wind speed, and wind direction profile from the model. For winds, only consider 4 altitudes: 1.2, 2.4, 3.6 and 4.8 km.

8.2 One day in February in southern Oklahoma: A winter storm is expected, the official forecast calling for a possible mix of snow, ice pellets, and freezing rain at ground. We would like to use the data from the radar to help determine which it is.

On the right is a Doppler PPI at 2.5° elevation. Range rings correspond to altitudes of 1 and 2 km.

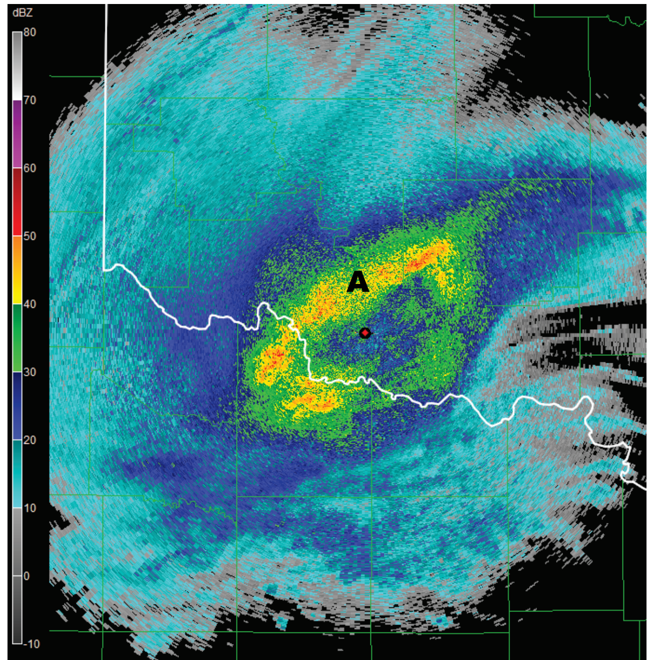
a) Derive the mean winds at the surface as well as at those two altitudes.

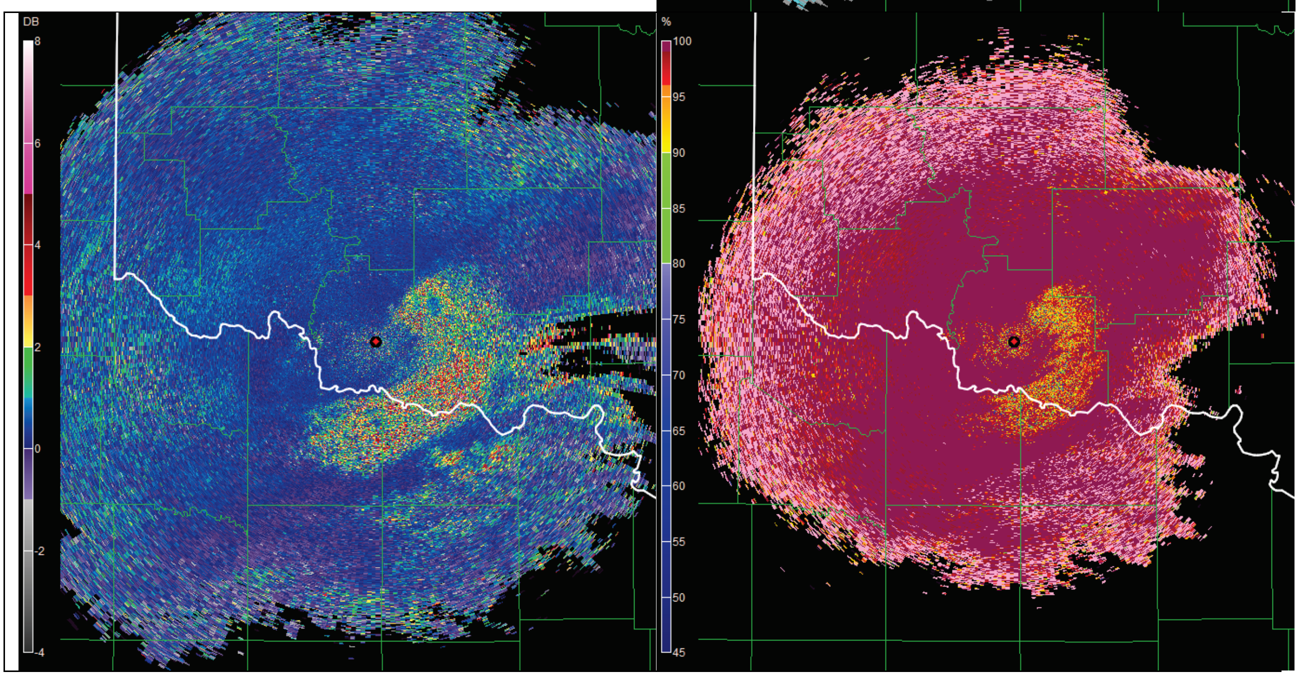
b) What does this profile suggests in the terms of cold or warm air advection close to the surface?



c) At the same time, 2.5° PPIs of Z, Z_{dr}, and ρ_{co} were also obtained and are plotted here. The small red lozenge shows the location of the radar. Based on the available information, what type of precipitation is observed by the radar at **A** at the altitude where measurements are taken (about 1 km above the surface)? Justify your assessment.

d) And what type of precipitation do you expect to see at the surface under **A**? Briefly explain your reasoning.





