Chapter 1: Introduction to Signals

Problem 1.1:

i) $z[m,n,k]$ is a three dimensional (3D) DT signal. The independent variables are given by m, n, and k, while z is the dependent variable. Digital video is an example of a 3D DT signal of the form $z[m,n,k]$. The intensity z of the pixels in a frame is a function of the spatial coordinates (m,n) and frame number k.

ii) $I(x,y,z,t)$ is a four dimensional (4D) CT signal. The independent variables are given by x, y, z, and t, while I is the dependent variable. Atmospheric pressure is an example of a 4D DT signal of the form $I(x,y,z,t)$ if recorded continuously in time and space. The atmospheric pressure I is a function of the spatial coordinates (x,y,z) and time t. ■

Problem 1.2:

The CT signals can be plotted using the following MATLAB code. The CT signals are plotted in Fig. S1.2. The students should also try plotting them by hand. ■

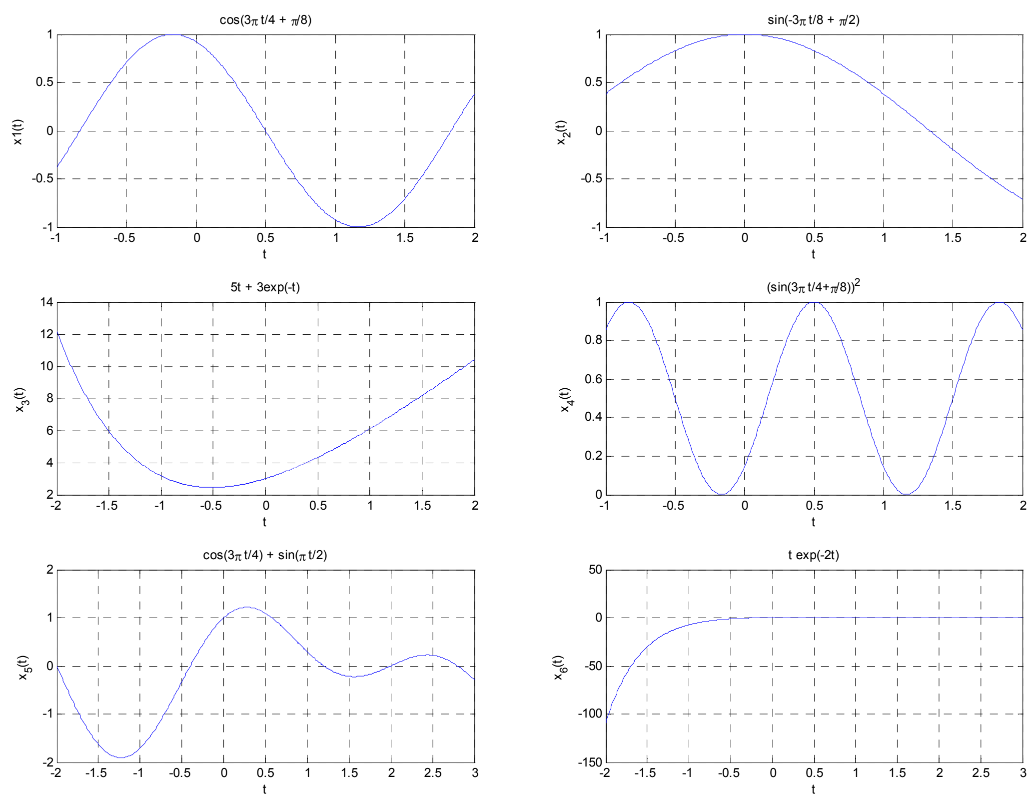


Fig S1.2: CT signals plotted for Problem 1.2.

```
% MATLAB code for Problem 1.2
clf                                       % clear figure
% signal defined in part (i)
t1 =-1:0.01:2  ;
x1 = cos(3*pi*t1/4+pi/8) ;
subplot(3,2,1), plot(t1, x1), grid on;
xlabel('t');                              % Label of X-axis
ylabel('x1(t)');                          % Label of Y-axis
title('cos(3\pi t/4 + \pi/8)');           % Title

% signal defined in part (ii)
t2 =-1:0.01:2  ;
x2 = sin(-3*pi*t2/8+pi/2) ;
subplot(3,2,2), plot(t2, x2), grid on;
xlabel('t');                              % Label of X-axis
ylabel('x_2(t)');                         % Label of Y-axis
title('sin(-3\pi t/8 + \pi/2)');          % Title

% signal defined in part (iii)
t3 =-2:0.01:2 ;
x3 = 5*t3+ 3*exp(-t3);
subplot(3,2,3), plot(t3, x3), grid on
xlabel('t');                              % Label of X-axis
ylabel('x_3(t)');                         % Label of Y-axis
title('5t + 3exp(-t)');                   % Title

% signal defined in part (iv)
t4 =-1:0.01:2;
x4 = sin(3*pi*t4/4+pi/8);
x4 =x4.*x4;
subplot(3,2,4), plot(t4, x4), grid on;
xlabel('t');                              % Label of X-axis
ylabel('x_4(t)');                         % Label of Y-axis
title('(sin(3\pi t/4+\pi/8))^2');         % Title


% signal defined in part (v)
t5 =-2:0.01:3  ;
x5 = cos(3*pi*t5/4) + sin(pi*t5/2);
subplot(3,2,5), plot(t5, x5), grid on;
xlabel('t');                              % Label of X-axis
ylabel('x_5(t)');                         % Label of Y-axis
title('cos(3\pi t/4) + sin(\pi t/2)');    % Title


% signal defined in part (vi)
t6 =-2:0.01:3 ;
x6 = t6.*exp(-2*t6) ;
subplot(3,2,6), plot(t6, x6), grid on;
xlabel('t');                              % Label of X-axis
```

```
ylabel('x_6(t)');                         % Label of Y-axis
title('t exp(-2t)');                      % Title

print -dtiff plot.tiff;                   % Save the figure as a TIFF
                                          file
```

Problem 1.3:

(i) The value of x1[k] for $-5 \le k \le 5$ is shown in the following table.

k	–5	–4	–3	–2	–1	0	1	2	3	4	5
$x1[k]$	0.38	–0.92	0.92	–0.38	–0.38	0.92	–0.92	0.38	0.38	–0.92	0.92

The sketch of x1[k] is shown in the top left figure in Fig. S1.3.

The other functions can be plotted in a similar way. However, we use MATLAB to plot the six DT. Fig. S1.3 contains the subplots for these sequences followed by the MATLAB code used to generate them. ∎

