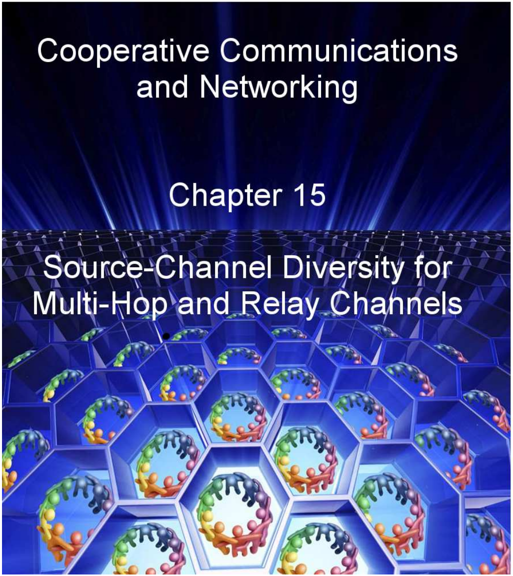
Cooperative Communications and Networking
Chapter 15
Source-Channel Diversity for Multi-Hop and Relay Channels

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Introduction

- A key challenge in the design of real-time wireless multimedia systems is the presence of fading coupled with strict delay constraints. A very effective answer to this problem is the use of diversity achieving techniques.
- We focuse on studying systems that exhibit diversity of three forms:
  - Source coding diversity.
  - Channel coding diversity.
  - user-cooperation diversity (implemented through either relay channels or multi-hop channels, each with amplify-and-forward or decode-and-forward user cooperation).
- We will differentiate between multi-hop and relay channels
  - A multi-hop channel where there is no direct path between the source and destination; i.e. the information path between source and destination contains one or more relaying nodes.
  - A relay channel as that where there is a direct communication path between source and destination as well as one or more paths through relaying nodes.

System Model

- Communication is performed over a complex, additive white Gaussian noise (AWGN) fading channel. I is the maximum average mutual information between the channel input and output, for the channel under consideration $I = \log(1 + |h|^2 SNR)$.
- It will be convenient for us to work with the random function e^I, which has a cumulative distribution function (cdf) F_{e^I} that can be approximated at high SNR as

$$F_{e^I}(t) \approx c \left(\frac{t}{SNR} \right)^p .$$

  For the case of Rayleigh fading we have $p = 1$.
- For K source samples and N channel inputs, we denote by $\beta \triangleq N/K$, the bandwidth expansion factor or processing gain.

System Model (cont.)

- We assume that K is large enough to average over the statistics of the source but N is not sufficiently large to average over the statistics of the channel, i.e., we assume block fading wireless channel.
- The source signal average end-to-end distortion is the figure of merit. Thus, performance will be measured in terms of the expected distortion $E[D] = E[d(\mathbf{s}, \hat{\mathbf{s}})]$, where

$$d(\mathbf{s}, \hat{\mathbf{s}}) = (1/K) \sum_{k=1}^{K} d(s_k, \hat{s}_k).$$

- We will assume $d(s_k, \hat{s}_k)$ to be the mean-squared distortion measure.

System Model (cont.)

- Our figure of merit is called the *distortion exponent* and is defined as

$$\Delta \triangleq - \lim_{SNR \to \infty} \frac{\log E[D]}{\log SNR}.$$

- We will consider two types of source encoders: a *single description* (SD) and a *dual description* source encoder, i.e. the source encoder generates either one or two coded descriptions of the source.
- For single description source encoder the R-D function can be approximated without loss of generality, as

$$R = \frac{1}{2\beta} \log \left(\frac{1}{D} \right).$$

Dual Description Source Encoder

- In dual description encoders, source samples are encoded into two descriptions.
- Each description can either be decoded independently of the other, when the other is unusable at the receiver, or combined to achieve a reconstruction of the source with lower distortion, when both descriptions are received correctly.
- Let D_1 and D_2 be the reconstructed distortions associated with descriptions 1 and 2, respectively, when each is decoded alone. Let D_0 be the source distortion when both description are combined and jointly decoded. R_1 and D_1, and R_2 and D_2 are related through,

$$R_1 = \frac{1}{2\beta} \log\left(\frac{1}{D_1}\right), \quad R_2 = \frac{1}{2\beta} \log\left(\frac{1}{D_2}\right).$$

Dual Description Source Encoder (cont.)