9 Radar estimation of precipitation

9.1 You would like to derive rainfall rates using low elevation PPI scans from your weather surveillance radar.

Where would you expect the uncertainties of the radar derived rainfall rates to be larger: At closer range or further away from the radar? Explain your answer.

9.2 We have learned from experience that the practical minimum resolution needed to reasonably describe the effects of a meteorological phenomenon of size x and duration t is to sample it with a spatial resolution Δx of the order of $x/5$ and a temporal resolution Δt of the order of $t/5$. In that context, consider the response time and basin scale plot of Fig. 9.3 to answer the following questions.

a) What are the minimum spatial and temporal resolution required to describe or predict the flooding that may occur for 1) the watershed of a large river that drains an area comparable to that covered by a radar (100,000 km^2), 2) the watershed of a smaller monitored tributary of that river (2,000 km^2), 3) the watershed of a typical mountain creek (100 km^2), and 4) the drainage system of a city neighborhood (2 km^2)?

b) What types of weather phenomena are most effectively able to flood each of these four watersheds?

c) How well can radar monitor flood potential for these four watersheds? Consider both issues of resolution and of ability to observe events close to the surface.

d) Consider raingauges as an alternative technology for flood monitoring. Typical raingauges can take good point measurements at 5 min resolution. Comment on their ability to perform well on these four watersheds, and if yes, calculate their required spacing. Finally, how many raingauges would be needed to monitor all such watersheds that can be found in the area monitored by a radar?

9.3 a) In the warmer Gulf of Mexico states of the U.S., it is common for radars to use a "tropical" Z-R relationship of $Z = 250R^{1.2}$ to better estimate rainfall. For showery conditions (reflectivity of 45 dBZ), how much does this change estimated rainfall compared to the more usually used $Z = 300R^{1.4}$? Why is this expected to improve rainfall estimates in these areas?

b) In the mountainous states of the U.S., especially in areas dominated by upslope flow, it is common for radars to use what is referred to as a "cool stratiform" Z-R relationship of $Z = 75R^2$ to better estimate winter rainfall. For typical conditions (reflectivity of 25 dBZ), how much does this change estimated rainfall compared to the more usually used $Z = 300R^{1.4}$? Why is this expected to improve rainfall estimates in these areas?

9.4 What is the effect of aggregation on reflectivity and precipitation?

a) To answer this question, first consider an unusual storm made of only two snowflakes aloft. How will the reflectivity of these snowflakes change with height between two scenarios: i) If the two snowflakes do not aggregate; and, ii) if the two snowflakes aggregate. If it helps the thinking process, assign numbers to the snowflake properties, e.g., 1 mm snowflakes at 3 km altitude falling at 1 m/s.

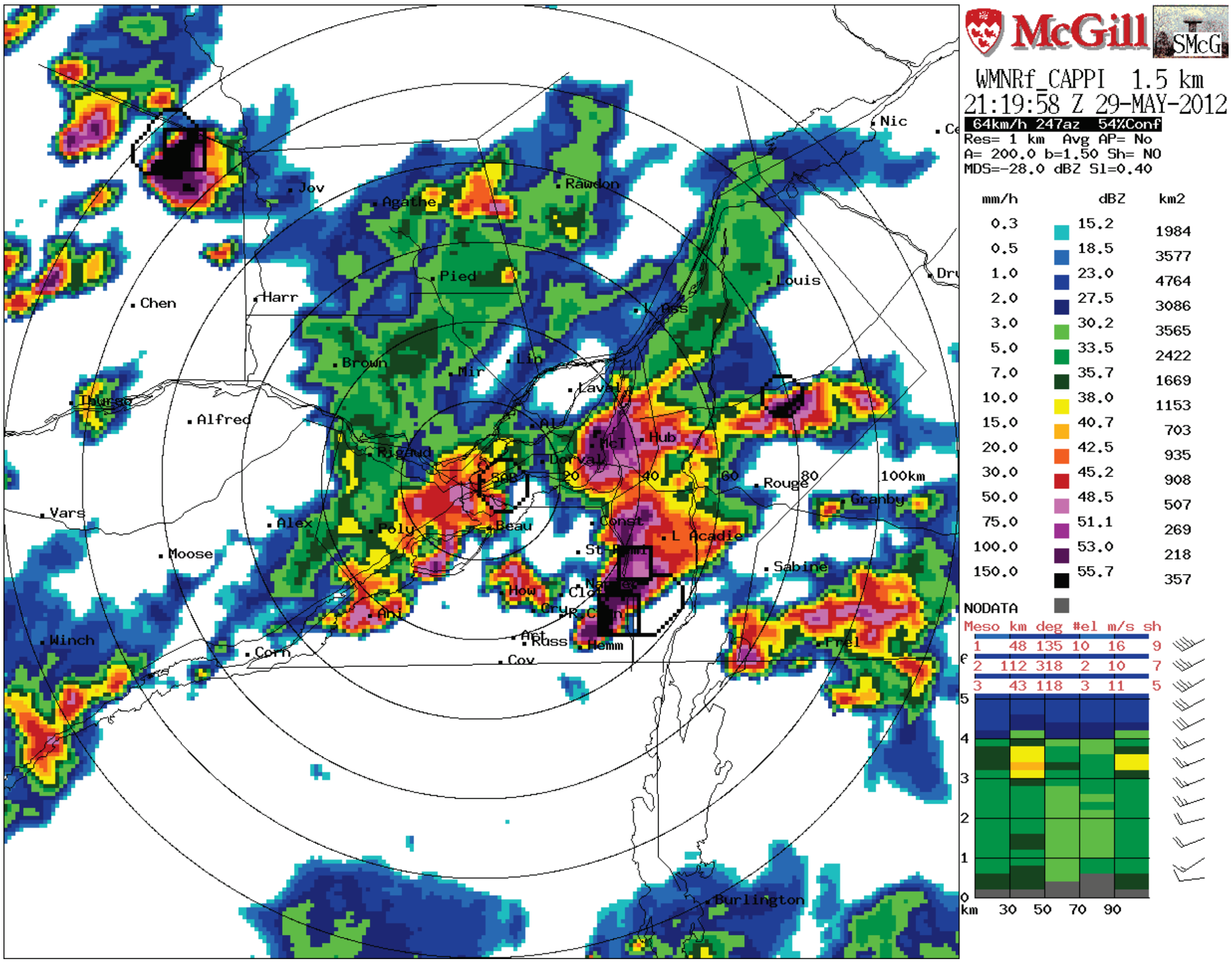
b) How is the precipitation rate at the ground going to change, and how is the total precipitation accumulation also going to change?