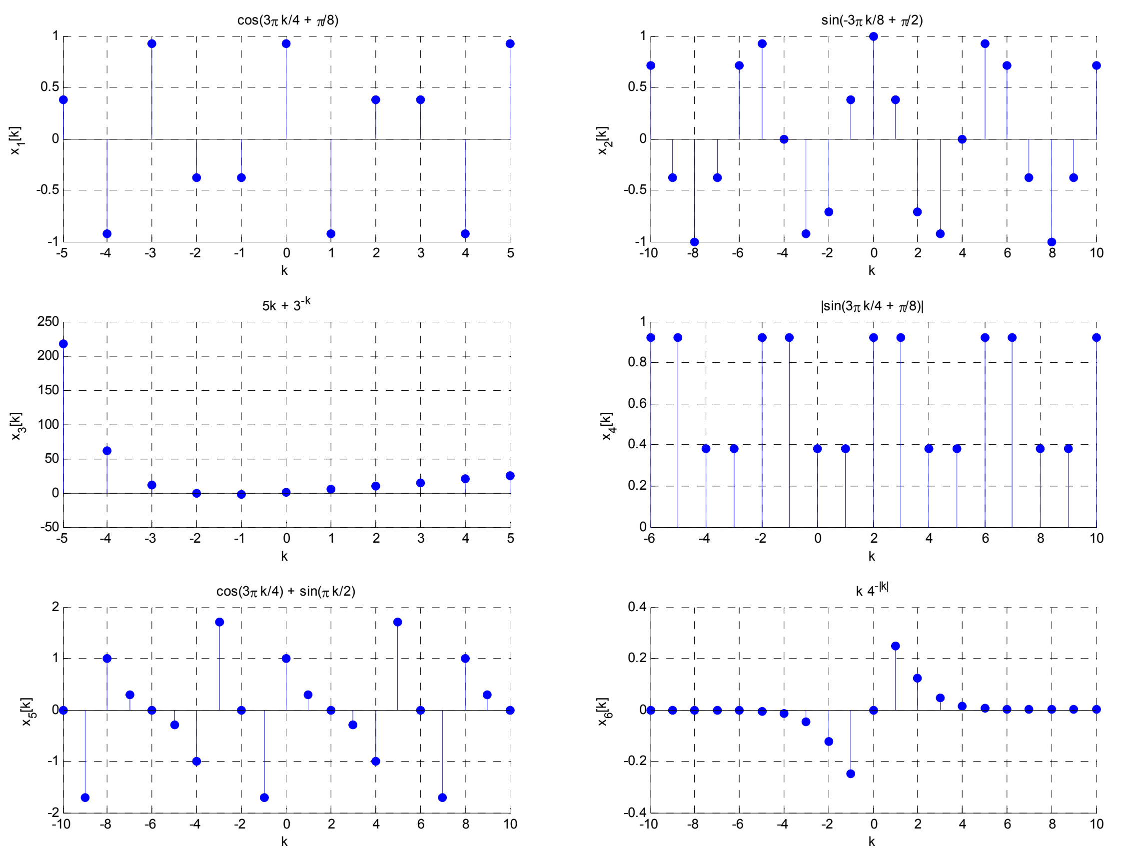


Fig S1.3: DT signals for P1.3

```
% MATLAB code for Problem 1.3
clf                                       % clear figure
% signal defined in part (i)
k1 =-5:5 ;
```

```
x1 = cos(3*pi*k1/4+pi/8);
subplot(3,2,1), stem(k1, x1, 'filled'),
grid on;                                  % Label of X-axis
xlabel('k');                              % Label of Y-axis
ylabel('x_1[k]');                         % Title
title(' cos(3\pi k/4 + \pi/8)');

% signal defined in part (ii)
k2 =-10:10  ;
x2 = sin(-3*pi*k2/8+pi/2);
subplot(3,2,2), stem(k2, x2, 'filled'),  % Label of X-axis
grid on;                                  % Label of Y-axis
xlabel('k');                              % Title
ylabel('x_2[k]');
title('sin(-3\pi k/8 + \pi/2)');

% signal defined in part (iii)
k3 =-5:5  ;
x3 = 5*k3+ 3.^(-k3);                      % Label of X-axis
subplot(3,2,3), stem(k3, x3, 'filled'),  % Label of Y-axis
grid on;                                  % Title
xlabel('k');
ylabel('x_3[k]');
title('5k + 3^{-k}');

% signal defined in part (iv)
k4 =-6:10  ;                              % Label of X-axis
x4 = abs(sin(3*pi*k4/4+pi/8)) ;           % Label of Y-axis
subplot(3,2,4), stem(k4, x4, 'filled'),  % Title
grid on;
xlabel('k');
ylabel('x_4[k]');

title('|sin(3\pi k/4 + \pi/8)|');
axis([-6  10  0  1]);
                                          % Label of X-axis
% signal defined in part (v)              % Label of Y-axis
k5 =-10:10  ;                             % Title
x5 = cos(3*pi*k5/4) + sin(pi*k5/2) ;
subplot(3,2,5), stem(k5, x5, 'filled'),
grid on;
xlabel('k');

ylabel('x_5[k]');                         % Label of X-axis
                                          % Label of Y-axis
title('cos(3\pi k/4) + sin(\pi k/2)');    % Title

% signal defined in part (vi)
k6 =-10:10  ;
x6 = k6.*4.^(-abs(k6)) ;
subplot(3,2,6), stem(k6, x6, 'filled'),
grid;
```

```
xlabel('k');

ylabel('x_6[k]');
title('k 4^{-|k|}');
```

Problem 1.4:

Because $x_1(t)$ has a fundamental period of T_1, and $x_2(t)$ has a fundamental period of T_2,

$$x_1(t) = x_1(t+T_1) \quad \text{and} \quad x_2(t) = x_2(t+T_2).$$

Evaluating the $g(t+nT_1)$, we obtain,

$$g(t+nT_1) = ax_1(t+nT_1) + bx_2(t+nT_1) = ax_1(t+nT_1) + bx_2(t+mT_2) = ax_1(t) + bx_2(t) = g(t),$$

which proves that $g(t)$ is periodic with period nT_1.

Problem 1.5:

(i) All CT sinusoidal signals are periodic. The function $x1(t)$ can be simplified as follows:

$$x1(t) = \sin\left(-5\pi t/8 + \pi/2\right) = \sin\left(\pi/2 - 5\pi t/8\right) = \cos\left(5\pi t/8\right) = \cos\left(\omega_0 t\right), \omega_0 = 5\pi/8.$$

Therefore, $x1(t)$ is periodic with fundamental period

$$T_1 = \tfrac{2\pi}{\omega_0} = \tfrac{2\pi}{5\pi/8} = \tfrac{16}{5}.$$

(ii) $x2(t) = \left|\sin(-5\pi t/8 + \pi/2)\right| = \left|\cos(5\pi t/8)\right|$.

The signal $x2(t + T)$ can be simplified as follows:

$$\begin{aligned} x2(t+T) &= \left|\cos(5\pi t/8 + 5\pi T/8)\right| \\ &= \left|\cos(5\pi t/8)\right| = x2(t) \quad \text{if} \quad 5\pi T/8 = \pi \text{ or if } T = 8/5 \end{aligned}$$

In other words, $x2(t)$ is periodic with $T_2 = 8/5$.

(iii) Looking at the individual terms

$$x3(t) = \underbrace{\sin\left(6\pi t/7\right)}_{\substack{periodic \\ T_1 = \frac{2\pi}{6\pi/7} = \frac{7}{3}}} + 2\underbrace{\cos\left(3t/5\right)}_{\substack{periodic \\ T_2 = \frac{2\pi}{3/5} = \frac{10\pi}{3}}}$$

Because $\dfrac{T_1}{T_2} = \dfrac{7/3}{10\pi/3} = \dfrac{7}{10\pi} \neq \text{rational number}$, $x_3(t)$ is not a periodic signal.

(iv) All CT complex exponentials are periodic.

Therefore $x4(t) = \exp\left(j\left(5t + \pi/4\right)\right)$ is also periodic with fundamental period $T_4 = \frac{2\pi}{5}$.

(v) Looking at the individual terms

$$x5(t) = \underbrace{\exp\left(j\,3\pi t/8\right)}_{\substack{periodic \\ T_1=\frac{2\pi}{3\pi/8}=\frac{16}{3}}} + \underbrace{\exp\left(\pi t/86\right)}_{not\ periodic}$$

We observe that the second term is not periodic. Therefore, the overall function $x5(t)$ is not periodic.

(vi) The function $x6(t)$ can be simplified as follows

$$\begin{aligned} x6(t) &= 2\cos\left(\tfrac{4\pi t}{5}\right) * \sin^2\left(\tfrac{16t}{3}\right) = 2\cos\left(\tfrac{4\pi t}{5}\right)\times\tfrac{1}{2}\left(1-\cos\left(\tfrac{32t}{3}\right)\right) \\ &= \cos\left(\tfrac{4\pi t}{5}\right) - \cos\left(\tfrac{4\pi t}{5}\right)\cos\left(\tfrac{32t}{3}\right) = \cos\left(\tfrac{4\pi t}{5}\right) - \tfrac{1}{2}\left[\cos\left(\tfrac{4\pi}{5}-\tfrac{32}{3}\right)t + \cos\left(\tfrac{4\pi}{5}+\tfrac{32}{3}\right)t\right] \\ &= \underbrace{\cos\left(\tfrac{4\pi t}{5}\right)}_{\substack{periodic \\ T_1=\frac{5}{2}}} - \tfrac{1}{2}\underbrace{\cos\left(\tfrac{12\pi-160}{15}t\right)}_{\substack{periodic \\ T_2=\frac{30\pi}{12\pi-160}}} - \tfrac{1}{2}\underbrace{\cos\left(\tfrac{12\pi+160}{15}t\right)}_{\substack{periodic \\ T_3=\frac{30\pi}{12\pi+160}}} \end{aligned}$$

$x6(t)$ will be periodic if all possible combinations T_1 / T_2, T_1 / T_3, and T_2 / T_3 are rational numbers.

Since

$$\frac{T_1}{T_2} = \tfrac{5}{2}\times\tfrac{12\pi-160}{30\pi} = \tfrac{12\pi-160}{12\pi} = 1 - \tfrac{40}{3\pi} \neq \text{rational number},$$

$x6(t)$ is not a periodic signal.

(vii) $x7(t) = \underbrace{1}_{\text{constant}} + \underbrace{\sin 20t}_{\substack{periodic \\ T_1=\frac{2\pi}{20}=\frac{\pi}{10}}} + \underbrace{\cos(30t+\pi/3)}_{\substack{periodic \\ T_2=\frac{2\pi}{30}=\frac{\pi}{15}}}$

Since

$$\frac{T_1}{T_2} = \tfrac{\pi}{10}\times\tfrac{15}{\pi} = \tfrac{3}{2} = \text{rational number},$$

$x7(t)$ is periodic. The fundamental period of $x7(t)$ is $2T_1 = 3T_2 = \frac{\pi}{5}$. ▌

Problem 1.6:

(i) $x1[k] = 5\times(-1)^k = 5e^{j\pi k}$.

For the complex exponential term, $2\pi/\omega_0 = 2$, which is a rational number. Hence, $x1[k]$ is periodic with a period of $K_1 = 2m\pi/\omega_0 = 2$ by setting $m = 1$.

(ii) Considering the two terms separately in $x2[k]$,

$$x2[k] = \underbrace{\exp\left(j\left(7\pi k/4\right)\right)}_{\substack{\frac{2\pi}{\Omega}=\frac{2\pi}{7\pi/4}=\frac{8}{7}=rational, \\ periodic\ \text{signal}\ with\ K=8}} + \underbrace{\exp\left(j\left(3k/4\right)\right)}_{\substack{\frac{2\pi}{\Omega}=\frac{2\pi}{3/4}=\frac{8\pi}{3}\neq rational, \\ aperiodic\ \text{signal}}}$$

we note that the 2nd complex exponential term $\exp(j(3k/4))$ is not periodic. Signal $x2[k]$ is, therefore, not periodic.

