```
In[58]:= Needs["Graphics`PlotField`"]
```

à Question 12

(i) (a)

In[59]:= Solve[{-x + $\frac{x\,y}{100}$ == 0, 2 y - $\frac{2\,x\,y}{25}$ == 0}, {x, y}]

Out[59]= {{x → 0, y → 0}, {x → 25, y → 100}}

(ii) (a)

In[60]:= fx1 = D[-x + $\frac{x\,y}{100}$, x] /. {x -> 0, y -> 0}

Out[60]= -1

In[61]:= fy1 = D[-x + $\frac{x\,y}{100}$, y] /. {x -> 0, y -> 0}

Out[61]= 0

In[62]:= gx1 = D[2 y - $\frac{2\,x\,y}{25}$, x] /. {x -> 0, y -> 0}

Out[62]= 0

In[63]:= gy1 = D[2 y - $\frac{2\,x\,y}{25}$, y] /. {x -> 0, y -> 0}

Out[63]= 2

In[64]:= matrixA1 = $\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$

Out[64]= {{-1, 0}, {0, 2}}

In[65]:= fx1 = D[-x + $\frac{x\,y}{100}$, x] /. {x -> 25, y -> 100}

Out[65]= 0

In[66]:= fy1 = D[-x + $\frac{x\,y}{100}$, y] /. {x -> 25, y -> 100}

Out[66]= $\frac{1}{4}$

In[67]:= gx1 = D[2 y - $\frac{2\,x\,y}{25}$, x] /. {x -> 25, y -> 100}

Out[67]= -8

In[68]:= gy1 = D[2 y - $\frac{2\,x\,y}{25}$, y] /. {x -> 25, y -> 100}

Out[68]= 0

In[69]:= `matrixA2 =` $\begin{pmatrix} 0 & \frac{1}{4} \\ -8 & 0 \end{pmatrix}$

Out[69]= $\{\{0, \frac{1}{4}\}, \{-8, 0\}\}$

(iii) (a)

```
In[70]:= Eigenvalues[matrixA1]

Out[70]= {-1, 2}

In[71]:= Eigenvectors[matrixA1]

Out[71]= {{1, 0}, {0, 1}}

In[72]:= Eigenvalues[matrixA2]
```

Out[72]= $\{-i\sqrt{2}, i\sqrt{2}\}$

```
In[73]:= Eigenvectors[matrixA2]
```

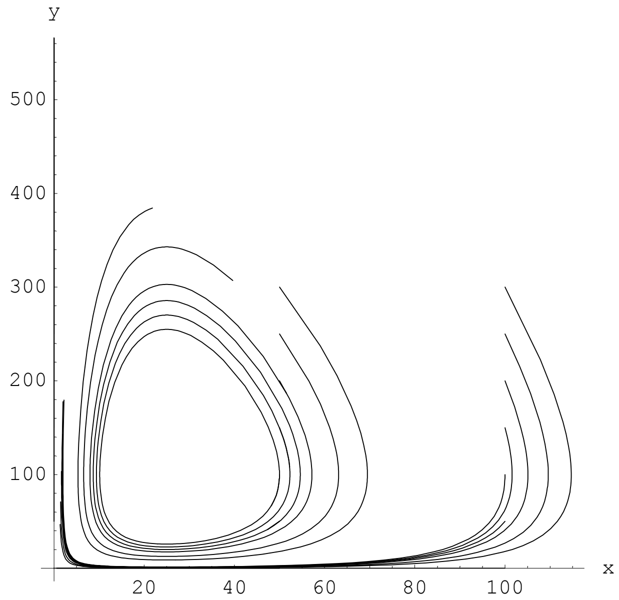
Out[73]= $\{\{\frac{i}{4\sqrt{2}}, 1\}, \{-\frac{i}{4\sqrt{2}}, 1\}\}$

(iv) (a)

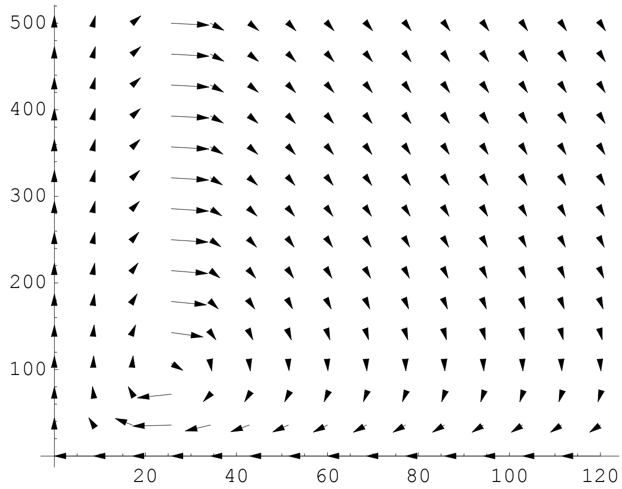
Note
The following procedure for plotting multiple trajectories along with the direction field is adapted from Commbes, K.R. et al (2nd ed 1998) Differential Equations with *Mathematica*. John Wiley & Sons, Inc., (Revised for *Mathematica* 3.0), Chapter 12.

```
In[74]:= nsol1[tval_, a_, b_, t1_] := ({x[t], y[t]} /.
      First[NDSolve[{x'[t] == -x[t] + x[t] y[t]/100, y'[t] == 2 y[t] - 2 x[t] y[t]/25,
        x[0] == a, y[0] == b}, {x[t], y[t]}, {t, 0, t1}]]) /. t -> tval
   nphase1[t1_] := ParametricPlot[Evaluate[Flatten[
       Table[nsol1[t, a, b, t1], {a, 0, 100, 50}, {b, 0, 300, 50}], 1]],
     {t, 0, t1}, AspectRatio -> 1, AxesLabel -> {"x", "y"}]
```