c) Generalize the answers to a) and b) to explain how the reflectivity and precipitation rate of a storm will differ if significant aggregation occurs compared to a similar storm where aggregation is mostly absent.

9.5 The image below is part of a sequence that illustrates the passage of a storm that caused some flooding downtown Montreal. At the McGill University campus, one could see the following reflectivity sequence: 20:50: 34 dBZ; 21:00: 43 dBZ; 21:10: 50 dBZ; 21:20: 54 dBZ; 21:30: 39 dBZ.

a) Estimate the radar-derived accumulation at McGill for this event.

b) Given that these data were collected by the McGill S-band radar at 1.5 km altitude, and given the type of event considered, do you believe that the result obtained significantly underestimates, significantly overestimates, or estimates closely the precipitation observed at ground. Justify your answer.



10 Nowcasting

10.1 What tool(s) are being used for severe weather nowcasting in your national weather service or civil protection agency? What are its (or their) bases?

10.2. Using the electronic supplement e07.1, determine where in the United States nowcasting systems based on extrapolation may perform reasonably, and where expert systems taking initiation into account are necessary.

10.3 [Longer problem; this exercise is best done after reading Appendix A.4.1 and A.4.2]
Read the data file `eA_3_HighResolutionXYData.nc` containing a time sequence of reflectivity.

a) Determine the average speed and direction of the weather patterns between 16:18 and 16:23 by computing the correlation between the two maps for different displacement Δx and Δy of the second map and finding which (Δx, Δy) maximizes correlation. Show your code and results.

b) Use that velocity to predict what the reflectivity patterns should look like at 16:24, 16:28 and 16:38. Compare with what actually happened and quantitatively evaluate the mismatch.

c) How do the errors above compare to the errors that would have been made if we had assumed persistence, i.e., if we had supposed that the rain at 16:24, 16:28 and 16:38 would have been identical to the one at 16:23? Tabulate your results for b) and c).

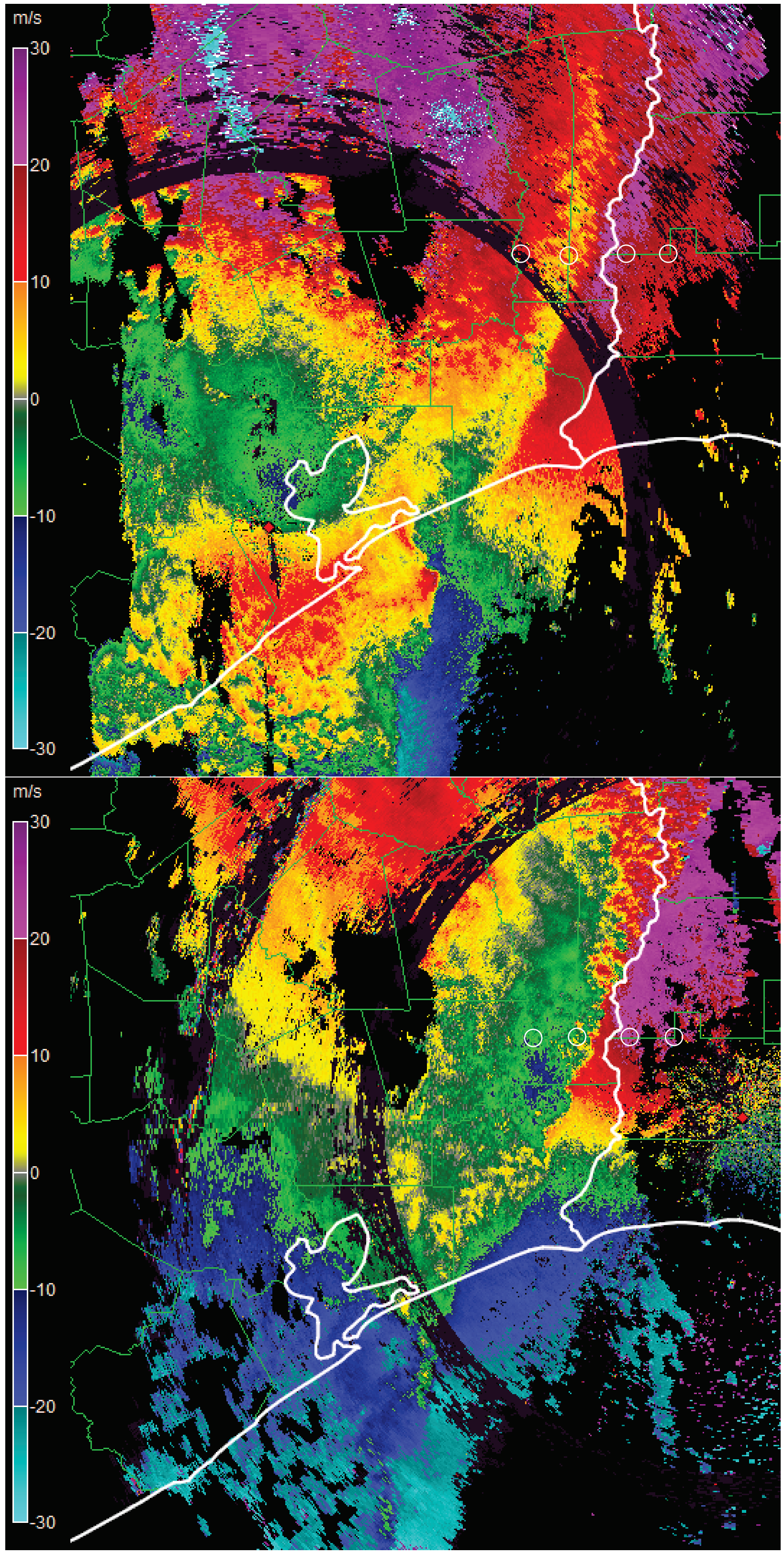
11 Additional radar measurements and retrievals