(iii) Considering the two terms separately in $x3[k]$,

$$x3[k] = \underbrace{\exp\left(j\left(7\pi k/4\right)\right)}_{\substack{\frac{2\pi}{\Omega}=\frac{2\pi}{7\pi/4}=\frac{8}{7}=rational, \\ periodic\ \text{signal}\ with\ K=8}} + \underbrace{\exp\left(j\left(3\pi k/4\right)\right)}_{\substack{\frac{2\pi}{\Omega}=\frac{2\pi}{3\pi/4}=\frac{8}{3}=rational, \\ periodic\ \text{signal with}\ K=8}}$$

we note that both complex exponential terms are periodic with the same period $K = 8$. Signal $x3[k]$ is, therefore, periodic with an overall period of 8.

(iv) Considering the two terms separately in $x4[k]$,

$$x4[k] = \underbrace{\sin(3\pi k/8)}_{\substack{\frac{2\pi}{\Omega}=\frac{2\pi}{3\pi/8}=\frac{16}{3}=rational,\\ periodic\ \text{signal}\ with\ K=16}} + \underbrace{\cos(63\pi k/64)}_{\substack{\frac{2\pi}{\Omega}=\frac{2\pi}{63\pi/64}=\frac{128}{63}=rational,\\ periodic\ \text{signal}\ with\ K=128}}$$

we note that both complex exponential terms are periodic with two different period of 16 and 128. Since the ratio of the two periods is 1/8, a rational number, therefore, $x4[k]$ is a periodic signal. The fundamental period is given by $16n = 128m$, which equals 128 by setting $n = 8$ and $m = 1$.

(v) Considering the two terms separately in $x5[k]$,

$$x5[k] = \underbrace{\exp(j(7\pi k/4))}_{\substack{\frac{2\pi}{\Omega}=\frac{2\pi}{7\pi/4}=\frac{8}{7}=rational,\\ periodic\ \text{signal}\ with\ K=8}} + \underbrace{\cos(4\pi k/7+\pi)}_{\substack{\frac{2\pi}{\Omega}=\frac{2\pi}{4\pi/7}=\frac{7}{2}=rational,\\ periodic\ \text{signal}\ with\ K=7}}$$

we note that both complex exponential terms are periodic with two different period of 8 and 7. Since the ratio of the two periods is 8/7, a rational number, therefore, $x5[k]$ is a periodic signal. The fundamental period is given by $8n = 7m$, which equals 56 by setting $n = 7$ and $m = 8$.

(vi) Considering the two terms separately in $x6[k]$,

$$x6[k] = \sin(3\pi k/8)\cos(63\pi k/64) = \underbrace{\tfrac{1}{2}\sin(87\pi k/64)}_{\substack{2\pi/\Omega_1=128/87\Rightarrow \text{rational}\\ \text{periodic signal with } K=128}} - \underbrace{\tfrac{1}{2}\sin(39\pi k/64)}_{\substack{2\pi/\Omega_2=128/39\Rightarrow \text{rational}\\ \text{periodic signal with } K=128}}$$

we note that both complex exponential terms are periodic with the same period $K = 128$. Signal $x6[k]$ is, therefore, periodic with an overall period of 128. ∎

Problem 1.7:

(i) $$x1(t) = \cos(\pi t)\sin(3\pi t) = \underbrace{\tfrac{1}{2}\sin(4\pi t)}_{perioidic\ with\ T_0=1/2} + \underbrace{\tfrac{1}{2}\sin(2\pi t)}_{perioidic\ with\ T_0=1}$$

We note that x1(t) is periodic with the fundamental period $T = 1$. Since periodic signals are always power signals, $x1(t)$ is a power signal.

The total energy E_{x1} in $x1(t)$ is infinite.

Based on Problem 1.10, the average power in a sinusoidal signal $x(t) = A_1 \sin(\omega_1 t + \phi_1) + A_2 \sin(\omega_2 t + \phi_2)$ is given by $(A_1)^2/2 + (A_2)^2/2$ if $\omega_1 \neq \omega_2$. The average power in $x1(t)$ is, therefore, given by $1/8 + 1/8 = 1/4$.

(ii) For the CT signal $x2(t) = \exp(-2t)$,

the total energy and average power are given by

Total Energy: $$E_{x2} = \int_{-\infty}^{\infty} e^{-4t}dt = -\tfrac{1}{4}\left[e^{-4t}\right]_{-\infty}^{\infty} = \tfrac{1}{4}e^{4\infty} = \infty$$

Average Power: $$P_{x2} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} e^{-4t}dt = \lim_{T\to\infty}\frac{1}{2T}\left[\frac{e^{-4t}}{(-4)}\right]_{-T}^{T} = \lim_{T\to\infty}\frac{1}{8T}\left[e^{4T} - e^{-4T}\right].$$

Applying the L'Hopital's rule

$$P_{x2} = \lim_{T\to\infty} \tfrac{1}{8}\left[4e^{4T} + 4e^{-4T}\right] = \infty .$$

Since the signal has infinite energy and infinite power, the signal is neither an energy signal nor a power signal.

(iii) Since $x3(t)$ is a complex signal, the total energy and average power are given by

Energy: $$E_{x3} = \int_{-\infty}^{\infty} \left|e^{-j2t}\right| dt = \int_{-\infty}^{\infty} 1\, dt = \infty .$$

Power: $$P_{x3} = \lim_{T\to\infty} \tfrac{1}{2T} \int_{-T}^{T} \left|e^{-j2t}\right| dt = \lim_{T\to\infty} \tfrac{1}{2T} \int_{-T}^{T} 1\, dt = = \lim_{T\to\infty} \tfrac{1}{2T}\left[T-(-T)\right] = 1 .$$

The signal $x3(t)$ is a power signal.

(iv) The energy in $x4(t)$ is finite and given by

$$E_{x4} = \int_{-\infty}^{\infty} e^{-2t} u(t) dt = \left[\tfrac{e^{-2t}}{(-2)}\right]_0^{\infty} = -\tfrac{1}{2}\left[0-1\right] = \tfrac{1}{2} .$$

The average power is zero and $x4(t)$ is an energy signal.

(v) Since $x5(t)$ is a finite duration signal with finite magnitude, it must be an energy signal. The total energy in $x5(t)$ is given by

$$E_{x5} = \int_{-3}^{3} \cos^2(3\pi t) dt = \tfrac{1}{2} \int_{-3}^{3} \left[1+\cos(6\pi t)\right] dt = \tfrac{1}{2}\left[t + \tfrac{1}{6\pi}\sin(6\pi t)\right]_{-3}^{3} = 3.$$

The signal $x5(t)$ has finite (non-zero) energy, and hence is an energy signal. Average power P_{x5} in $x5(t)$ is zero.

(vi) Since $x6(t)$ is a finite duration signal with finite magnitude, it must be an energy signal. The total energy in $x6(t)$ is given by

$$E_{x6} = \int_{0}^{2} t^2 dt + \int_{2}^{4} (4-t)^2\, dt = \tfrac{t^3}{3}\Big|_0^2 - \tfrac{(4-t)^3}{3}\Big|_2^4 = \tfrac{8}{3} - \left[0 - \tfrac{8}{3}\right] = \tfrac{16}{3} .$$

Since $x6(t)$ has finite energy, it is an energy signal. Average power P_{x6} in $x6(t)$ is zero. ▌

Problem 1.8:

(i) $$x1[k] = \cos(\pi k/4)\sin(3\pi k/8) = \underbrace{\tfrac{1}{2}\sin(5\pi k/8)}_{\textit{perioidic with } N_0=16} + \underbrace{\tfrac{1}{2}\sin(\pi k/8)}_{\textit{perioidic with } N_0=16}$$

We note that $x1[k]$ is periodic with an overall period of $N_0 = 16$. Since periodic signals are always power signals, $x1[k]$ is a power signal. Based on Problem 1.10, the average power in a sinusoidal sequence $x[k] = A_1 \sin(\omega_1 k + \phi_1) + A_2 \sin(\omega_2 k + \phi_2)$ is given by $(A_1)^2/2 + (A_2)^2/2$ if $\omega_1 \neq \omega_2$. the average power is given by $P_{x1} = 1/4 + 1/4 = 1/2$. The total energy E_{x1} in $x1[t]$ is infinite.

(ii) Since $x2[k]$ is a finite duration signal of length 11 with finite magnitude, it must be an energy signal. The total energy in $x2[k]$ is calculated as follows.

$$E_{x2} = \sum_{k=-10}^{0}\cos^2(3\pi k/16) = \sum_{k=-10}^{0}\frac{1+\cos(3\pi k/8)}{2} = \sum_{k=-10}^{0}\tfrac{1}{2} + \sum_{k=-10}^{0}\frac{\cos(3\pi k/8)}{2}$$

$$= \tfrac{11}{2} + \tfrac{1}{4}\sum_{k=-10}^{0} e^{j3\pi k/8} + \tfrac{1}{4}\sum_{k=-10}^{0} e^{-j3\pi k/8}$$

Using the GP series, we obtain

$$\sum_{k=-10}^{0} e^{j3\pi k/8} = \frac{e^{-j30\pi/8}(1-e^{j33\pi/8})}{(1-e^{j3\pi/8})} = 0.3244 + j0.1344$$

and
$$\sum_{k=-10}^{0} e^{-j3\pi k/8} = \frac{e^{j30\pi/8}(1-e^{-j33\pi/8})}{(1-e^{-j3\pi/8})} = 0.3244 - j0.1344\ .$$

The total energy is, therefore, given by $E_{x2} = 5.5 + 0.1622 = 5.6622$.

The average power P_{x2} in $x2[k]$ is zero.