- All the schemes we will consider in this work present the same communication conditions to each description. Therefore, it will be reasonable to assume $R_1 = R_2 = R_{md}/2$. Under this condition, it was shown that the following bounds can be derived

$$(4D_0 D_1)^{-1/(2\beta)} \lessapprox e^{R_{md}} \lessapprox (2D_0 D_1)^{-1/(2\beta)},$$

  This is under the low distortion scenario corresponding to $D_1 + D_2 - D_0 < 1$.

- In the case of the high distortion scenario, $D_1 + D_2 - D_0 > 1$, the R-D function equals

$$R_{md} = \frac{1}{2\beta} \log\left(\frac{1}{D_0}\right).$$

  There will be no loss for using dual description source encoder compared to SD source encoder.

Multi-Hop Amplify-and-Forward Protocol

- The multi-hop channel is a channel where there is no direct path between the source and destination.

Single Relay

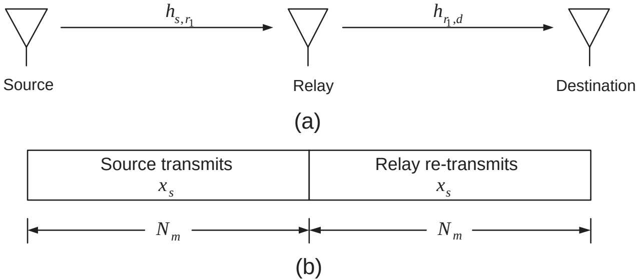

Figure: Two-hop single relay system (a) system model (b) time frame structure.

Multi-Hop Single Relay Amplify-and-Forward Protocol

- The received signal at the destination is given by

  $y_d = h_{r_1,d}\alpha_1 y_{r_1} + n_{r_1,d} = h_{r_1,d}\alpha_1 h_{s,r_1}\sqrt{P}x_s + h_{r_1,d}\alpha_1 n_{s,r_1} + n_{r_1,d},$

  where $\alpha_1 \leq \sqrt{\frac{P}{P|h_{s,r_1}|^2+N_0}}$ is the power constraint at the relay node.
- At high SNR, we have

$$I(x_s, y_d) \approx \log\left(1 + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right)$$

$$\approx \log\left(\frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right).$$

- The harmonic mean of two nonnegative random variables can be upper and lower bounded as

$$\frac{1}{2}\min(Z_1, Z_2) \leq \frac{Z_1 Z_2}{Z_1 + Z_2} \leq \min(Z_1, Z_2).$$

Multi-Hop Single Relay Amplify-and-Forward Protocol (cont.)

- Define the random variable $Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$ then we have

$$\Pr\left[\min(Z_1, Z_2) < t\right] \leq \Pr\left[Z < t\right] \leq \Pr\left[\min(Z_1, Z_2) < 2t\right].$$

- From the definition of the channel model we can show

$$c_1 \left(\frac{t}{SNR}\right)^p \lessapprox F_Z(t) \lessapprox c_2 \left(\frac{t}{SNR}\right)^p,$$

where Z is the harmonic mean of the source-relay and relay-destination channels.

- The minimum expected end-to-end distortion can now be computed as

$$E[D] = \min_D \Big\{ \Pr\left[I(x_s, y_d) < R(D)\right] + D \ \Pr\left[I(x_s, y_d) \geq R(D)\right] \Big\},$$

Multi-Hop Single Relay Amplify-and-Forward Protocol (cont.)