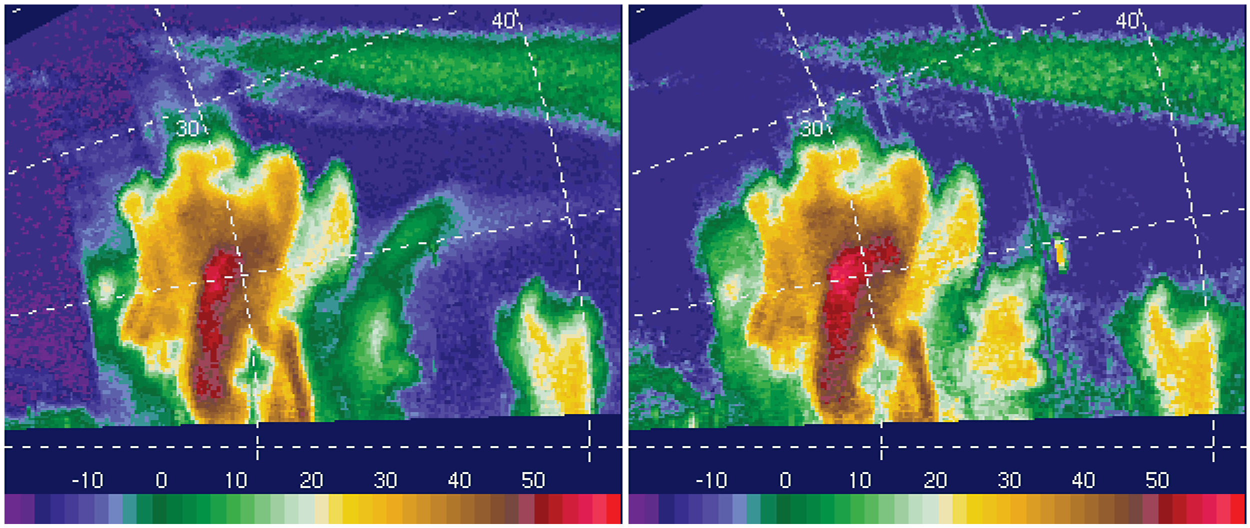
11.1 Consider these two 0.5° Doppler PPIs from adjacent radars near the Gulf of Mexico, the southernmost white line being the coast line. The north-to-south white line is the Mississippi valley.

a) Assuming that the radar aims horizontally, use the two Doppler images to estimate the winds at the four locations circled around the Mississippi valley.

b) If both radars measure radial velocity with an error of 1 m/s, what is the expected magnitude of the error on the retrieved tangential velocity for the point on the left?

c) What would be the error on the retrieved tangential component of the wind at the mouth of the Mississippi valley?