(iii) $\left|x3[k]\right| = \left|(-1)^k\right| = 1\ .$

We note that the signal $x3[k]$ is a power signal with an average power of 1. The total energy E_{x3} in $x3[k]$ is infinite.

(iv) $\left|x4[k]\right| = \left|\exp\left(j\left(\pi k/2 + \pi/8\right)\right)\right| = 1\ .$

We note that the signal $x4[k]$ is a power signal with an average power of 1. The total energy E_{x4} in $x4[k]$ is infinite.

(v) Since $x5[k]$ is a finite duration signal of length 16 with finite magnitude, it must be an energy signal. The total energy in $x5[k]$ is given by

$$E_{x5} = \sum_{k=0}^{10} 2^k + \sum_{k=11}^{15} 1 = \frac{(2^{11}-1)}{(2-1)} + 5 = 2052.$$

The average power P_{x5} in $x5[k]$ is zero. ▌

Problem 1.9:

The CT signal $x(t) = A\sin(\omega_0 t + \theta)$ is periodic with the fundamental period $T_0 = 2\pi/\omega_0$. Its average power is calculated as follows:

$$P_x = \tfrac{1}{T_0}\int_{T_0}|x(t)|^2\,dt = \tfrac{A^2}{T_0}\int_0^{T_0}\sin^2(\omega_0 t+\theta)dt = \tfrac{A^2}{2T_0}\int_0^{T_0}\left[1-\cos(2\omega_0 t+2\theta)\right]dt \qquad \left[\because \sin^2\theta = \tfrac{1}{2}\{1-\cos(2\theta)\}\right]$$

$$= \tfrac{A^2}{2T_0}\int_0^{T_0}dt - \tfrac{A^2}{2T_0}\int_0^{T_0}cos(2\omega_0 t+2\theta)dt = \tfrac{A^2}{2T_0}\times\left[T_0 - 0\right] - \tfrac{A^2}{2T_0}\times\tfrac{1}{2\omega_0}\left[\sin(2\omega_0 t+2\theta)\right]_0^{T_0}$$

$$= \tfrac{A^2}{2T_0}\times T_0 - \tfrac{A^2}{4\omega_0 T_0}\times\left[\sin(2\omega_0 T_0+2\theta)-\sin(2\theta)\right] = \tfrac{A^2}{2} - \tfrac{A^2}{4\omega_0 T_0}\times\left[\sin(4\pi+2\theta)-\sin(2\theta)\right] \qquad \left[\because T_0 = \tfrac{2\pi}{\omega_0},\ \ \omega_0 T_0 = 2\pi\right]$$

$$= \tfrac{A^2}{2} - \tfrac{A^2}{4\omega T_0}\times\left[\sin(2\theta)-\sin(2\theta)\right] = \tfrac{A^2}{2},$$

which proves the result. Note that the power of a sinusoid does not depend on its initial phase θ. ▮

Problem 1.10:

The CT signal $y(t) = A_1 \sin(\omega_1 t + \phi_1) + A_2 \sin(\omega_2 t + \phi_2)$ is the sum of two sinusoids and may not be always periodic. It is periodic only when ω_1/ω_2 is a rational number. To consider the general case, where $y(t)$ is not necessarily periodic, we will use the general formula to evaluate the power in the signal.

$$P_y = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |y(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |A_1 \sin(\omega_1 t+\phi_1) + A_2 \sin(\omega_2 t+\phi_2)|^2\, dt$$

$$= \underbrace{\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A_1^2 \sin^2(\omega_1 t+\phi_1)dt}_{=P_1} + \underbrace{\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A_2^2 \sin^2(\omega_2 t+\phi_2)dt}_{=P_2} + \underbrace{\lim_{T\to\infty} \frac{2A_1A_2}{2T}\int_{-T}^{T} \sin(\omega_1 t+\phi_1)\sin(\omega_2 t+\phi_2)dt}_{=P_3}$$

The right hand side of the above equation includes three integrals. The first integral P_1 represents the power of a periodic signal $A_1 \sin(\omega_1 t + \phi_1)$. Based on Problem 1.9, the average power P_1 is given by $(A_1)^2/2$. Similarly, the secong integral $P_2 = (A_2)^2/2$. The third integral is evaluated by substituting

$$2\sin(\omega_1 t+\phi_1)\sin(\omega_1 t+\phi_1) = \cos(\omega_1 t+\phi_1+\omega_2 t+\phi_2) - \cos(\omega_1 t+\phi_1-\omega_2 t-\phi_2)$$

to get

$$P_3 = \lim_{T\to\infty} \frac{A_1A_2}{2T}\int_{-T}^{T} \cos[(\omega_1+\omega_2)t + (\phi_1+\phi_2)]dt + \lim_{T\to\infty} \frac{A_1A_2}{2T}\int_{-T}^{T} \cos[(\omega_1-\omega_2)t + (\phi_1-\phi_2)]dt\,.$$

Case $\omega_1 \neq \omega_2$: In such a case, both integrals result in finite values giving

$$P_3 = \lim_{T\to\infty} \frac{A_1A_2}{2T} \times (\text{finite value}\,\#1) + \lim_{T\to\infty} \frac{A_1A_2}{2T}(\text{finite value}\,\#2) = 0\,.$$

Case $\omega_1 = \omega_2$: In such a case, we obtain

$$P_3 = \lim_{T\to\infty} \frac{A_1A_2}{2T} \times (\text{finite value \# 1}) + \lim_{T\to\infty} \frac{A_1A_2}{2T}\int_{-T}^{T} \cos[(\phi_1-\phi_2)]dt$$

$$= 0 + \lim_{T\to\infty} \frac{A_1A_2}{2T} 2T\cos[(\phi_1-\phi_2)] = A_1A_2\cos[(\phi_1-\phi_2)].$$

Combining the above results, we obtain

$$P_y = \begin{cases} \frac{A_1^2}{2} + \frac{A_2^2}{2} & \omega_1 \neq \omega_2 \\ \frac{A_1^2}{2} + \frac{A_2^2}{2} + A_1A_2\cos(\phi_1-\phi_2) & \omega_1 = \omega_2. \end{cases}$$ ▮

Problem 1.11:

The power of the CT signal $x(t)$ is calculated as follows:

$$P_x = |x(t)|^2 = x(t)x^*(t) = \left(De^{j\omega_0 t}\right)\left(D^* e^{-j\omega_0 t}\right) = DD^* = |D|^2,$$

which proves the result. ▮

Problem 1.12

The average power of the CT signal $x(t)$ is given by

$$P_x = \lim_{T\to\infty} \tfrac{1}{T} \int_{-0.5T}^{0.5T} |x(t)|^2\, dt = \lim_{T\to\infty} \tfrac{1}{T} \int_{-0.5T}^{0.5T} x(t)x^*(t)dt = \lim_{T\to\infty} \tfrac{1}{T} \int_{-0.5T}^{0.5T} \left(\sum_{n=1}^{N} D_n e^{j\omega_n t}\right)\left(\sum_{m=1}^{N} D_m^* e^{-j\omega_m t}\right) dt,$$

$$= \lim_{T\to\infty} \tfrac{1}{T} \int_{-0.5T}^{0.5T} \sum_{n=1}^{N}\sum_{m=1}^{N} D_n D_m^* e^{j(\omega_n-\omega_m)t} dt$$

Changing the order of the integral and summation, we obtain

$$P_x = \sum_{m=1}^{N}\sum_{n=1}^{N} D_n D_m^* \left(\lim_{T\to\infty} \tfrac{1}{T} \int_{-0.5T}^{0.5T} e^{j(\omega_n-\omega_m)t} dt \right)$$

The above integral has two different sets of values for $\omega_n = \omega_m$ and $\omega_n \neq \omega_m$.

Case I ($\omega_n = \omega_m$): $\quad \lim_{T\to\infty} \tfrac{1}{T} \int_{-0.5T}^{0.5T} e^{j(\omega_n-\omega_m)t} dt = \lim_{T\to\infty} \tfrac{1}{T} \int_{-0.5T}^{0.5T} 1\, dt = \lim_{T\to\infty} \tfrac{1}{T} \times T = 1$

Case II ($\omega_n \neq \omega_m$):

$$\lim_{T\to\infty} \tfrac{1}{T} \int_{-0.5T}^{0.5T} e^{j(\omega_n-\omega_m)t} dt = \lim_{T\to\infty} \tfrac{1}{T} \left[\tfrac{e^{j(\omega_n-\omega_m)t}}{j(\omega_n-\omega_m)} \right]_{-0.5T}^{0.5T} = \lim_{T\to\infty} \tfrac{1}{j(\omega_n-\omega_m)T} \left[2j\sin(0.5(\omega_n-\omega_m)T)\right]$$

$$= \lim_{T\to\infty} \tfrac{2}{(\omega_n-\omega_m)T} \times [\text{finite value}] = 0$$

Combining the two cases, we obtain,

$$P_x = \sum_{m=1}^{N} \sum_{\substack{n=1\\ n=m}}^{N} D_n D_m^*(1) + \sum_{m=1}^{N} \sum_{\substack{n=1\\ n\neq m}}^{N} D_n D_m^*(0) = \sum_{m=1}^{N} |D_m|^2,$$

which proves the result. ▌

Problem 1.13:

Note that the energy of the signal in one period ($T = 1$) is given by

$$E_x = \int_0^1 |x(t)|^2\, dt = \sum_{m=0}^{\infty} \left(\int_{2^{-2m-1}}^{2^{-2m}} |x(t)|^2\, dt \right) = \sum_{m=0}^{\infty} \left(\int_{2^{-2m-1}}^{2^{-2m}} 1\, dt \right) = \sum_{m=0}^{\infty} \left[2^{-2m} - 2^{-2m-1} \right]$$

$$= \sum_{m=0}^{\infty} (1/4)^m - 0.5 \sum_{m=0}^{\infty} (1/4)^m = 0.5 \sum_{m=0}^{\infty} (1/4)^m = 0.5 \times \frac{1}{1-\frac{1}{4}} = \frac{2}{3}.$$

Therefore, the average power is given by, $P_x = 2/3$ (as period=1). ▌

Problem 1.14:

(i) $\quad x1(t) = \underbrace{\underbrace{2\sin(2\pi t)}_{=odd} \underbrace{\left[2 + \underbrace{\cos(4\pi t)}_{=even} \right]}_{=even}}_{=odd}$

We note that $x1(t)$ is a product of an odd term with an even term. Overall, $x1(t)$ is, therefore, an odd function.