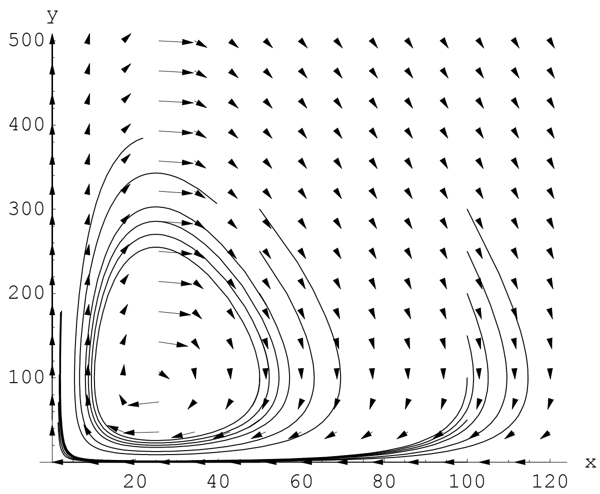
```
In[76]:= lines1 = nphase1[5];
```



In[77]:= **arrows1 = PlotVectorField**$\left[\left\{-x + \frac{x\,y}{100},\ 2\,y - \frac{2\,x\,y}{25}\right\}\right.$**, {x, 0, 120},**
{y, 0, 500}, Axes -> True, ScaleFunction -> (1 &), AspectRatio -> 0.8];



```
In[78]:= Show[arrows1, lines1, PlotRange -> {0, 500}, AxesLabel -> {"x", "y"}];
```



Which shows oscillations around the fixed point (x, y) = (25, 100).

(i) (b)

In[79]:= $\text{Solve}\left[\left\{-\frac{1}{2}x+\frac{x\,y}{\frac{1}{4}+y}==0,\ y-y^2-\frac{x\,y}{\frac{1}{4}+y}==0\right\},\{x,y\}\right]$

Out[79]= $\left\{\{x\to 0,\ y\to 0\},\ \{x\to 0,\ y\to 1\},\ \left\{x\to\frac{3}{8},\ y\to\frac{1}{4}\right\}\right\}$

(ii) (b)

In[80]:= $\text{fx1}=\text{D}\left[-\frac{x}{2}+\frac{x\,y}{\frac{1}{4}+y},\ x\right]\ /.\ \{x\to 0,\ y\to 0\}$

Out[80]= $-\frac{1}{2}$

In[81]:= $\text{fy1}=\text{D}\left[-\frac{x}{2}+\frac{x\,y}{\frac{1}{4}+y},\ y\right]\ /.\ \{x\to 0,\ y\to 0\}$

Out[81]= 0

In[82]:= $\text{gx1}=\text{D}\left[y-y^2-\frac{x\,y}{\frac{1}{4}+y},\ x\right]\ /.\ \{x\to 0,\ y\to 0\}$

Out[82]= 0

In[83]:= $\text{gy1}=\text{D}\left[y-y^2-\frac{x\,y}{\frac{1}{4}+y},\ y\right]\ /.\ \{x\to 0,\ y\to 0\}$

Out[83]= 1

In[84]:= $\text{matrixB1}=\begin{pmatrix}-\frac{1}{2} & 0\\ 0 & 1\end{pmatrix}$

```
General::spell1 :
 Possible spelling error: new symbol name "matrixB1" is similar to existing symbol "matrixA1".
```

Out[84]= $\left\{\left\{-\frac{1}{2},\ 0\right\},\ \{0,\ 1\}\right\}$

In[85]:= **fx1 = D[$-\frac{x}{2}+\frac{x\,y}{\frac{1}{4}+y}$, x] /. {x -> 0, y -> 1}**

Out[85]= $\frac{3}{10}$

In[86]:= **fy1 = D[$-\frac{x}{2}+\frac{x\,y}{\frac{1}{4}+y}$, y] /. {x -> 0, y -> 1}**

Out[86]= 0

In[87]:= **gx1 = D[$y-y^2-\frac{x\,y}{\frac{1}{4}+y}$, x] /. {x -> 0, y -> 1}**

Out[87]= $-\frac{4}{5}$

In[88]:= **gy1 = D[$y-y^2-\frac{x\,y}{\frac{1}{4}+y}$, y] /. {x -> 0, y -> 1}**

Out[88]= -1

In[89]:= **matrixB2 =** $\begin{pmatrix} \frac{3}{10} & 0 \\ -\frac{4}{5} & -1 \end{pmatrix}$

```
General::spell1 :
 Possible spelling error: new symbol name "matrixB2" is similar to existing symbol "matrixA2".
```

Out[89]= $\{\{\frac{3}{10}, 0\}, \{-\frac{4}{5}, -1\}\}$

In[90]:= **fx1 = D[$-\frac{x}{2}+\frac{x\,y}{\frac{1}{4}+y}$, x] /. {x -> $\frac{3}{8}$, y -> $\frac{1}{4}$}**

Out[90]= 0

In[91]:= **fy1 = D[$-\frac{x}{2}+\frac{x\,y}{\frac{1}{4}+y}$, y] /. {x -> $\frac{3}{8}$, y -> $\frac{1}{4}$}**

Out[91]= $\frac{3}{8}$

In[92]:= **gx1 = D[$y-y^2-\frac{x\,y}{\frac{1}{4}+y}$, x] /. {x -> $\frac{3}{8}$, y -> $\frac{1}{4}$}**

Out[92]= $-\frac{1}{2}$

In[93]:= **gy1 = D[$y-y^2-\frac{x\,y}{\frac{1}{4}+y}$, y] /. {x -> $\frac{3}{8}$, y -> $\frac{1}{4}$}**

Out[93]= $\frac{1}{8}$

In[94]:= **matrixB3 =** $\begin{pmatrix} 0 & \frac{3}{8} \\ -\frac{1}{2} & \frac{1}{8} \end{pmatrix}$

Out[94]= $\{\{0, \frac{3}{8}\}, \{-\frac{1}{2}, \frac{1}{8}\}\}$

(iii) (b)

