unperturbed structure, the expansion coefficients are not constants of propagation but vary with z as the optical field propagates through the structure:

$$\mathbf{E}(\mathbf{r}) = \sum_{\nu} A_{\nu}(z)\hat{\boldsymbol{\mathcal{E}}}_{\nu}(x, y) \exp(\mathrm{i}\beta_{\nu}z), \tag{4.5}$$

$$\mathbf{H}(\mathbf{r}) = \sum_{\nu} A_{\nu}(z)\hat{\boldsymbol{\mathcal{H}}}_{\nu}(x, y) \exp(\mathrm{i}\beta_{\nu}z). \tag{4.6}$$

Because the power in a normal mode is given by $P_{\nu} = |A_{\nu}|^2$, according to (3.27), the z dependence of $A_{\nu}(z)$ in the above indicates that the power of a mode that is coupled to another mode does not remain a constant of propagation. Thus, coupling of modes leads to exchange of mode power.

4.1.1 Single-Structure Mode Coupling

We first consider the coupling between normal modes in a single optical structure, such as a single waveguide, that is subject to some perturbation. By single structure, we mean that the entire optical structure is considered in defining the normal modes characterized by normalized mode fields $\left(\hat{\boldsymbol{\mathcal{E}}}_{\nu}, \hat{\boldsymbol{\mathcal{H}}}_{\nu}\right)$ of propagation constants β_{ν}. The structure can be a simple structure, such as a homogeneous medium, a single interface, or a single waveguide; or it can be a compound structure that consists of multiple interfaces or multiple waveguides. In any event, no matter how complicated the structure might be, it is considered as a single entity and is described with a single $\boldsymbol{\epsilon}(\mathbf{r})$ to define the normal modes.

A spatially dependent perturbation to the structure at a frequency of ω can be represented by a single perturbing polarization, $\Delta\mathbf{P}(\mathbf{r})$, so that the equations in (4.1) and (4.2) are modified as

$$\nabla \times \mathbf{E} = \mathrm{i}\omega\mu_0\mathbf{H}, \tag{4.7}$$

$$\nabla \times \mathbf{H} = -\mathrm{i}\omega\boldsymbol{\epsilon}\cdot\mathbf{E} - \mathrm{i}\omega\Delta\mathbf{P}. \tag{4.8}$$

Any optical field propagating in this perturbed structure can be expanded as (4.5) and (4.6) while its propagation is governed by these two equations with $\Delta\mathbf{P} \neq 0$. Meanwhile, the normal mode fields defined by the unperturbed structure, which are defined by (4.1) and (4.2), also satisfy these two equations with $\Delta\mathbf{P} = 0$.

Applying (4.7) and (4.8) to two arbitrary sets of fields, $(\mathbf{E}_1, \mathbf{H}_1)$ and $(\mathbf{E}_2, \mathbf{H}_2)$, with respective perturbations of $\Delta\mathbf{P}_1$ and $\Delta\mathbf{P}_2$, we find the *Lorentz reciprocity theorem*:

$$\nabla\cdot\left(\mathbf{E}_1 \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_1\right) = -\mathrm{i}\omega\left(\mathbf{E}_1\cdot\Delta\mathbf{P}_2^* - \mathbf{E}_2^*\cdot\Delta\mathbf{P}_1\right), \tag{4.9}$$

which holds for any two sets of fields that are respectively associated with two arbitrary perturbations. To derive the couple-mode equation, we take $(\mathbf{E}_1, \mathbf{H}_1)$ to be the optical field propagating in the perturbed structure with $\Delta\mathbf{P}_1 = \Delta\mathbf{P}$, which can be expanded as (4.5) and (4.6), and $(\mathbf{E}_2, \mathbf{H}_2)$ to be the normal mode fields $\left(\hat{\boldsymbol{\mathcal{E}}}_{\nu}, \hat{\boldsymbol{\mathcal{H}}}_{\nu}\right)$ defined by the unperturbed structure