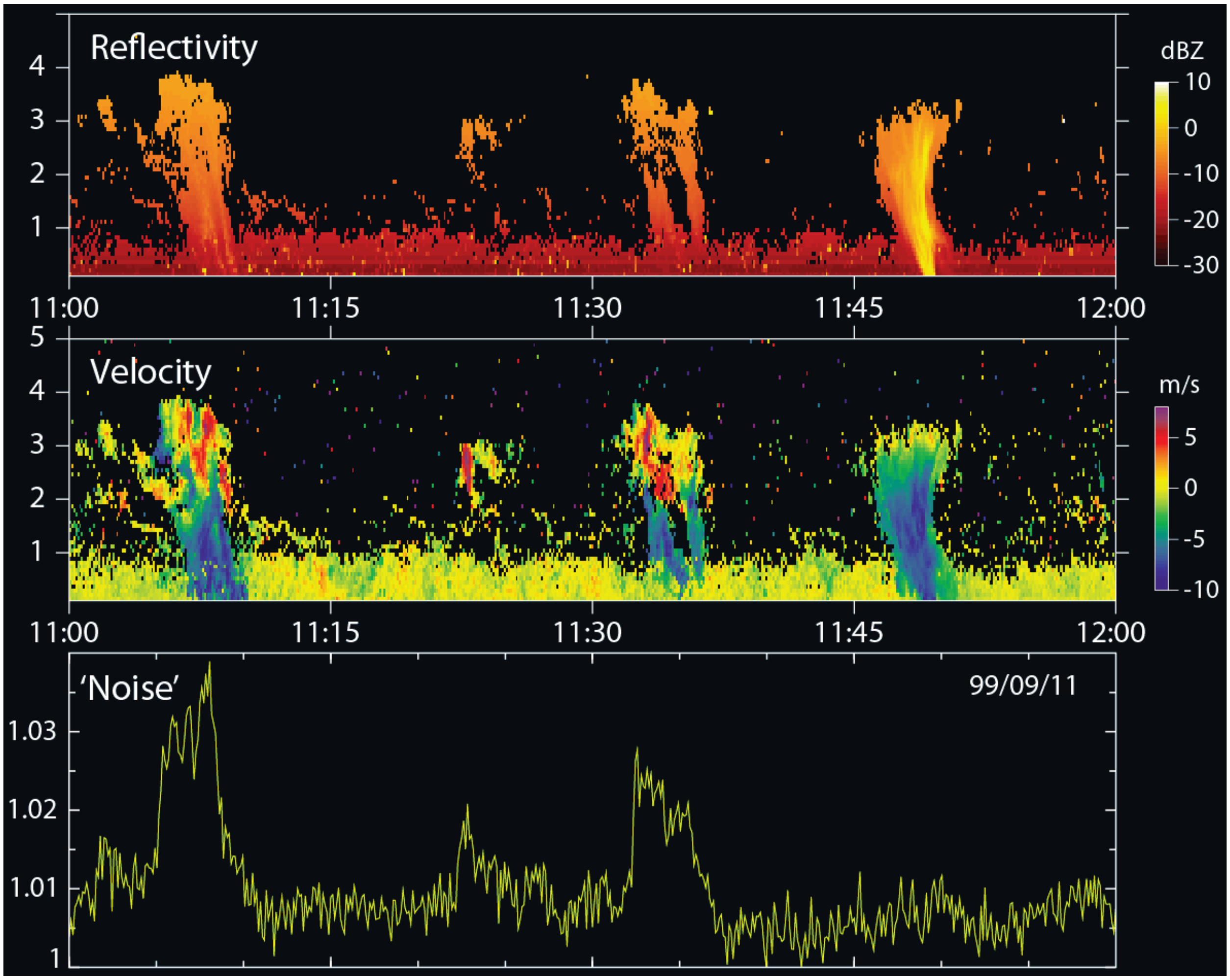




11.2 The RHIs shown above were collected simultaneously by a dual-wavelength radar working at X-band (left) and S-band (right), both systems having the same beam widths. Three cells of different sizes can be identified, the middle one being much weaker at X-band than at S-band.

a) Speculate on why the middle cell appears weaker at X-band and why all the other weather echoes are of similar reflectivity, explaining your reasoning.

b) At S-band, near the top of the middle cell, a point echo caused by an airplane is well observed, but is not clearly seen at X-band. Given that the metal the airplane is made of is an excellent reflector of microwaves at both frequencies, why is there such a difference in measured reflectivity?



11.3 As briefly mentioned at the end of Chapter 11, radar can also indirectly detect emissions of microwaves as an increase in noise level. The image above shows a time-height section of reflectivity, one of vertical velocity, and a time series of the noise power received in arbitrary units, all measured at X-band by a vertically pointing radar. What that lower plot reveals is that, at some specific times, something is emitting (and hence attenuating) more the X-band microwaves than at other times, that something being correlated with the radar echoes observed.

Given the echo pattern and strength observed, and using the expected attenuation from atmospheric constituents (see Fig. 2.12 or supplement e02.5), is the enhanced attenuation in this particular case primarily driven by changes in water vapor, the presence of cloud water, or that of precipitation? Justify your answer. Surface temperature is around 20°C, while dew point temperature is 12°C. In your answer, reflect on and contrast the difference in signature between the first three echoes and the last one.

12 Cloud and spaceborne radars

12.1 Imagine a cloud with 10^6 droplets of 10 μm diameter per liter and rain with 1 millimeter-size drop per liter.

a) Compute the reflectivity expected from these hydrometeors at 10 cm wavelength, using $||K||^2 = 0.93$.

b) Consider the radar cross-section of hydrometeors at X-band in Fig. 2.2, and let us assume that for a given D/λ, the ratio of $\sigma_b(D/\lambda)/\sigma_b$(Rayleigh) seen at X-band on that plot also applies to shorter wavelengths (note that $4\sigma_b$(Rayleigh)$/(\pi D^2) = ||K||^2$). Use this assumption and the plot to estimate the reflectivity of the cloud and rain at Ka-band, W-band, and for $\lambda = 1$ mm. State your assumptions.

12.2 Let us reexamine Fig. 12.1, focusing more specifically on the base of echoes with reflectivities of approximately −40 dBZ. Because cloud droplets grow dominantly from cloud base to cloud top (note the mostly positive velocities and the gradual intensification of reflectivity with height), this −40 dBZ corresponds to the minimum detectable echo intensity (or signal) at 2.5 km altitude.

a) Let us assume that a typical 10^6 droplets per liter have formed. Given the reflectivity observed at the base of the echo, what is the mean size of cloud droplets when they become detectable by radar?

b) At 3 km altitude, the same number of droplets now has on average a reflectivity near −32 dBZ (see Fig. 12.1). Assuming that the diameter of cloud droplets grow in height as $D = az_c^{0.5}$, where a is a constant and z_c is the height with respect to cloud base, estimate the altitude of cloud base, or where the cloud base is with respect to the base of the radar echo.