(ii) $$x2(t)=\underbrace{\underbrace{t^2}_{=even}+\underbrace{\cos(3t)}_{=even}}_{=even}$$

We note that $x2(t)$ is a sum of two even terms. Overall, $x2(t)$ is, therefore, an even function.

(iii) $$x3(t)=\underbrace{\underbrace{e^{-3t}}_{\neq even,odd}\underbrace{\sin(3\pi t)}_{odd}}_{\neq even,odd}$$

We note that $x3(t)$ is a product of a neither-even-nor-odd term with an odd term. Overall, $x3(t)$ is, therefore, a neither-even-nor-odd function.

To evaluate the even and off components of $x3(t)$, we evaluate

$$x3(-t)=e^{3t}\sin(-3\pi t)=-e^{3t}\sin(3\pi t).$$

The even and odd components are given by

Even Component:

$$x3_{even}(t)=\tfrac{1}{2}\left[x3(t)+x3(-t)\right]=\tfrac{1}{2}\left[e^{-3t}\sin(3\pi t)-e^{3t}\sin(3\pi t)\right]=\tfrac{1}{2}\left(e^{-3t}-e^{3t}\right)\sin(3\pi t).$$

Odd Component:

$$x3_{odd}(t)=\tfrac{1}{2}\left[x3(t)-x3(-t)\right]=\tfrac{1}{2}\left[e^{-3t}\sin(3\pi t)+e^{3t}\sin(3\pi t)\right]=\tfrac{1}{2}\left(e^{-3t}+e^{3t}\right)\sin(3\pi t).$$

The even and odd components of $x3(t)$ are shown in Fig. S1.14.1.

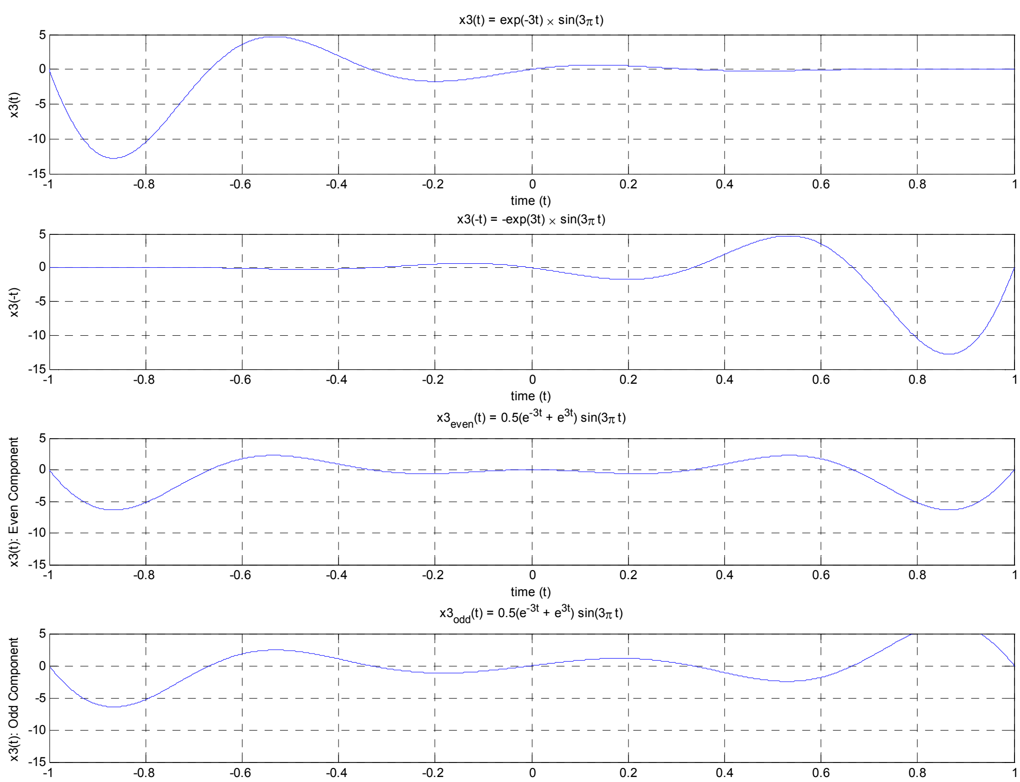


Fig. S1.14.1: CT functions $x3(t)$, its reflection $x3(-t)$, and its even and odd components for Problem 1.14(iii). Only the range between $(-1 \le t \le 1)$ is plotted.

(iv) $$x4(t) = \underbrace{\underbrace{t}_{=odd}\ \underbrace{\sin(5t)}_{=odd}}_{=even}$$

We note that $x4(t)$ is a product of two odd terms. Overall, $x4(t)$ is, therefore, an even function.

(v) $$x5(t) = \underbrace{\underbrace{t}_{=odd}\ \underbrace{u(t)}_{\neq even,odd}}_{\neq even,odd}$$

We note that $x5(t)$ is a product of an odd term with a neither-even-nor-odd term. Overall, $x5(t)$ is, therefore, a neither-even-nor-odd function.

To evaluate the even and off components of $x5(t)$, we evaluate

$$x5(-t) = -tu(-t).$$

The even and odd components of $x5(t)$ are given by

Even Component: $$x5_{even}(t) = \tfrac{1}{2}\left[x5(t) + x5(-t)\right] = \tfrac{1}{2}tu(t) - \tfrac{1}{2}tu(-t) = \tfrac{1}{2}|t|.$$

Odd Component: $$x5_{odd}(t) = \tfrac{1}{2}\left[x5(t) - x5(-t)\right] = \tfrac{1}{2}tu(t) + \tfrac{1}{2}tu(-t) = \tfrac{1}{2}t.$$

The even and odd components for $x5$(t) are plotted in Fig. S1.14.2 within the range $(-1 \le t \le 1)$.

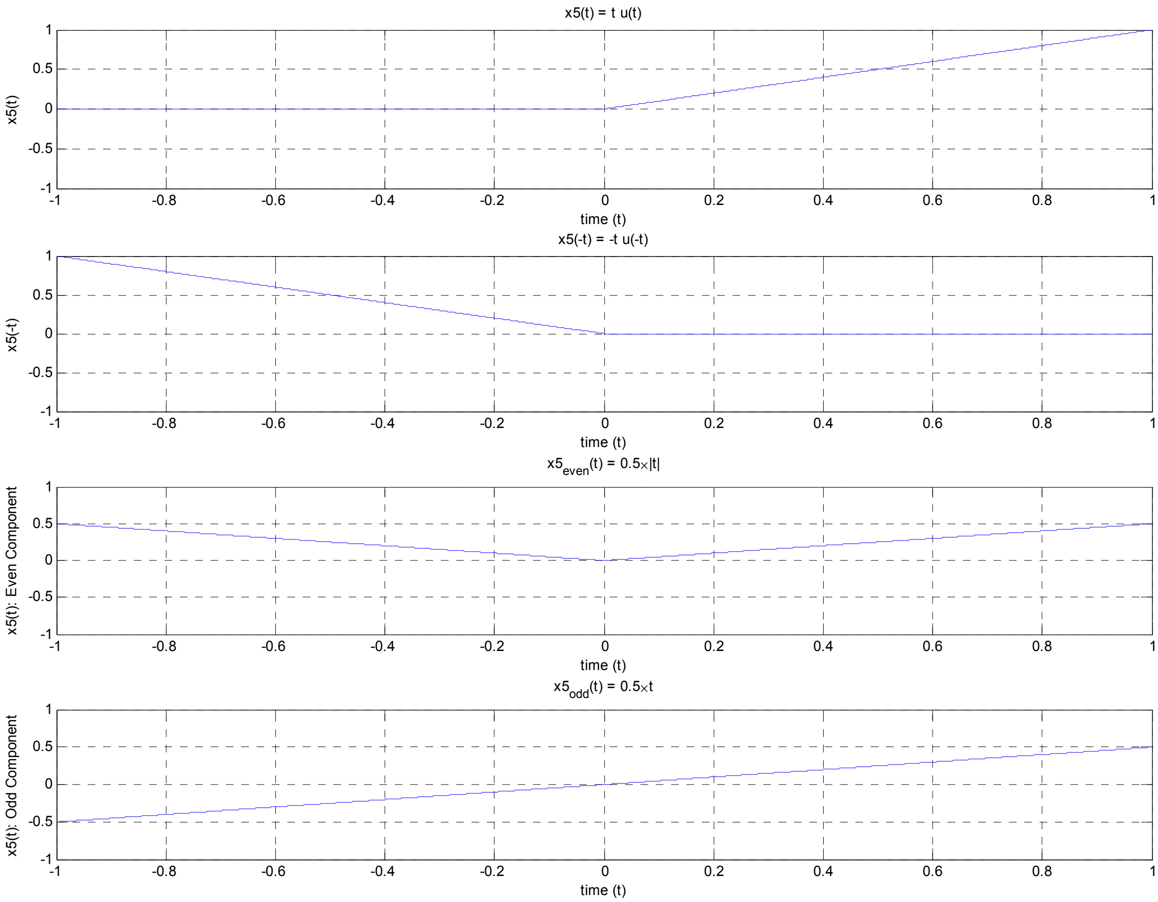


Fig. S1.14.2: CT functions $x5(t)$, its reflection $x5(-t)$, and its even and odd components for Problem 1.14(v). Only the range between $(-1 \leq t \leq 1)$ is plotted.