- For sufficiently large SNRs ,we have

$$\min_D \left\{ c_1 \left(\frac{\exp(R(D))}{SNR} \right)^p + D \right\} \lessapprox E[D] \lessapprox \min_D \left\{ c_2 \left(\frac{\exp(R(D))}{SNR} \right)^p + D \right\}.$$

- We have $\exp(R(D)) = D^{\frac{-1}{2\beta_m}}$ which leads to

$$\min_D c_1 \frac{D^{\frac{-p}{2\beta_m}}}{SNR^p} + D \lessapprox E[D] \lessapprox \min_D c_2 \frac{D^{\frac{-p}{2\beta_m}}}{SNR^p} + D.$$

- Differentiating the lower bound and setting equal to zero we get

$$C_{LB}\ SNR^{\frac{-2\beta_m p}{2\beta_m + p}} \lessapprox E[D] \lessapprox C_{UB}\ SNR^{\frac{-2\beta_m p}{2\beta_m + p}},$$

Multi-Hop Single Relay Amplify-and-Forward Protocol (cont.)

- **Theorem:** The distortion exponent of the two-hop single relay amplify-and-forward protocol is

$$\Delta_{SH-1R-AMP} = \frac{2p\beta_m}{p + 2\beta_m},$$

where $\beta_m = N_m/K$.

Multi-Hop Amplify-and-Forward Protocol with M relay nodes

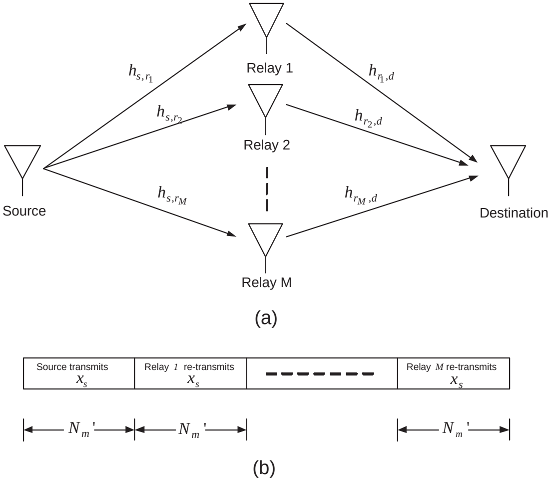

Figure: Two-hop M relays system (a) system model (b) time frame structure.

Multi-Hop Amplify-and-Forward Protocol with M relay nodes (cont.)

- The destination selects the signal of the highest quality (highest SNR) to recover the source signal.
- **Theorem:** The distortion exponent of the two-hop M relays selection channel coding diversity amplify-and-forward protocol is

$$\Delta_{SH-MR-AMP} = \frac{4Mp\beta_m}{M(M+1)p + 4\beta_m}.$$

- The distortion exponent shows a tradeoff between the diversity and the source encoder performance. Increasing the number of relay nodes increases the diversity of the system at the expense of using lower rate source encoder (higher distortion under no outage).

Multi-Hop Amplify-and-Forward Protocol with M relay nodes (cont.)

- The optimum number of relay nodes is

$$\frac{\partial}{\partial M}\Delta_{SH-MR-AMP} = 0 \longrightarrow M_{opt} = 2\sqrt{\frac{\beta_m}{p}}.$$

- The number of relays decreases, for a fixed β_m, as p increases. For higher channel quality the system performance is limited by the distortion introduced by the source encoder in the absence of outage. Then, as p increases, the optimum number of relays decreases to allow for the use of a better source encoder with lower source encoding distortion. The system is said to be a **quality limited system**.
- As β_m increases (higher bandwidth), for a fixed p, the performance will be limited by the outage event. As β_m increases, the optimum number of relays increases to achieve better outage performance. The system is said to be an **outage limited system**.

