The nonvanishing magnetic field components $\mathcal{H}_x$ and $\mathcal{H}_z$ of the TE mode are found from $\mathcal{E}_y$ by using (3.83) and (3.84), respectively. The mode field $\mathcal{E}_y$ in (3.113) is not normalized because it extends to infinity in both positive and negative x directions. For all x, $\mathcal{E}_y$ in (3.113) is the superposition of the incident, reflected, and transmitted fields resulting from two incident waves: one from medium 1 that has a field amplitude of $\boldsymbol{\mathcal{E}}_{i1} = \hat{y}\cos\psi_2 e^{i\psi_1}/2$ and a wavevector of $\mathbf{k}_{i1} = -h_1\hat{x} + \beta\hat{z}$, and the other from medium 2 that has $\boldsymbol{\mathcal{E}}_{i2} = \hat{y}\cos\psi_1 e^{i\psi_2}/2$ and $\mathbf{k}_{i2} = h_2\hat{x} + \beta\hat{z}$. Note that (3.114) eliminates one free phase parameter so that the phase relation between the two incident waves in the composition of the TE mode field is determined.

For the TM mode, the $\mathcal{H}_y$ field profile satisfying the boundary conditions that $\mathcal{H}_y$ and $n^{-2}\partial\mathcal{H}_y/\partial x$ are continuous at $x = 0$ is

$$\mathcal{H}_y(x) = \begin{cases} \cos\psi_2 \cos(h_1 x - \psi_1), & x > 0, \\ \cos\psi_1 \cos(h_2 x - \psi_2), & x < 0, \end{cases} \tag{3.115}$$

where the two phase factors ψ_1 and ψ_2 are related by

$$\frac{h_1}{n_1^2}\tan\psi_1 = \frac{h_2}{n_2^2}\tan\psi_2. \tag{3.116}$$

The nonvanishing electric field components $\mathcal{E}_x$ and $\mathcal{E}_z$ of the TM mode are found from $\mathcal{H}_y$ by using (3.85) and (3.86), respectively. The mode field $\mathcal{H}_y$ in (3.115) is not normalized because it extends to infinity in both positive and negative x directions. For all x, $\mathcal{H}_y$ in (3.115) is the superposition of the incident, reflected, and transmitted fields resulting from two incident waves: one from medium 1 that has a field amplitude of $\boldsymbol{\mathcal{H}}_{i1} = \hat{y}\cos\psi_2 e^{i\psi_1}/2$ and a wavevector of $\mathbf{k}_{i1} = -h_1\hat{x} + \beta\hat{z}$, and the other from medium 2 that has $\boldsymbol{\mathcal{H}}_{i2} = \hat{y}\cos\psi_1 e^{i\psi_2}/2$ and $\mathbf{k}_{i2} = h_2\hat{x} + \beta\hat{z}$. The relation in (3.116) eliminates one free phase parameter so that the phase relation between the two incident waves in the composition of the TM mode field is determined.

EXAMPLE 3.11

The glass plate with a refractive index of 1.5 at the $\lambda = 1$ μm wavelength given in Example 3.10 is now immersed in water, which has a refractive index of 1.33. Find the parameters of the radiation modes at the water–glass interface corresponding to internal reflection at the two different incident angles of 45° and 75°, respectively. What is the penetration depth of the evanescent tail into the water if a radiation mode is found to be a one-sided radiation mode at a particular incident angle? What are the phase shifts on reflection at the interface for TE and TM waves, respectively?

Solution:

In this problem, $n_1 = 1.5$ and $n_2 = 1.33$ so that the critical angle of the interface is $\theta_c = \sin^{-1}(1.33/1.5) = 62.5°$ and the Brewster angle for internal reflection is $\theta_B = \tan^{-1}(1.33/1.5) = 41.6° < \theta_c$. At $\lambda = 1$ μm,

$$k_1 = \frac{2\pi n_1}{\lambda} = 9.42 \times 10^6\ \text{m}^{-1} \quad \text{and} \quad k_2 = \frac{2\pi n_2}{\lambda} = 8.36 \times 10^6\ \text{m}^{-1}.$$