(vi) The function x6(t) is a neither-even-nor-odd function.

To evaluate the even and off components of $x6(t)$, we evaluate

$$x6(-t) = \begin{cases} -3t & 0 \leq -t \leq 2 \\ 6 & 2 \leq -t \leq 4 \\ 3(t+6) & 4 \leq -t \leq 6 \\ 0 & \text{elsewhere} \end{cases} = \begin{cases} -3t & -2 \leq t \leq 0 \\ 6 & -4 \leq t \leq -2 \\ 3(t+6) & -6 \leq t \leq -4 \\ 0 & \text{elsewhere.} \end{cases}$$

The even and odd components of $x6(t)$ are given by

Even Component:

$$x6_{even}(t) = \tfrac{1}{2}\left[\begin{cases} 3t & 0 \le t \le 2 \\ 6 & 2 \le t \le 4 \\ 3(-t+6) & 4 \le t \le 6 \\ 0 & \text{elsewhere} \end{cases} + \begin{cases} -3t & -2 \le t \le 0 \\ 6 & -4 \le t \le -2 \\ 3(t+6) & -6 \le t \le -4 \\ 0 & \text{elsewhere} \end{cases}\right] = \tfrac{1}{2}\begin{cases} 3(t+6) & -6 \le t \le -4 \\ 6 & -4 \le t \le -2 \\ -3t & -2 \le t \le 0 \\ 3t & 0 \le t \le 2 \\ 6 & 2 \le t \le 4 \\ 3(-t+6) & 4 \le t \le 6 \\ 0 & \text{elsewhere.} \end{cases}$$

Odd Component:

$$x6_{odd}(t) = \tfrac{1}{2}\left[\begin{cases} 3t & 0 \le t \le 2 \\ 6 & 2 \le t \le 4 \\ 3(-t+6) & 4 \le t \le 6 \\ 0 & \text{elsewhere} \end{cases} - \begin{cases} -3t & -2 \le t \le 0 \\ 6 & -4 \le t \le -2 \\ 3(t+6) & -6 \le t \le -4 \\ 0 & \text{elsewhere} \end{cases}\right] = \tfrac{1}{2}\begin{cases} -3(t+6) & -6 \le t \le -4 \\ -6 & -4 \le t \le -2 \\ 3t & -2 \le t \le 0 \\ 3t & 0 \le t \le 2 \\ 6 & 2 \le t \le 4 \\ 3(-t+6) & 4 \le t \le 6 \\ 0 & \text{elsewhere.} \end{cases}$$

The even and odd components for $x6(t)$ are plotted in Fig. S1.14.3 within the range $(-6 \le t \le 6)$. ■

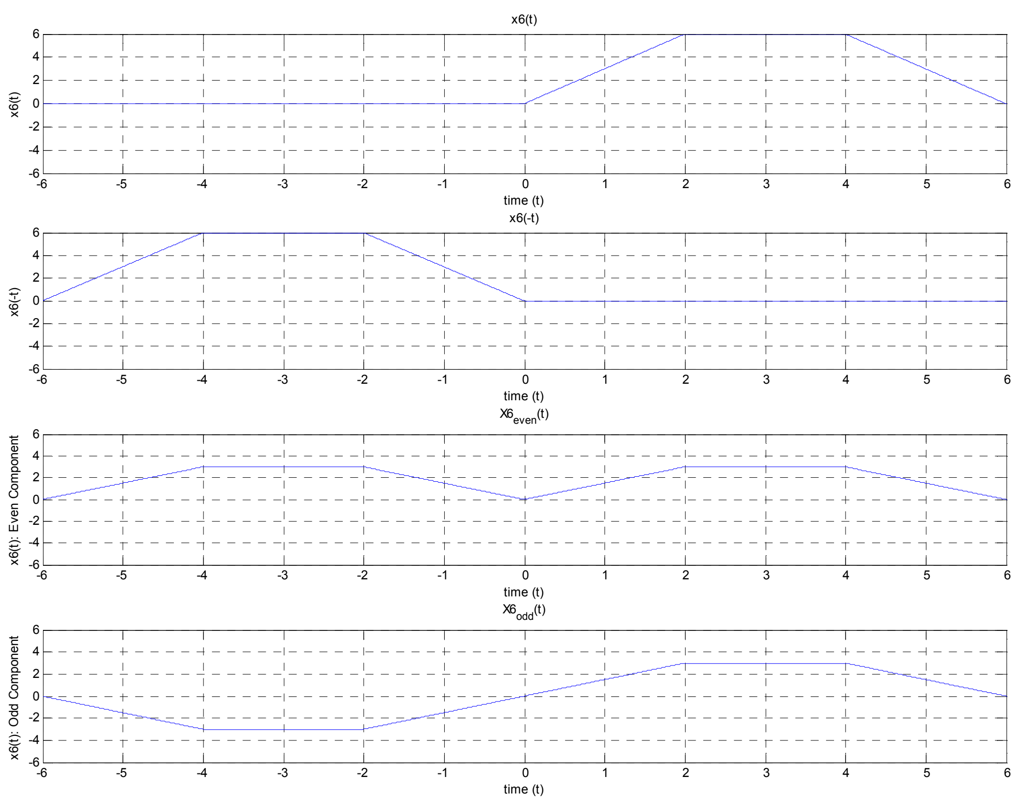


Fig. S1.14.3: CT functions $x6(t)$, its reflection $x6(-t)$, and its even and odd components for Problem 1.14(vi). Only the range between $(-6 \le t \le 6)$ is plotted.

Problem 1.15:

(i) $x1[k] = \underbrace{\sin(4k)}_{=odd} + \underbrace{\cos(2\pi k/3)}_{=even}$

We note that the DT signal $x1[k]$ is a sum of an odd term with an even term. Overall, $x1[k]$ is, therefore, a neither-even-nor-odd function.

The even and odd components of $x1[k]$ are given by

Even component: $x1_{even}[k] = \frac{1}{2}\{x1[k] + x1[-k]\} = \cos(2\pi k/3)$.

Odd component: $x1_{odd}[k] = \frac{1}{2}\{x1[k] - x1[-k]\} = \sin(4k)$.

The even and odd components are plotted in Fig. S1.15.1 followed by the Matlab code used to generate the two components.