12.3 Let us reexamine Fig. 12.3. Near echo top, say above 5 km, ice crystals are small enough to be Rayleigh scatterers at W-band. Any difference in reflectivity between X-band and W-band radars must hence be due to attenuation.

a) Estimate visually the two-way attenuation at W-band through the precipitation at 0:02.

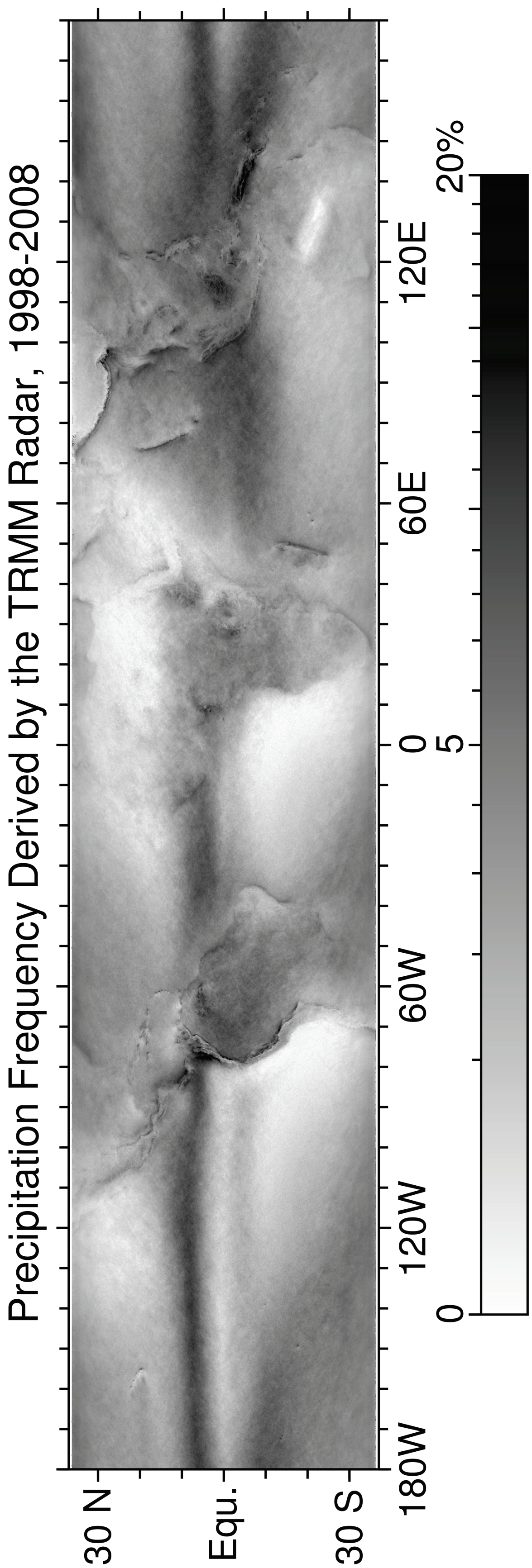
b) How intense is precipitation near the surface at that time? How does it compare with a typical rain rate of widespread precipitation (1-2 mm/h) and of convective precipitation (50 mm/h)? What implication does this have on the ability of W-band radars to make measurements in the presence of rainfall?

12.4 Now time to dream about radar-based weather surveillance from space, considering first radars on low polar orbit, and then on geostationary orbit.

a) Imagine radars with characteristics like the GPM precipitation Ku-band system (0.7°beam, minimum detectable signal of 17 dBZ) on board of polar orbiting satellites at 400 km altitude (93 min orbit) making measurements over a swath 245 km wide. Because the Earth rotates underneath the satellite, each satellite makes measurements over different locations after each orbit. Near the Equator is where the distance between swaths is maximized. In that context, how long will it take before each location on the planet has been sampled at least once? Explain your reasoning. As a result, if we dream of hourly revisits globally, how many such satellites would be required?

b) Let us be bolder and dream of a set of geostationary Ka-band radars. If we are to have a 3-dB beam width resolution on the ground of 5 km, how big an antenna would be required?

c) One of the reasons radar swaths on polar-orbiting satellites are limited to approximately ±15° is because ground targets contaminate measurements increasingly higher as you get to the edge of the swath. Significant contamination by ground echoes occur when clutter is within 1.5 beam widths from the beam axis. Up to what altitude would one expect to see clutter at the satellite's longitude at i) 20° and ii) 40° latitude? Pay attention to the measurement geometry before answering.