In[95]:= **Eigenvalues[matrixB1]**

Out[95]= $\left\{-\frac{1}{2}, 1\right\}$

In[96]:= **Eigenvectors[matrixB1]**

Out[96]= {{1, 0}, {0, 1}}

In[97]:= **Eigenvalues[matrixB2]**

Out[97]= $\left\{-1, \frac{3}{10}\right\}$

In[98]:= **Eigenvectors[matrixB2]**

Out[98]= $\left\{\{0, 1\}, \left\{-\frac{13}{8}, 1\right\}\right\}$

In[99]:= **Eigenvalues[matrixB3]**

Out[99]= $\left\{\frac{1}{16}\left(1 - i\sqrt{47}\right), \frac{1}{16}\left(1 + i\sqrt{47}\right)\right\}$

In[100]:= **Eigenvectors[matrixB3]**

Out[100]= $\left\{\left\{\frac{1}{8} i \left(-i + \sqrt{47}\right), 1\right\}, \left\{-\frac{1}{8} i \left(i + \sqrt{47}\right), 1\right\}\right\}$

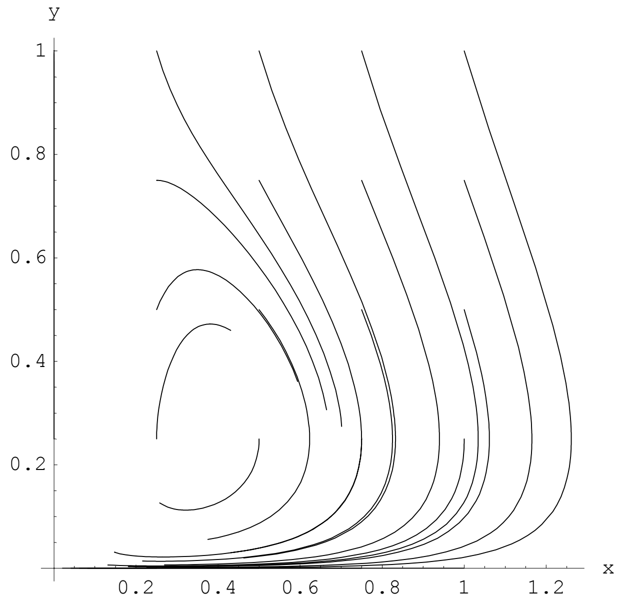
(iv) (b)

In[101]:=

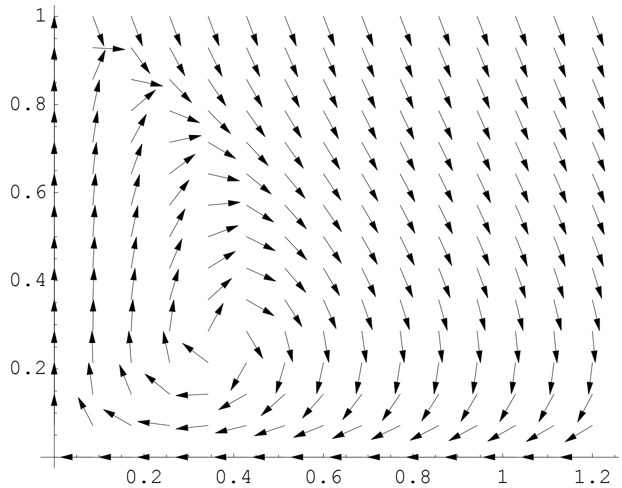
$$\texttt{nsol2[tval\_, a\_, b\_, t1\_] :=}\ \left(\{x[t], y[t]\} \text{ /. First}\left[\text{NDSolve}\left[\left\{x'[t] == -\frac{1}{2} x[t] + \frac{x[t]\, y[t]}{\frac{1}{4} + y[t]},\ y'[t] == y[t] - y[t]^2 - \frac{x[t]\, y[t]}{\frac{1}{4} + y[t]},\ x[0] == a,\ y[0] == b\right\}, \{x[t], y[t]\}, \{t, 0, t1\}\right]\right]\right) \text{ /. } t \to tval$$

```
nphase2[t1_] := ParametricPlot[Evaluate[Flatten[
    Table[nsol2[t, a, b, t1], {a, 0, 1, .25}, {b, 0, 1, .25}], 1]],
  {t, 0, t1}, AspectRatio -> 1, AxesLabel -> {"x", "y"}]
```

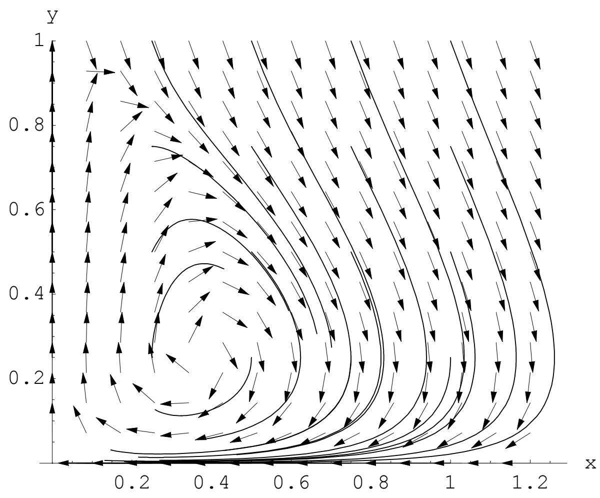
In[103]:= **lines2 = nphase2[5];**



In[104]:= **arrows2 = PlotVectorField**$\left[\left\{-\frac{1}{2}\,x+\frac{x\,y}{\frac{1}{4}+y},\ y-y^2-\frac{x\,y}{\frac{1}{4}+y}\right\}\right.$**, {x, 0, 1.2},**
{y, 0, 1}, Axes -> True, ScaleFunction -> (1 &), AspectRatio -> 0.8];



```
In[105]:= Show[arrows2, lines2, PlotRange -> {0, 1}, AxesLabel -> {"x", "y"}];
```



Which shows a limit cycle around the fixed point $(x, y) = (\frac{3}{8}, \frac{1}{4})$.