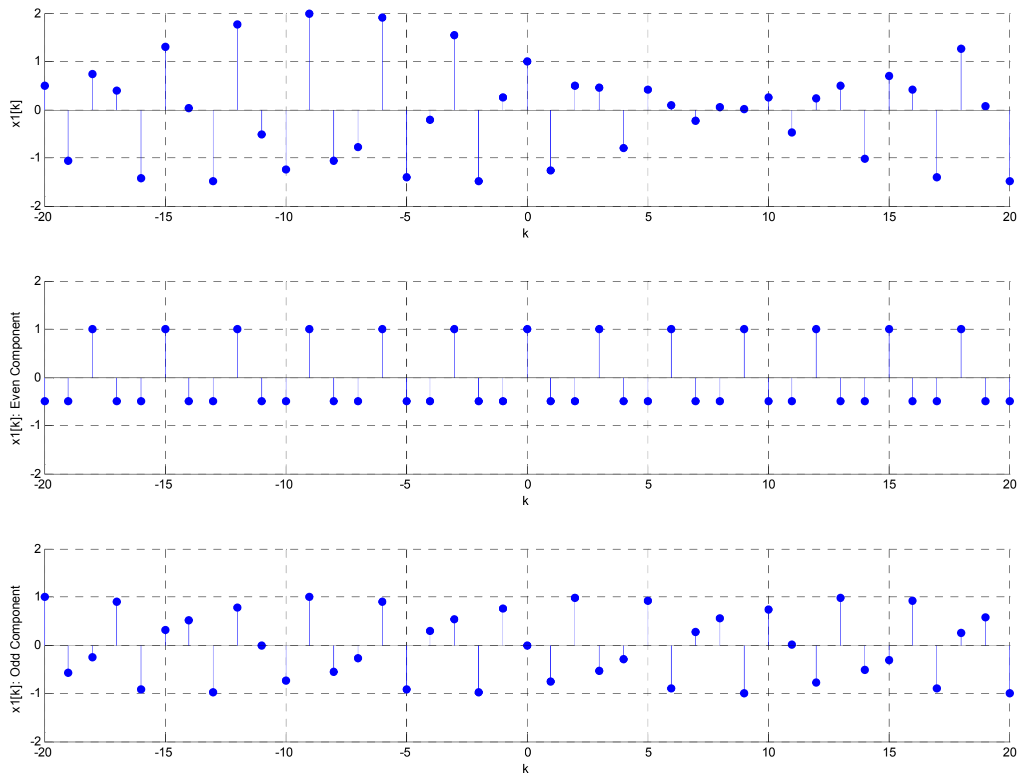


Fig. S1.15.1: Odd and Even components of $x1$[k] in Problem 1.15(i) for $(-20 \le k \le 20)$.

```
% MATLAB code for Problem 1.15(i)
% clear figure
clf
% signal defined in part (i)
k1 =-20:20;
x1 = sin(4*k1) + cos(2*pi*k1/3);
subplot(3,1,1), stem(k1, x1, 'filled'), grid on
xlabel('k');                        % Label of X-axis
ylabel('x1[k] ')                    % Label of Y-axis
axis([-20, 20, -2, 2]) ;
```

```
%
k1 =-20:20;
x1_even = cos(2*pi*k1/3);
subplot(3,1,2), stem(k1, x1_even, 'filled'), grid on
xlabel('k');                         % Label of X-axis
ylabel('x1[k]: Even Component')      % Label of Y-axis
axis([-20, 20, -2, 2]) ;

% signal defined in part (i)
x1_odd = sin(4*k1);
subplot(3,1,3), stem(k1, x1_odd, 'filled'), grid on
xlabel('k');                         % Label of X-axis
ylabel('x1[k]: Odd Component ')      % Label of Y-axis
axis([-20, 20, -2, 2]) ;
print -dtiff plot.tiff ;             % Save the figure as a TIFF file
```

(ii) $x2[k] = \underbrace{\sin(\pi k/3000)}_{=odd} + \underbrace{\cos(2\pi k/3)}_{=even}$

We note that $x2[k]$ is the sum of an even with an odd component. Therefore, the DT signal is neither even nor odd.

The even and odd components of $x2[k]$ are given by

Even component: $x2_{even}[k] = \frac{1}{2}\{x2[k] + x2[-k]\} = \cos(2\pi k/3).$

Odd component: $x2_{odd}[k] = \frac{1}{2}\{x2[k] - x2[-k]\} = \sin(\pi k/3000).$

The even and odd components are plotted in Fig. S1.15.2. Note that the odd component is close to 0 for the plotted values of k. This is because $\sin(\pi k/3000) \approx \sin(0) = 0$ for $(-20 \le k \le 20)$.

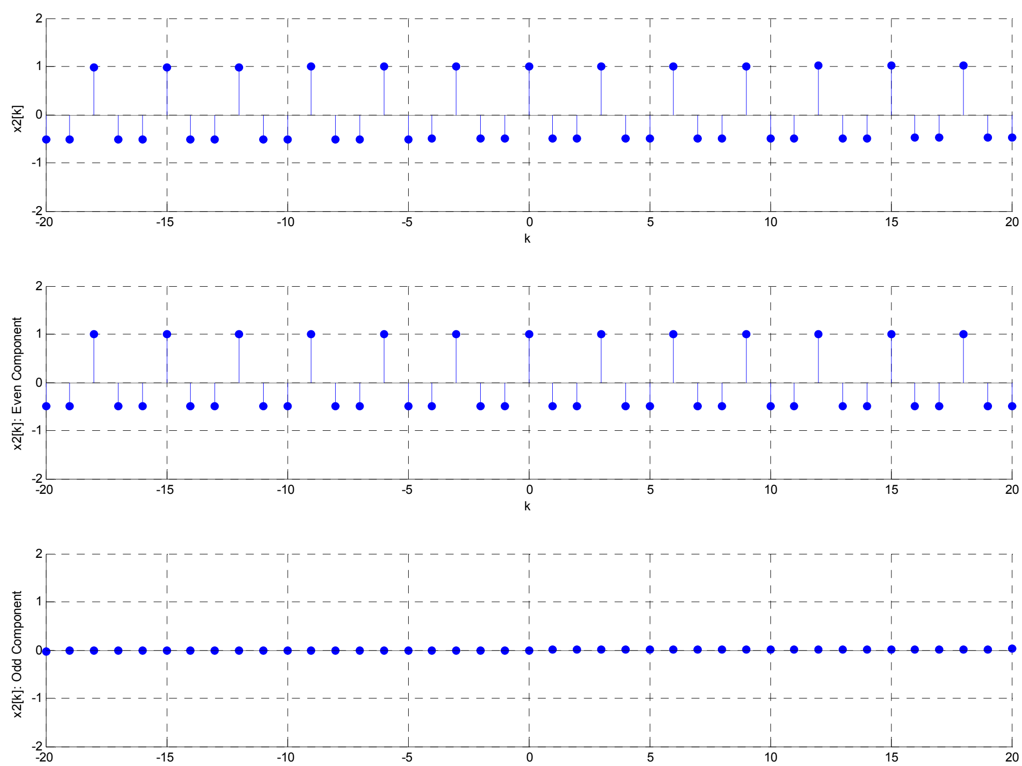


Fig. S1.15.2: Odd and Even components of $x2[k]$ in Problem 1.15(ii) for $(-20 \le k \le 20)$.

(iii)

$$x3[k] = \exp(j7\pi k/4) + \cos(4\pi k/7 + \pi) = \cos(7\pi k/4) + j\sin(7\pi k/4) - \cos(4\pi k/7)$$
$$= \underbrace{\cos(7\pi k/4) - \cos(4\pi k/7)}_{=even} + j\underbrace{\sin(7\pi k/4)}_{=odd}$$

Therefore, the DT signal is neither even nor odd.

The even and odd components of $x3[k]$ are given by

Even component: $x3_{even}[k] = \tfrac{1}{2}\{x3[k] + x3[-k]\} = \cos(7\pi k/4) - \cos(4\pi k/7).$

Odd component: $x3_{odd}[k] = \tfrac{1}{2}\{x3[k] - x3[-k]\} = j\sin(7\pi k/4).$

The even and odd components are plotted in Fig. S1.15.3. Since $x3[k]$ is complex, we plot the real and imaginary components of $x3[k]$ separately. Although the real component of $x3[k]$ is even and the imaginary component is odd, $x3[k]$ is neither-even-nor-odd. This is the reason why the even component of $x3[k]$ is the same as its real component and the odd component is the same as the imaginary component.

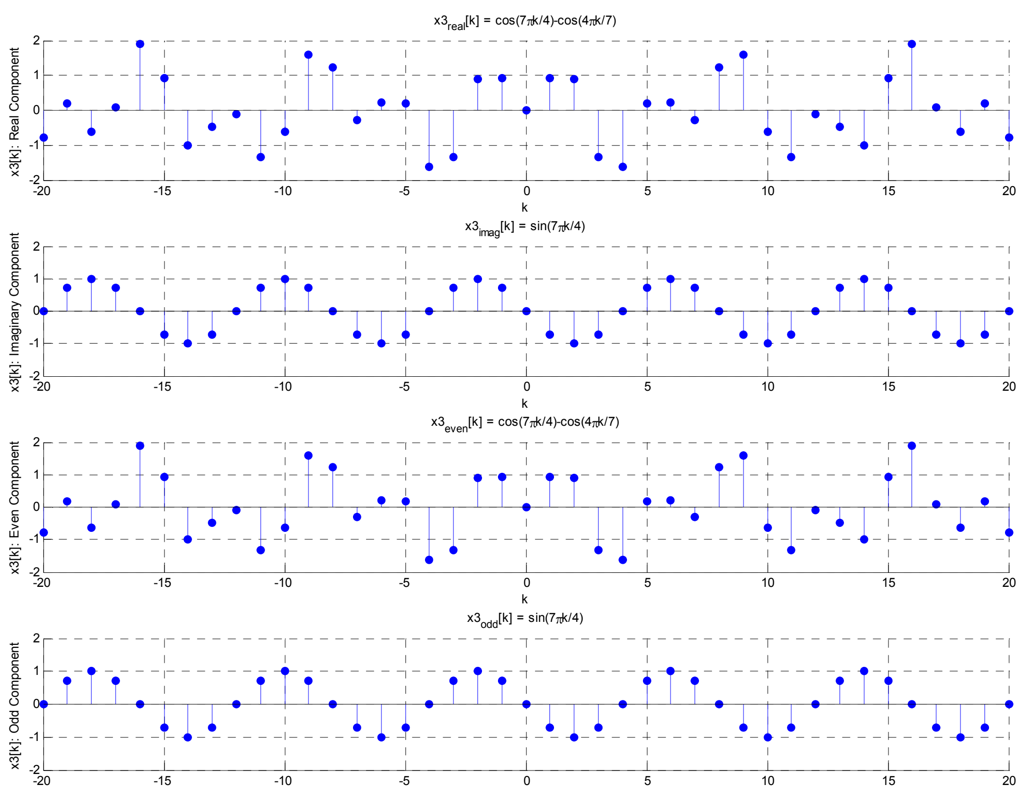


Fig. S1.15.3: Odd and Even components of $x3[k]$ in Problem 1.15(iii) for $(-20 \le k \le 20)$.

iv) $$x4[k] = \underbrace{\underbrace{\sin(3\pi k/8)}_{=odd}\underbrace{\cos(63\pi k/64)}_{=even}}_{=odd}$$

We note that $x4[k]$ is a product of an odd function with an even function. Therefore, the DT signal $x4[k]$ is odd.