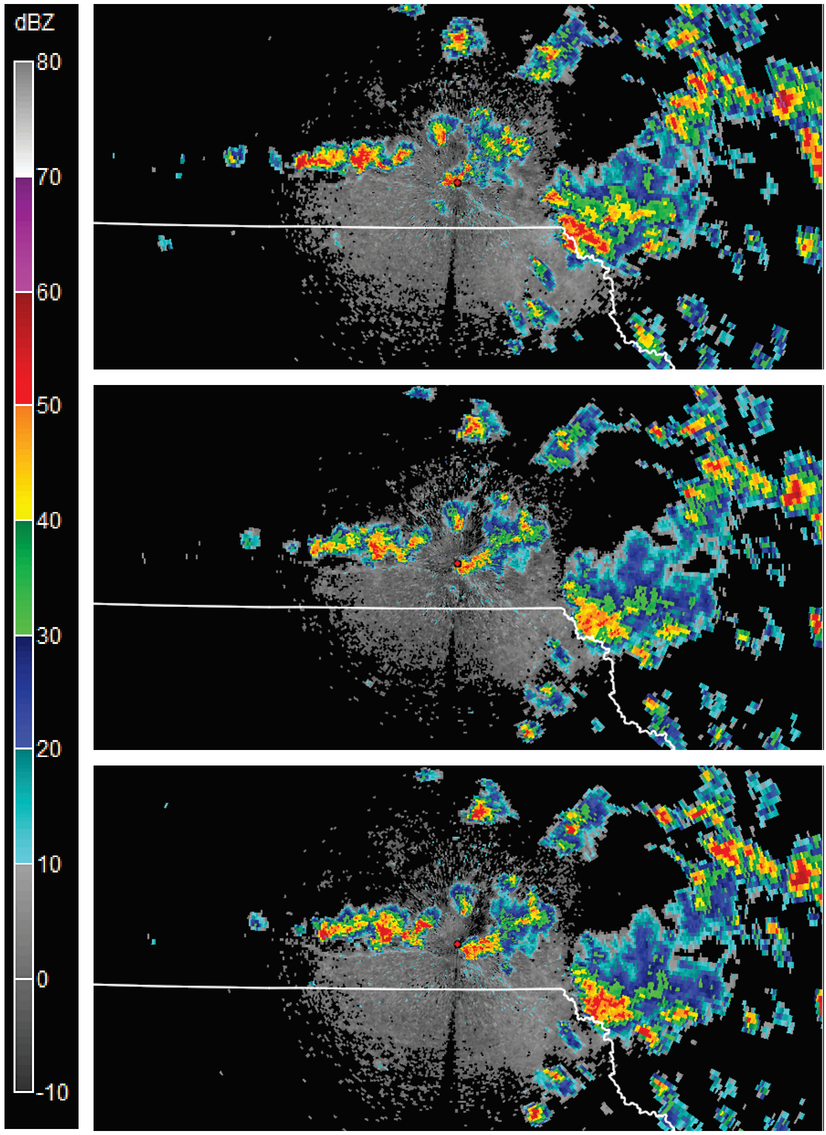
12.5 Figure 12.8, reproduced here, shows the frequency of precipitation detected by the TRMM Precipitation Radar over an 11 year period.

Find two unexpected signatures beyond the ones listed in the textbook figure caption and describe your findings. If you can explain them using meteorological reasoning or as artifacts of radar measurements, do so; if you cannot, explain why. [This question is very open-ended by design]

13 What does radar really measure?

13.1 Consider the receiver system described in supplement e13.2. The receiver was designed to amplify a very weak signal of 7×10^{-16} W arriving at the antenna to a manageable signal of 5.4×10^{-10} W at the digitizer. Missing from the discussion in e13.2 is the fact that amplifiers and digitizers have upper limits beyond which they cannot amplify or get correct digital values; this limit is known as the saturation power of the receiver. Let us assume that the receiver in question had a saturation power at the digitizer of 6×10^{-3} W. Given a dual-polarized WSR-88D using its long transmit pulse ($\tau = 4.5$ µs, $P_t = 375$ kW, $D_a = 8.5$ m, $\lambda = 10.5$ cm), what is the maximum reflectivity that can be detected at 1 km range with this receiver?

13.2 Attenuation by precipitation includes attenuation done by water on the radome, forming a layer of absorbing material. That attenuation is very difficult to estimate directly and is hence often poorly known. On slide 4 of electronic supplement e07.1, between 18:00 and 18:15, and another time 30 minutes later, an intense cell moves over the radar. As a result, the range up to which weak insect echoes can be detected shrinks momentarily, as illustrated on the right before (top), during (middle), and after (bottom) the cell passes over the radar. This reduction is a result of the attenuation by the small intense cell itself and by the water on the radome.



a) Estimate the ratio of the range of up to which insects are observed to the south during the passage of the cell and that before and after the passage of the cell over the radar.

b) Use that ratio to estimate the sensitivity loss caused by attenuation.

c) Using Fig. 2.12, contrast that sensitivity loss with the attenuation expected from that cell at S-band, equivalent to the attenuation caused by about 5 km of 50 mm/h rain. What is the inferred resulting radome attenuation caused by this cell?

13.3 [Longer problem] The reflectivity as a function of distance x (in km) of a simplified hail cell surrounded by insect echoes is $dBZ = 5 + 60\exp\left[-\left(\frac{x}{10}\right)^2\right]$, the x axis being locally perpendicular to the radar beam axis. This reflectivity pattern, assumed independent of height, is observed by two radars with a 1° beam, one at 50 km, and one at 200 km range.

a) The antenna of the two radars have the performance of a WSR-88D antenna with $f_{\text{lobes}} = 0.25$ and $\phi_{\text{lobes}} = \theta_{\text{lobes}} = 10°$. Based on (13.2), plot the one-way antenna pattern at as a function of $\delta\theta$ (from −15 to 15°) for $\delta\phi = 0$.

b) Use (13.2) to numerically solve for how the storm scene is measured by the two radars. Plot your results in two diagrams, one showing the true and measured reflectivity at 50 km as a function of x, and another for the same plots at 200 km. Assume no azimuthal averaging and ignore beam blockage issues. Discuss your findings.

c) An alternative radar technology is being proposed; it will have superior technology but poorer sidelobe performance such that $f_{\text{lobes}} = 0.5$, everything else being equal. Plot the antenna pattern of this new radar at as a function of $\delta\theta$ for $\delta\phi = 0$ on the same diagram as in a).

d) Use (13.2) to numerically solve for how the hail echo would be distorted by the two new radars at 50 km and 200 km, overlaying the plot of the measured reflectivity at each range on the plots made in b). Compare the reflectivity measured by both radar technologies, commenting on their respective ability to get good data from the insect echoes near the hail cell as would be needed to measure the flow of air in or out of the cell.