(i) (c)

```
In[106]:= Solve[{x - x^2 - x y == 0, y - 2 x y - 2 y^2 == 0}, {x, y}]

Out[106]= {{x → 0, y → 0}, {x → 0, y → 1/2}, {x → 1, y → 0}}
```

(ii) (c)

```
In[107]:= fx1 = D[x - x^2 - x y, x] /. {x -> 0, y -> 0}

Out[107]= 1

In[108]:= fy1 = D[x - x^2 - x y, y] /. {x -> 0, y -> 0}

Out[108]= 0

In[109]:= gx1 = D[y - 2 x y - 2 y^2, x] /. {x -> 0, y -> 0}

Out[109]= 0

In[110]:= gy1 = D[y - 2 x y - 2 y^2, y] /. {x -> 0, y -> 0}

Out[110]= 1

In[111]:= matrixC1 = (1 0
                      0 1)

        General::spell : Possible spelling error: new symbol
           name "matrixC1" is similar to existing symbols {matrixA1, matrixB1}.

Out[111]= {{1, 0}, {0, 1}}

In[112]:= fx1 = D[x - x^2 - x y, x] /. {x -> 0, y -> 1/2}

Out[112]= 1/2
```

In[113]:= **fy1 = D[x - x^2 - x y, y] /. {x -> 0, y -> $\frac{1}{2}$}**

Out[113]= 0

In[114]:= **gx1 = D[y - 2 x y - 2 y^2, x] /. {x -> 0, y -> $\frac{1}{2}$}**

Out[114]= -1

In[115]:= **gy1 = D[y - 2 x y - 2 y^2, y] /. {x -> 0, y -> $\frac{1}{2}$}**

Out[115]= -1

In[116]:= **matrixC2 =** $\begin{pmatrix} \frac{1}{2} & 0 \\ -1 & -1 \end{pmatrix}$

```
General::spell : Possible spelling error: new symbol
   name "matrixC2" is similar to existing symbols {matrixA2, matrixB2}.
```

Out[116]= {{$\frac{1}{2}$, 0}, {-1, -1}}

In[117]:= **fx1 = D[x - x^2 - x y, x] /. {x -> 1, y -> 0}**

Out[117]= -1

In[118]:= **fy1 = D[x - x^2 - x y, y] /. {x -> 1, y -> 0}**

Out[118]= -1

In[119]:= **gx1 = D[y - 2 x y - 2 y^2, x] /. {x -> 1, y -> 0}**

Out[119]= 0

In[120]:= **gy1 = D[y - 2 x y - 2 y^2, y] /. {x -> 0, y -> 0}**

Out[120]= 1

In[121]:= **matrixC3 =** $\begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}$

```
General::spell1 :
 Possible spelling error: new symbol name "matrixC3" is similar to existing symbol "matrixB3".
```

Out[121]= {{-1, -1}, {0, 1}}

(iii) (c)

In[122]:= **Eigenvalues[matrixC1]**

Out[122]= {1, 1}

In[123]:= **Eigenvectors[matrixC1]**

Out[123]= {{0, 1}, {1, 0}}

In[124]:= **Eigenvalues[matrixC2]**

Out[124]= {-1, $\frac{1}{2}$}

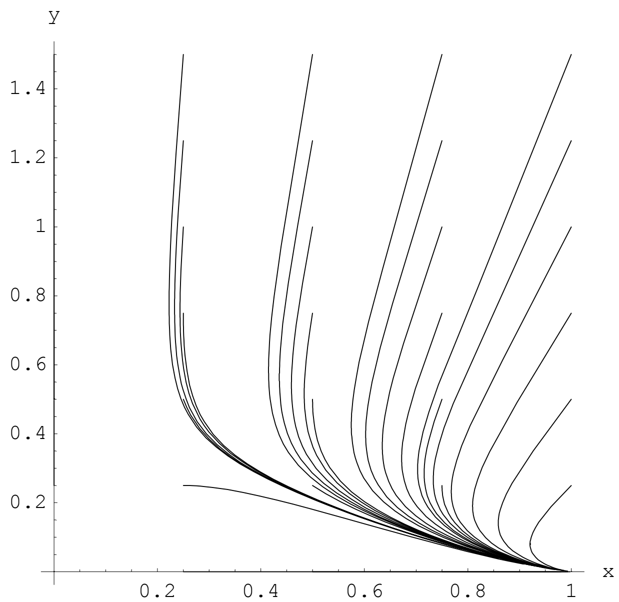
```
In[125]:= Eigenvectors[matrixC2]
```

Out[125]= $\left\{\{0, 1\}, \left\{-\frac{3}{2}, 1\right\}\right\}$

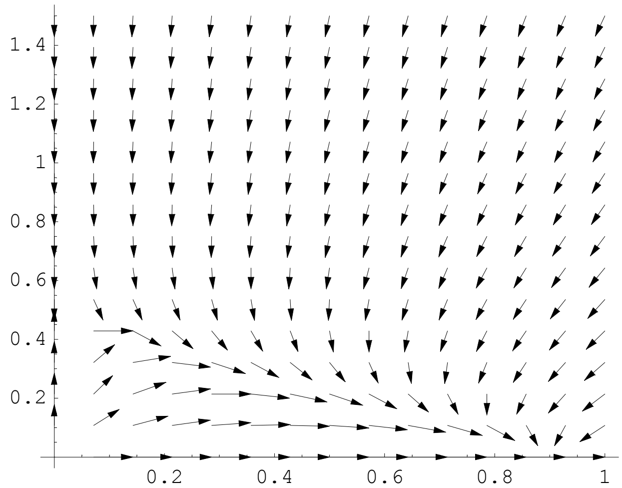
```
In[126]:= Eigenvalues[matrixC3]

Out[126]= {-1, 1}

In[127]:= Eigenvectors[matrixC3]

Out[127]= {{1, 0}, {-1, 2}}
```

(iv) (c)