Optimal Channel Coding Diversity with 2 Relays

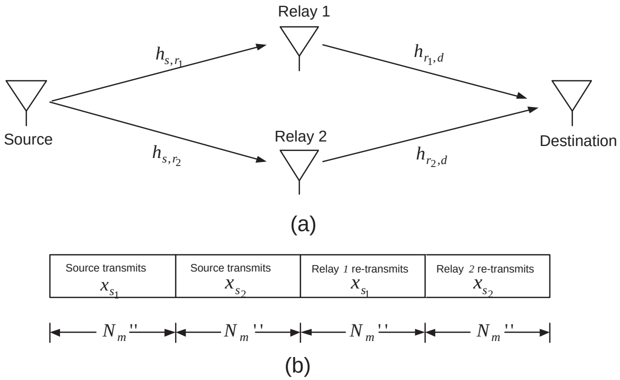

Figure: Two-hop 2 relays optimal channel coding diversity (source coding diversity) system (a) system model (b) time frame structure.

Optimal Channel Coding Diversity with 2 Relays (Cont.)

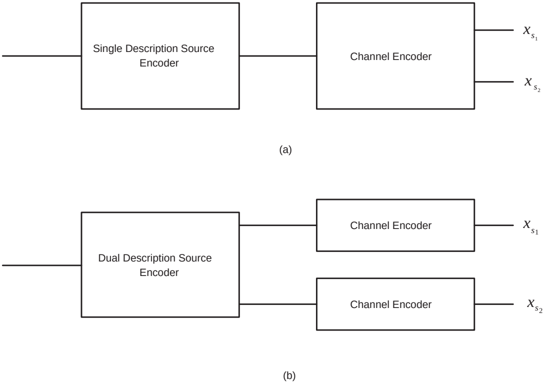

Figure: Two relays source and channel encoding for (a) optimal channel coding diversity (b) source coding diversity.

Optimal Channel Coding Diversity with 2 Relays (cont.)

- The mutual information is

$$I \approx \log\left(1+\frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right) + \log\left(1+\frac{|h_{s,r_2}|^2 SNR |h_{r_2,d}|^2 SNR}{|h_{s,r_2}|^2 SNR + |h_{r_2,d}|^2 SNR}\right).$$

- **Theorem:** The distortion exponent of the two-hop two-relay optimal channel coding diversity amplify-and-forward system is

$$\Delta_{SH-2R-OPTCH-AMP} = \frac{2p\beta_m}{p+\beta_m}.$$

Source Coding Diversity with 2 Relays

- **Theorem:** The distortion exponent of the two-hop 2 relays source coding diversity amplify-and-forward protocol is

$$\Delta_{SH-2R-SRC-AMP} = \max\left[\frac{4p\beta_m}{3p+2\beta_m}, \frac{2p\beta_m}{p+2\beta_m}\right].$$

Multi-Hop Decode-and-Forward Protocol

- For the single relay case we can formulate the outage as

$$P_{outage} = \Pr\left[\min(I(x_s, y_{r_1}), I(x_{r_1}, y_d)) < R(D)\right].$$

  Hence, in terms of outage probability, the multi-hop decode-and-forward protocol outperforms the multi-hop amplify-and-forward protocol.

- **Theorem:** The distortion exponent of the multi-hop decode-and-forward schemes are:
  - for the two-hop single relay

$$\Delta_{SH-1R-DEC} = \frac{2p\beta_m}{p + 2\beta_m},$$

  - for the two-hop M relays selection channel coding diversity

$$\Delta_{SH-MR-DEC} = \frac{4Mp\beta_m}{M(M+1)p + 4\beta_m},$$

  - for the two-hop 2 relays source coding diversity

$$\Delta_{SH-2R-SRCDEC} = \max\left[\frac{4p\beta_m}{3p + 2\beta_m}, \frac{2p\beta_m}{p + 2\beta_m}\right].$$

Optimal Channel Coding Diversity with 2 Relays

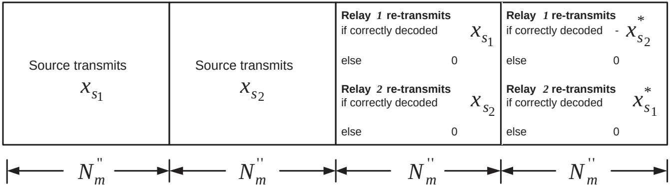

Figure: Two-hop 2 relays decode-and-forward optimal channel coding diversity system' time frame structure.