13.4 [Even longer problem] Supplement e13.4 contains the raw reflectivity data that was used in Fig. 13.5. Instructions on how to read that data are in the file `e13_4_Fig13_5_DataInfo.txt`.

a) Assume a radar with a 1° beam and no sidelobes making measurements of that field while scanning and averaging in azimuth over 0.5°. Using (13.1), simulate what the radar should observe at 30 km and 120 km range if it took such measurements at every 0.01° elevation and azimuth angle. Plot the results. If you are successful, and if I was, the resulting field should look like Fig. 13.5b and Fig. 13.5c.

b) Assume that winds cause the true Doppler velocity to be only a function of height such that $v_{DOP} = 4z$, z being height in km. Neglecting aliasing issues, compute and plot the resulting measured Doppler velocity at 30 and 120 km range. Contrast it with the true Doppler profile.

A Mathematics and statistics of radar meteorology

A.1 The first elevation of a typical weather surveillance radar is 0.5º.

a) Assuming that the 4/3 Earth radius approximation holds, at what altitude will the center of the radar beam be at 120 km? 360 km? (radius of the Earth a_e is approximately 6371 km)

b) A better approximation is to assume an exponential profile for $(n-1)$ with height. A simplified implementation of radio propagation through an atmosphere with an exponential $N(z)$ is used in the supplement eA.1. Assuming that the refractivity $N(z) = 350 \exp(-z/6700)$ (z in m), a profile expected in widespread rainy conditions, compare the results obtained from the spreadsheet with those from the 4/3 Earth approximation.

c) The wind varies linearly with height such that $v(z) = z/200$. Assuming that the average reflectivity gradient at 360 km range is −10 dB/km and that the radar antenna has a one-way half-power beam width of 1°, determine the Doppler velocity measured when the radar points due north and contrast it with what would be measured if there was no reflectivity gradient. What does this result tell us about our ability to properly measure winds at far range? Useful tip: $\int_{-\infty}^{\infty} e^{-ax^2-2bx}dx = \sqrt{\frac{\pi}{a}}\, e^{\frac{b^2}{a}}$.

A.2 Read the data file `eA_3_HighResolutionXYData.nc`. Focus on the first scan (16:18 UTC) only, and ignore the mask information.

a) At the native resolution of the map (250 m by 250 m), compute a rainfall map using $Z = 300\, R^{1.5}$ and the average estimated rainfall for the whole scan. Show your code.

b) Average pixels in dBZ over 1 km by 1 km areas; repeat the computation of rainfall above.

c) Average pixels in linear Z units over 1 km by 1 km areas; repeat the computation of rainfall.

d) Comment on the differences found, focusing on the average rainfall value for the whole field. In particular, explain why there is a difference between a) and b), and why there is a difference between a) and c).

A.3 Consider the following set of 11 (x,y) values: (0,0), (1,1), (2,2), (3,3), … (10,10).

a) Compute a linear regression between y and x values assuming x to be the independent variable;

b) Let us add "noise" to the y values, subtracting 0.2 to the odd values of y (1, 3, 5, 7, 9) and adding 0.2 to the even values of y (0, 2, 4,…, 10). Redo the regression. Comment on your findings, especially concerning the slope.

c) Restarting from the original set of values, add stronger "noise" to the y values, subtracting 2 to the odd values of y (1, 3, 5, 7, 9) and adding 2 to the even values of y (0, 2, 4,…, 10). Redo the regression, and comment on your findings.

d) Finally, restarting from the original set of values, add strong "noise" but this time to the x values, subtracting 2 to the odd values of x (1, 3, 5, 7, 9) and adding 2 to the even values of x (0, 2, 4,…, 10). Redo the regression, and comment on your findings.

e) What are the implications of these results on regressions that are done between two variables that 1) have intrinsic variability but come from the same volume, such as the reflectivity in a given radar sample volume and the rainfall in that same sample volume, and 2) come from different volumes, such as the reflectivity in a radar sample volume and the rainfall in a rain gauge?

A.4 Consider the following 8-sample complex time series simulating the received signal from a radar:

$S_1 = 20.245 + 5.1496\ i$ $\quad$ $S_2 = 0.76826 - 3.9296\ i$

$S_3 = -6.2033 + 2.5842\ i$ $\quad$ $S_4 = 1.8683 - 9.0584\ i$

$S_5 = 15.077 + 7.9737\ i$ $\quad$ $S_6 = .2638 + 20.079\ i$

$S_7 = -3.9157 - 12.464\ i$ $\quad$ $S_8 = 11.110 - 8.0469\ i$

This time series is the result of the mixture of a signal with long correlation time and of noise that decorrelates completely from one sample to the next.

a) Estimate the ratio of the power from the coherent signal to that of the incoherent noise. State your assumptions.

b) Compute the power spectrum of that signal. Plot the power spectrum coefficients as a function of the frequency per sample interval from −½ to ½.

You may choose to do the calculations using a calculator or by writing computer code. If you solve the problem by calculator, show how you computed a) and one of the power spectrum coefficients of b). If you solve it using code, please join the code to your results.