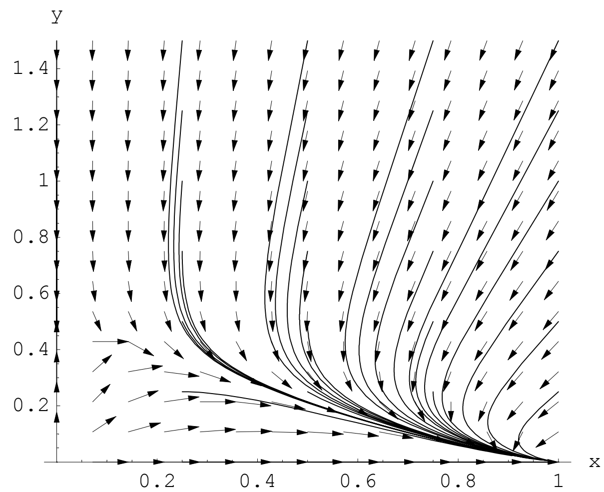
```
In[128]:= nsol3[tval_, a_, b_, t1_] :=
           ({x[t], y[t]} /. First[NDSolve[{x'[t] == x[t] - x[t]^2 - x[t] y[t],
                 y'[t] == y[t] - 2 x[t] y[t] - 2 y[t]^2, x[0] == a, y[0] == b},
                {x[t], y[t]}, {t, 0, t1}]]) /. t -> tval
          nphase3[t1_] := ParametricPlot[Evaluate[Flatten[
              Table[nsol3[t, a, b, t1], {a, 0, 1, .25}, {b, 0, 1.5, .25}], 1]],
            {t, 0, t1}, AspectRatio -> 1, AxesLabel -> {"x", "y"}]

In[130]:= lines3 = nphase3[5];
```



```
In[131]:= arrows3 = PlotVectorField[{x - x^2 - x y, y - 2 x y - 2 y^2}, {x, 0, 1},
           {y, 0, 1.5}, Axes -> True, ScaleFunction -> (1 &), AspectRatio -> 0.8];
```




```
In[132]:= Show[arrows3, lines3, PlotRange -> {0, 1.5}, AxesLabel -> {"x", "y"}];
```



Which shows competing predators at the fixed point $(x, y) = (1, 0)$.

à Question 13

(i)

Critical point found by solving

```
In[133]:= Solve[{0 == 1.4 (1 - y) x, 0 == 0.6 (1 - 4 y + x) y}, {x, y}]

Out[133]= {{x → 0., y → 0.}, {x → 0., y → 0.25}, {x → 3., y → 1.}}
```

(ii)

```
In[134]:= fx1 = D[1.4 (1 - y) x, x] /. {x -> 0, y -> 0}

Out[134]= 1.4
```

```
In[135]:= fy1 = D[1.4 (1 - y) x, y] /. {x -> 0, y -> 0}

Out[135]= 0

In[136]:= gx1 = D[0.6 (1 - 4 y + x) y, x] /. {x -> 0, y -> 0}

Out[136]= 0

In[137]:= gy1 = D[0.6 (1 - 4 y + x) y, y] /. {x -> 0, y -> 0}

Out[137]= 0.6
```

In[138]:= **matrixA1 =** $\begin{pmatrix} 1.4 & 0 \\ 0 & 0.6 \end{pmatrix}$

```
Out[138]= {{1.4, 0}, {0, 0.6}}

In[139]:= fx1 = D[1.4 (1 - y) x, x] /. {x -> 0, y -> 0.25}

Out[139]= 1.05

In[140]:= fy1 = D[1.4 (1 - y) x, y] /. {x -> 0, y -> 0.25}

Out[140]= 0

In[141]:= gx1 = D[0.6 (1 - 4 y + x) y, x] /. {x -> 0, y -> 0.25}

Out[141]= 0.15

In[142]:= gy1 = D[0.6 (1 - 4 y + x) y, y] /. {x -> 0, y -> 0.25}

Out[142]= -0.6
```

In[143]:= **matrixA2 =** $\begin{pmatrix} 1.05 & 0 \\ 0.15 & -0.6 \end{pmatrix}$

```
Out[143]= {{1.05, 0}, {0.15, -0.6}}

In[144]:= fx1 = D[1.4 (1 - y) x, x] /. {x -> 3, y -> 1}

Out[144]= 0

In[145]:= fy1 = D[1.4 (1 - y) x, y] /. {x -> 3, y -> 1}

Out[145]= -4.2

In[146]:= gx1 = D[0.6 (1 - 4 y + x) y, x] /. {x -> 3, y -> 1}

Out[146]= 0.6

In[147]:= gy1 = D[0.6 (1 - 4 y + x) y, y] /. {x -> 3, y -> 1}

Out[147]= -2.4
```

In[148]:= **matrixA3 =** $\begin{pmatrix} 0 & -4.2 \\ 0.6 & -2.4 \end{pmatrix}$

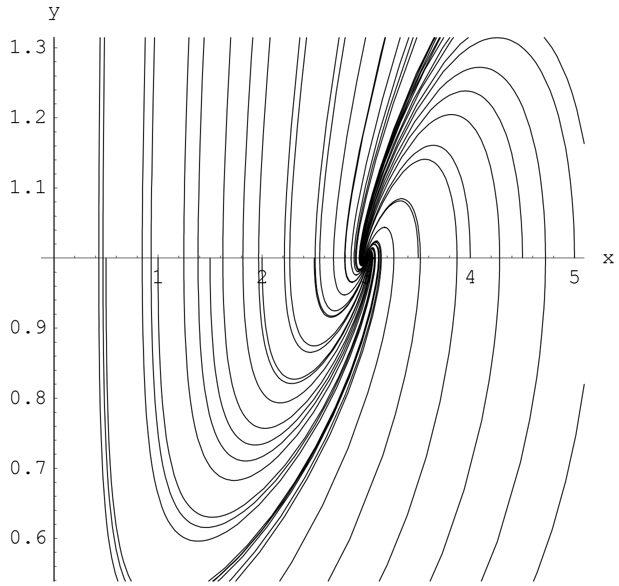
```
General::spell : Possible spelling error: new symbol
   name "matrixA3" is similar to existing symbols {matrixB3, matrixC3}.

Out[148]= {{0, -4.2}, {0.6, -2.4}}
```