- **Theorem:** The distortion exponent of the two-hop 2 relays optimal channel coding diversity decode-and-forward protocol is

$$\Delta_{SH-2R-OPTCH-DEC} = \frac{2p\beta_m}{p+\beta_m}.$$

Relay Channels

- For comparison purposes, we consider the case when the source transmits a single description source coded message over the source-destination channel without the help of any relay node. The distortion exponent is given by as

$$\Delta_{NO-DIV} = \frac{2p\beta_r}{p + 2\beta_r}$$

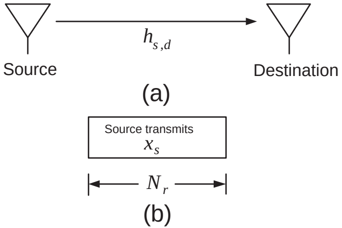

Figure: No diversity (direct transmission) system (a) system model (b) time frame structure.

Amplify-and-Forward Protocol over Relay Channels

- For the **single relay** case, The mutual information of this system is given by

$$I(x_s, y_d) = \log\left(1 + |h_{s,d}|^2 SNR + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR + 1}\right).$$

- **Theorem:** The distortion exponent of the single relay amplify-and-forward scheme is

$$\Delta_{RC-1R-AMP} = \frac{2p\beta_r}{2p + \beta_r}.$$

- Asymptotically comparing the distortion exponents for case of no diversity and a single relay we have

$$\lim_{\beta_r/p \to \infty} \frac{\Delta_{RC-1R-AMP}}{\Delta_{NO-DIV}} = 2,$$
$$\lim_{\beta_r/p \to 0} \frac{\Delta_{RC-1R-AMP}}{\Delta_{NO-DIV}} = \frac{1}{2}.$$

Amplify-and-Forward Protocol over Relay Channels (cont.)

- **Theorem:** The distortion exponent of M relay nodes amplify-and-forward protocol is

$$\Delta_{RC-MR-AMP} = \frac{2(M+1)p\beta_r}{2\beta_r + (M+1)^2 p}.$$

Optimal Channel Coding Diversity with 2 Relays

- The mutual information is given by

$$I \approx \log\left(1 + |h_{s,d}|^2 SNR + \frac{|h_{s,r_1}|^2 SNR |h_{r_1,d}|^2 SNR}{|h_{s,r_1}|^2 SNR + |h_{r_1,d}|^2 SNR}\right) + \log\left(1 + |h_{s,d}|^2 SNR + \frac{|h_{s,r_2}|^2 SNR |h_{r_2,d}|^2 SNR}{|h_{s,r_2}|^2 SNR + |h_{r_2,d}|^2 SNR}\right).$$

- **Theorem:** The distortion exponent of the 2 relays optimal channel coding diversity amplify-and-forward protocol is

$$\Delta_{RC-2R-OPTCH-AMP} = \frac{3p\beta_r}{3p + \beta_r}.$$

Source Coding Diversity with 2 Relays Employing Amplify-and-Forward Protocol

- **Theorem:** The distortion exponent of the 2 relays source coding diversity amplify-and-forward protocol is

$$\Delta_{RC-2R-SRC-AMP} = \max\left[\frac{2p\beta_r}{2p+\beta_r}, \frac{3p\beta_r}{4p+\beta_r}\right].$$