(iii)

```
In[149]:= Eigenvalues[matrixA1]

Out[149]= {1.4, 0.6}

In[150]:= Eigenvectors[matrixA1]

Out[150]= {{1., 0.}, {0., 1.}}

In[151]:= Eigenvalues[matrixA2]

Out[151]= {1.05, -0.6}

In[152]:= Eigenvectors[matrixA2]

Out[152]= {{0.995893, 0.0905357}, {0., 1.}}

In[153]:= Eigenvalues[matrixA3]

Out[153]= {-1.2 + 1.03923 i, -1.2 - 1.03923 i}

In[154]:= Eigenvectors[matrixA3]

Out[154]= {{0.935414 + 0. i, 0.267261 - 0.231455 i},
            {0.935414 + 0. i, 0.267261 + 0.231455 i}}

In[155]:= nsol13[tval_, a_, b_, t1_] :=
            ({x[t], y[t]} /. First[NDSolve[{x'[t] == 1.4 (1 - y[t]) x[t],
                  y'[t] == 0.6 (1 - 4 y[t] + x[t]) y[t], x[0] == a, y[0] == b},
                {x[t], y[t]}, {t, 0, t1}]]) /. t -> tval
          nphase13[t1_] := ParametricPlot[Evaluate[
              Flatten[Table[nsol13[t, a, b, t1], {a, 0, 6, .5}, {b, 0, 2, .5}], 1]],
             {t, 0, t1}, AspectRatio -> 1, AxesLabel -> {"x", "y"}]

      General::spell1 :
       Possible spelling error: new symbol name "nsol13" is similar to existing symbol "nsol3".

      General::spell1 :
       Possible spelling error: new symbol name "nphase13" is similar to existing symbol "nphase3".
```

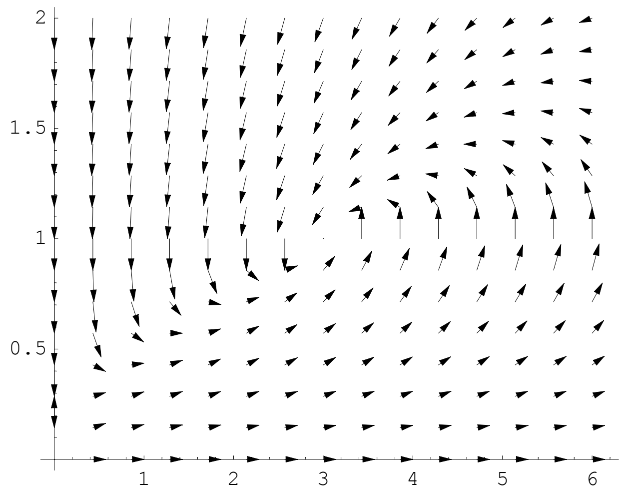
In[157]:= **lines13 = nphase13[5];**

```
General::spell1 :
 Possible spelling error: new symbol name "lines13" is similar to existing symbol "lines3".
```

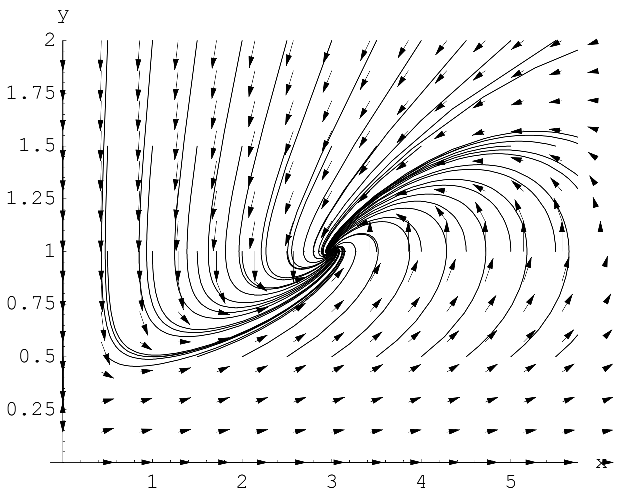


In[158]:= **arrows13 = PlotVectorField[{1.4 (1 - y) x, 0.6 (1 - 4 y + x) y}, {x, 0, 6}, {y, 0, 2}, Axes -> True, ScaleFunction -> (1 &), AspectRatio -> 0.8];**

```
General::spell1 :
 Possible spelling error: new symbol name "arrows13" is similar to existing symbol "arrows3".
```



```
In[159]:= Show[arrows13, lines13, PlotRange -> {0, 2}, AxesLabel -> {"x", "y"}];
```



Illustrates that the system is dominated by equilibrium point $(x, y) = (3, 1)$.

(iv)

A spreadsheet investigation of the system around point $(x, y) = (3, 1)$ reveals quite a different picture from the one above. There are two reasons for this. First, in the above analysis we have assumed the system is continuous. Discrete systems do not always behave in the same way that continuous systems behave, especially when they are non-linear. Second, the above stability was constructed from a linear approximation in the neighbourhood of the point $(x, y) = (3, 1)$. How small such a neighbourhood needs to be is not clear. This example illustrates the danger of using continuous linear approximations for non-linear discrete systems.

à Question 14

Subtracting x_t from the first equation and y_t from the second we obtain

$$\Delta x_t = x_{t+1} - x_t = 0.3\, x_t - 0.3\, x_t^2 - 0.15\, x_t\, y_t$$
$$\Delta y_t = y_{t+1} - y_t = 0.3\, y_t - 0.3\, y_t^2 - 0.15\, x_t\, y_t$$

(i)

In[160]:= $\text{Solve}\left[\left\{x\left(\frac{3}{10} - \frac{3}{10}x - \frac{3}{20}y\right) == 0,\ y\left(\frac{3}{10} - \frac{3}{10}y - \frac{3}{20}x\right) == 0\right\}, \{x, y\}\right]$

Out[160]= $\left\{\{x \to 0, y \to 0\}, \{x \to 0, y \to 1\}, \left\{x \to \frac{2}{3}, y \to \frac{2}{3}\right\}, \{x \to 1, y \to 0\}\right\}$

(ii)

In[161]:= $\text{fx1} = \text{D}\left[x\left(\frac{3}{10} - \frac{3}{10}x - \frac{3}{20}y\right), x\right]$ /. {x -> 0, y -> 0}

Out[161]= $\frac{3}{10}$

In[162]:= $\text{fy1} = \text{D}\left[x\left(\frac{3}{10} - \frac{3}{10}x - \frac{3}{20}y\right), y\right]$ /. {x -> 0, y -> 0}

Out[162]= 0

In[163]:= $gx1 = D\left[y\left(\frac{3}{10} - \frac{3}{10}y - \frac{3}{20}x\right), x\right] /. \{x \to 0, y \to 0\}$

Out[163]= 0

In[164]:= $gy1 = D\left[y\left(\frac{3}{10} - \frac{3}{10}y - \frac{3}{20}x\right), y\right] /. \{x \to 0, y \to 0\}$

Out[164]= $\frac{3}{10}$

In[165]:= $matrixA1 = \begin{pmatrix} \frac{3}{10} & 0 \\ 0 & \frac{3}{10} \end{pmatrix}$

Out[165]= $\left\{\left\{\frac{3}{10}, 0\right\}, \left\{0, \frac{3}{10}\right\}\right\}$

In[166]:= $fx1 = D\left[x\left(\frac{3}{10} - \frac{3}{10}x - \frac{3}{20}y\right), x\right] /. \{x \to 0, y \to 1\}$

Out[166]= $\frac{3}{20}$

In[167]:= $fy1 = D\left[x\left(\frac{3}{10} - \frac{3}{10}x - \frac{3}{20}y\right), y\right] /. \{x \to 0, y \to 1\}$

Out[167]= 0

In[168]:= $gx1 = D\left[y\left(\frac{3}{10} - \frac{3}{10}y - \frac{3}{20}x\right), x\right] /. \{x \to 0, y \to 1\}$

Out[168]= $-\frac{3}{20}$

In[169]:= $gy1 = D\left[y\left(\frac{3}{10} - \frac{3}{10}y - \frac{3}{20}x\right), y\right] /. \{x \to 0, y \to 1\}$

Out[169]= $-\frac{3}{10}$

In[170]:= $matrixA2 = \begin{pmatrix} \frac{3}{20} & 0 \\ -\frac{3}{20} & -\frac{3}{10} \end{pmatrix}$

Out[170]= $\left\{\left\{\frac{3}{20}, 0\right\}, \left\{-\frac{3}{20}, -\frac{3}{10}\right\}\right\}$

In[171]:= $fx1 = D\left[x\left(\frac{3}{10} - \frac{3}{10}x - \frac{3}{20}y\right), x\right] /. \left\{x \to \frac{2}{3}, y \to \frac{2}{3}\right\}$

Out[171]= $-\frac{1}{5}$

In[172]:= $fy1 = D\left[x\left(\frac{3}{10} - \frac{3}{10}x - \frac{3}{20}y\right), y\right] /. \left\{x \to \frac{2}{3}, y \to \frac{2}{3}\right\}$

Out[172]= $-\frac{1}{10}$

In[173]:= $gx1 = D\left[y\left(\frac{3}{10} - \frac{3}{10}y - \frac{3}{20}x\right), x\right] /. \left\{x \to \frac{2}{3}, y \to \frac{2}{3}\right\}$

Out[173]= $-\frac{1}{10}$

In[174]:= $gy1 = D\left[y\left(\frac{3}{10} - \frac{3}{10}y - \frac{3}{20}x\right), y\right] /. \left\{x \to \frac{2}{3}, y \to \frac{2}{3}\right\}$

Out[174]= $-\frac{1}{5}$

In[175]:= $matrixA3 = \begin{pmatrix} -\frac{1}{5} & -\frac{1}{10} \\ -\frac{1}{10} & -\frac{1}{5} \end{pmatrix}$

Out[175]= $\left\{\left\{-\frac{1}{5}, -\frac{1}{10}\right\}, \left\{-\frac{1}{10}, -\frac{1}{5}\right\}\right\}$

In[176]:= $fx1 = D\left[x\left(\frac{3}{10} - \frac{3}{10}x - \frac{3}{20}y\right), x\right] /. \{x \to 1, y \to 0\}$

Out[176]= $-\frac{3}{10}$

In[177]:= $fy1 = D\left[x\left(\frac{3}{10} - \frac{3}{10}x - \frac{3}{20}y\right), y\right] /. \{x \to 1, y \to 0\}$

Out[177]= $-\frac{3}{20}$

In[178]:= $gx1 = D\left[y\left(\frac{3}{10} - \frac{3}{10}y - \frac{3}{20}x\right), x\right] /. \{x \to 1, y \to 0\}$

Out[178]= 0

In[179]:= $gy1 = D\left[y\left(\frac{3}{10} - \frac{3}{10}y - \frac{3}{20}x\right), y\right] /. \{x \to 1, y \to 0\}$