Decode-and-Forward Relay Channel

- **Theorem:** The distortion exponents of the decode-and-forward relay channel are
  - For the single relay channel
  $$\Delta_{RC-1R-DEC} = \frac{2p\beta_r}{2p+\beta_r}.$$
  - For the M relays selection channel coding diversity
  $$\Delta_{RC-MR-DEC} = \frac{2(M+1)p\beta_r}{2\beta_r+(M+1)^2p}.$$
  - For the optimal channel coding with 2 relays, with the same time frame structure as in Fig. 5,
  $$\Delta_{RC-2R-OPTCH-AMP} = \frac{3p\beta_r}{3p+\beta_r}.$$
  - For the source coding diversity with 2 relays
  $$\Delta_{RC-2R-SRC-DEC} = \max\left[\frac{2p\beta_r}{2p+\beta_r}, \frac{3p\beta_r}{4p+\beta_r}\right].$$

The distortion exponent for the various systems as a function of β_m for the multi-hop channel

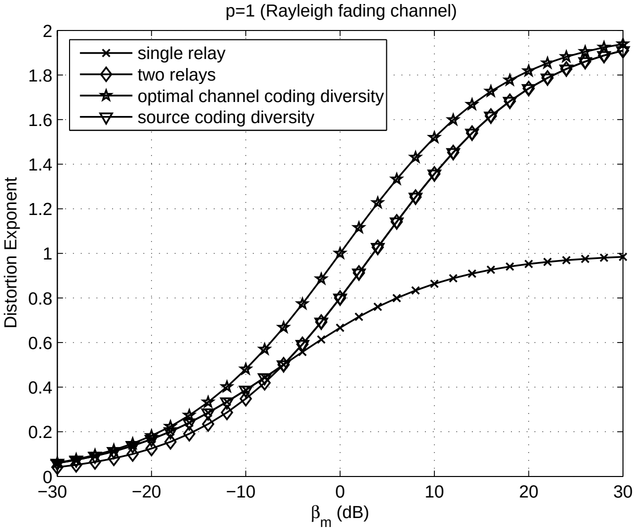

Figure: Distortion exponents for two-hop amplify-and-forward (decode-and-forward) protocol

The distortion exponent versus β_r for the various relay channel schemes

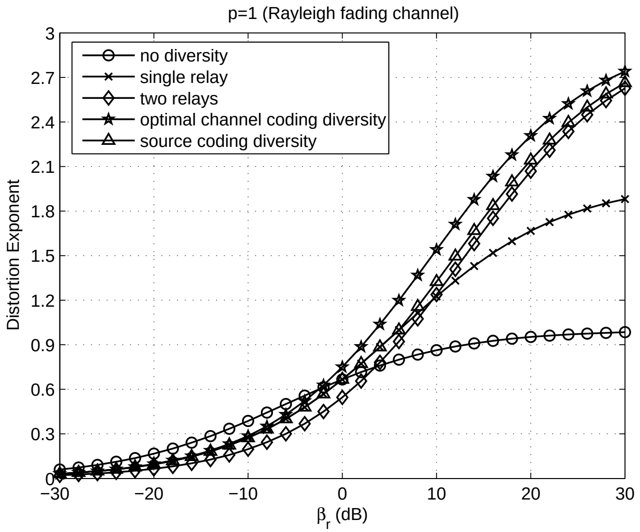

Figure: Distortion exponents for amplify-and-forward

Discussion and Conclusions

- The optimal channel coding diversity scheme always results in a higher distortion exponent than the source coding diversity scheme at any bandwidth expansion factor.
- Between source and channel coding, it is better to exploit diversity at the channel encoder level.
- There is a tradeoff between the quality (resolution) of the source encoder and the amount of cooperation (number of relays).
- At low bandwidth it is not the channel outage event, but the distortion introduced at the source coding stage is the dominant factor limiting the distortion exponent performance. Therefore, in these cases it is better not to cooperate and use a lower distortion source encoder.

Discussion and Conclusions (cont.)

- Similarly, we showed that as the bandwidth expansion factor increases, the distortion exponent is improved by increasing the number of relays because user cooperation diversity is the main limiting factor.
- In these cases, the system is said to be an outage limited system. Therefore, it is better to cooperate in these cases which results in minimizing the outage probability and, consequently, minimizing the end-to-end distortion.