Out[179]= $\frac{3}{20}$

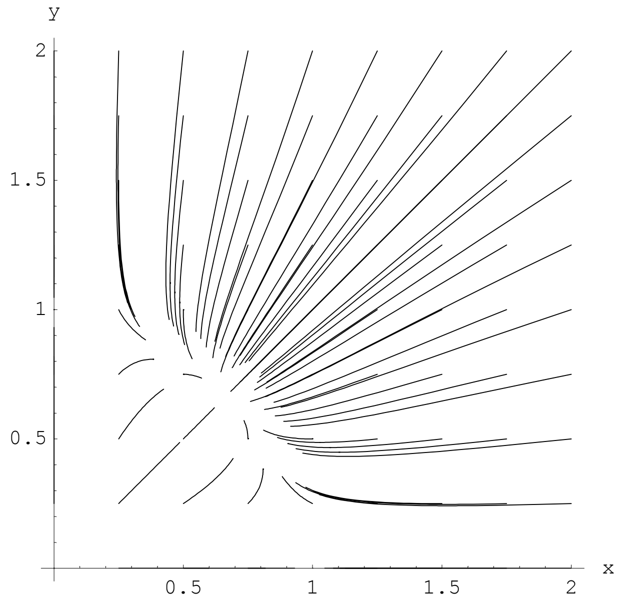
In[180]:= $matrixA4 = \begin{pmatrix} -\frac{3}{10} & -\frac{3}{20} \\ 0 & \frac{3}{20} \end{pmatrix}$

Out[180]= $\left\{\left\{-\frac{3}{10}, -\frac{3}{20}\right\}, \left\{0, \frac{3}{20}\right\}\right\}$

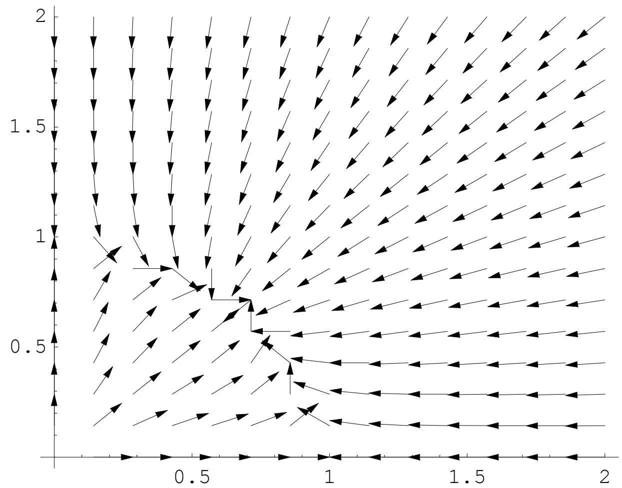
(iii)

In[181]:=
```
nsol14[tval_, a_, b_, t1_] :=
  ({x[t], y[t]} /. First[NDSolve[{x'[t] == x[t] (0.3 - 0.3 x[t] - 0.15 y[t]),
        y'[t] == y[t] (0.3 - 0.3 y[t] - 0.15 x[t]), x[0] == a, y[0] == b},
       {x[t], y[t]}, {t, 0, t1}]]) /. t -> tval
nphase14[t1_] := ParametricPlot[Evaluate[
    Flatten[Table[nsol14[t, a, b, t1], {a, 0, 2, .25}, {b, 0, 2, .25}], 1]],
   {t, 0, t1}, AspectRatio -> 1, AxesLabel -> {"x", "y"}]
```

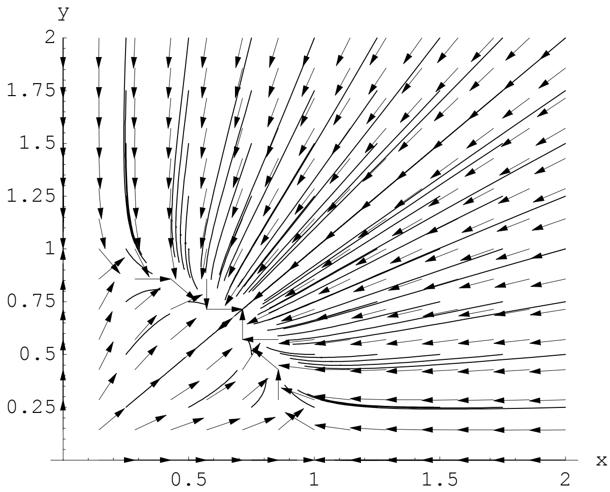
```
In[183]:= lines14 = nphase14[5];
```




```
In[184]:= arrows14 =
   PlotVectorField[{x (0.3 - 0.3 x - 0.15 y), y (0.3 - 0.3 y - 0.15 x)}, {x, 0, 2},
    {y, 0, 2}, Axes -> True, ScaleFunction -> (1 &), AspectRatio -> 0.8];
```



```
In[185]:= Show[arrows14, lines14, PlotRange -> {0, 2}, AxesLabel -> {"x", "y"}];
```



à Question 15

■ (i)

The system takes the form

$$x_1(t) = 0.42\, x_3(t)$$
$$x_2(t) = 0.6\, x_1(t)$$
$$x_3(t) = 0.75\, x_2(t) + 0.95\, x_3(t)$$

■ (iii)

```
In[186]:= mA = {{0, 0, 0.42}, {0.6, 0, 0}, {0, 0.75, 0.95}}

Out[186]= {{0, 0, 0.42}, {0.6, 0, 0}, {0, 0.75, 0.95}}

In[187]:= Eigenvalues[mA]

Out[187]= {1.10483, -0.0774172 + 0.406292 i, -0.0774172 - 0.406292 i}
```

Note also,

```
In[188]:= R = Sqrt[(-0.0774172)^2 + 0.406292^2]

Out[188]= 0.413602
```

So the system is stable.

■ (iv)

From eigenvalue 1.10483 we obtain the growth rate of the bison population of 10.5%.

■ (v)

```
In[189]:= Eigenvectors[mA]

Out[189]= {{0.348901, 0.189477, 0.917805},
           {-0.0927328 + 0.486669 ⅈ, 0.718699 + 0. ⅈ, -0.453692 - 0.179412 ⅈ},
           {-0.0927328 - 0.486669 ⅈ, 0.718699 + 0. ⅈ, -0.453692 + 0.179412 ⅈ}}

In[190]:= sumeig = 0.348901 + 0.189477 + 0.917805

Out[190]= 1.45618

In[191]:= {0.348901, 0.189477, 0.917805} / sumeig

Out[191]= {0.2396, 0.130119, 0.630281}
```

Hence, calves settle down to 24%, yearlings to 13% and adults to 63%.