v) Computing the time reversed form of

$$x5[k]=\begin{cases}(-1)^k & k\ge 0\\ 0 & k<0\end{cases}$$

we obtain
$$x5[-k]=\begin{cases}(-1)^{-k} & -k\ge 0\\ 0 & -k<0\end{cases}=\begin{cases}(-1)^k & k\le 0\\ 0 & k>0\end{cases}=\begin{cases}0 & k>0\\ (-1)^k & k\le 0\end{cases}.$$

Since $x5[k] \neq \pm\, x5[-k]$, the DT signal $x5[k]$ is neither-even-nor-odd. The even and odd components of $x5[k]$ are given by

Even component: $$x5_{even}[k]=\tfrac{1}{2}\{x5[k]+x5[-k]\}=\tfrac{1}{2}\begin{cases}(-1)^k & k<0\\ 2 & k=0\\ (-1)^k & k>0\end{cases}=\begin{cases}1 & k=0\\ \tfrac{1}{2}(-1)^k & k\neq 0\end{cases}$$

Odd component: $$x5_{odd}[k]=\tfrac{1}{2}\{x5[k]-x5[-k]\}=\begin{cases}-(-1)^k & k<0\\ 0 & k=0\\ (-1)^k & k>0.\end{cases}$$

The even and odd components are plotted in Fig. S1.15.4. ▌

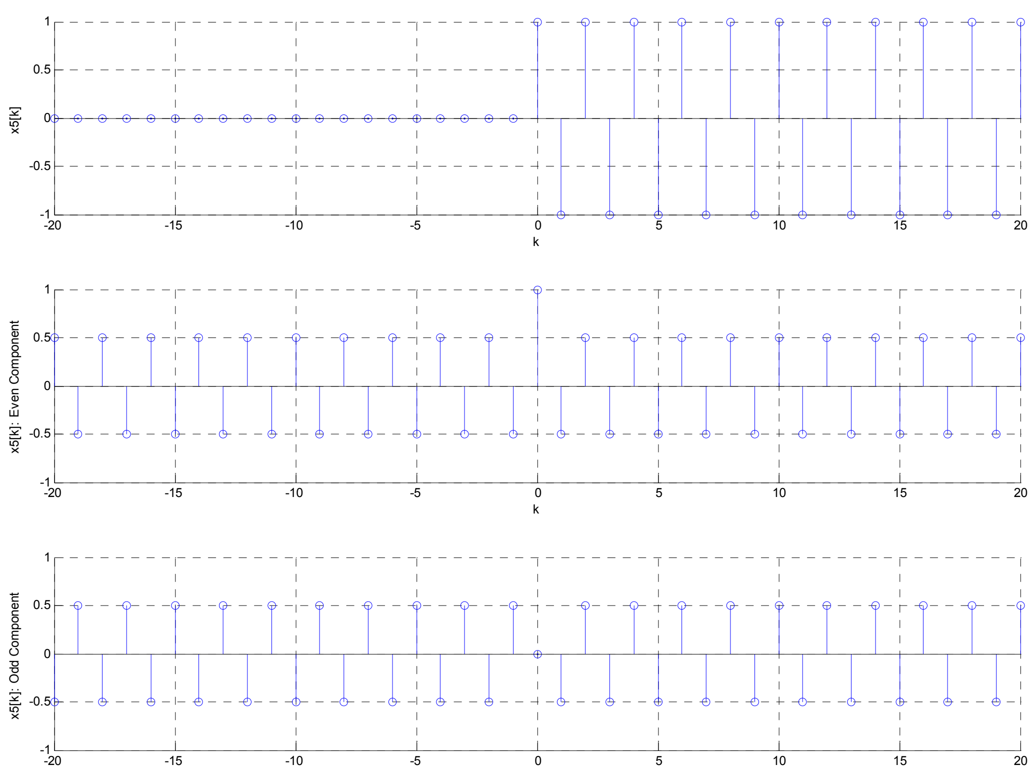


Fig. S1.15.4: Odd and Even components of $x5[k]$ in Problem 1.15(v) for $(-20 \le k \le 20)$.

Problem 1.16:

(a) Assume $x(t)$ to be an even function for $T = T_e$. Using $x(t) = x(-t)$, we get

$$\underbrace{3\sin\left(\tfrac{2\pi t}{5} - \tfrac{2\pi T_e}{5}\right)}_{x(t)} = \underbrace{3\sin\left(-\tfrac{2\pi t}{5} - \tfrac{2\pi T_e}{5}\right)}_{x(-t)} = -3\sin\left(\tfrac{2\pi t}{5} + \tfrac{2\pi T_e}{5}\right)$$

or,
$$3\sin\left(\tfrac{2\pi t}{5} - \tfrac{2\pi T_e}{5}\right) = 3\sin\left(\tfrac{2\pi t}{5} + \tfrac{2\pi T_e}{5} + (2m+1)\pi\right).$$

The above expression implies that

$$-\tfrac{2\pi T_e}{5} = \tfrac{2\pi T_e}{5} + (2m+1)\pi,$$

or,
$$T_e = \tfrac{5(2m+1)}{4}$$

with $m \in Z^+$.

(b) Assume $x(t)$ to be an odd function for $T = T_o$. Using $x(t) = -x(-t)$, we obtain

$$\underbrace{3\sin\left(\tfrac{2\pi t}{5} - \tfrac{2\pi T_o}{5}\right)}_{x(t)} = -\underbrace{3\sin\left(-\tfrac{2\pi t}{5} - \tfrac{2\pi T_o}{5}\right)}_{x(-t)} = 3\sin\left(\tfrac{2\pi t}{5} + \tfrac{2\pi T_o}{5}\right)$$

or,
$$3\sin\left(\tfrac{2\pi t}{5} - \tfrac{2\pi T_o}{5}\right) = 3\sin\left(\tfrac{2\pi t}{5} + \tfrac{2\pi T_o}{5} - 2m\pi\right).$$

The above expression implies that

$$-\tfrac{2\pi T_o}{5} = \tfrac{2\pi T_o}{5} - 2m\pi,$$

or,
$$T_o = \tfrac{5m}{2}.$$

with $m \in Z^+$.

Problem 1.17:

(a) Neither-even-nor-odd; aperiodic; and energy signal.

Energy = $5^2 \times (0.5) + 5^2 \times (0.5) = 25$ and Power = 0.

(b) Odd signal; periodic signal with period 1; and power signal.

Power = $[2.5^2 \times (0.5) + 2.5^2 \times (0.5)]/1 = 6.25$ and Energy = ∞.

(c) Neither-even-nor-odd; aperiodic; and energy signal.

Energy:
$$E_{x3} = \int_{-\infty}^{\infty} \left(e^{-1.5t}u(t)\right)^2 dt = \int_0^{\infty} e^{-3t}\, dt = \left[-\frac{e^{-3t}}{3}\right]_0^{\infty} = \frac{1}{3}.$$

Power = 0.

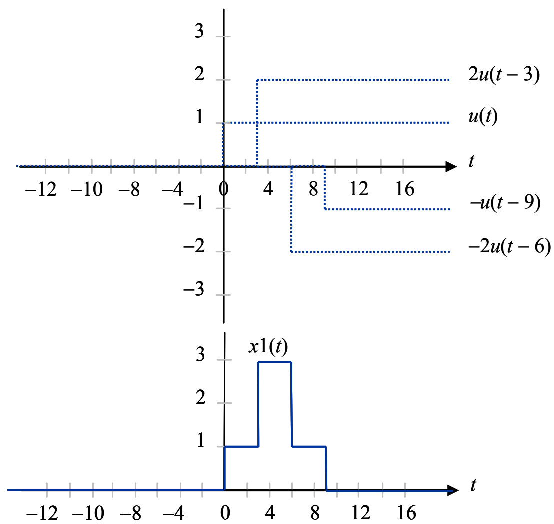
(d) Odd signal; periodic signal with period 3; and power signal.

Power: $$P = \frac{1}{3}\int_0^3 (\tfrac{5}{3}t - 2.5)^2\,dt = \frac{1}{3}\frac{(\tfrac{5}{3}t-2.5)^3}{3(5/3)}\bigg|_0^3 = \frac{1}{15}\left[2.5^3 - (-2.5)^3\right] = \frac{25}{12}.$$

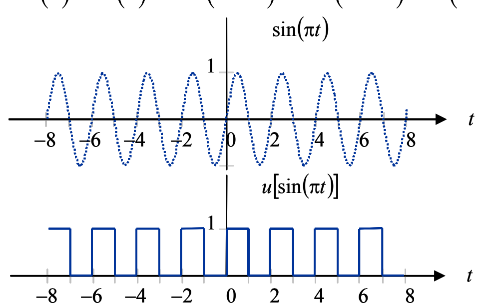
Energy = ∞.

Problem 1.18:

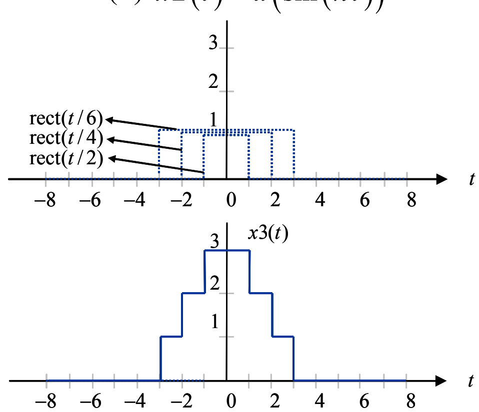
The waveforms of the signals are shown in Fig. S1.18, where the individual components are plotted in the top subplot followed by the overall signal.



(i) $x1(t) = u(t) + 2u(t-3) - 2u(t-6) - u(t-9)$

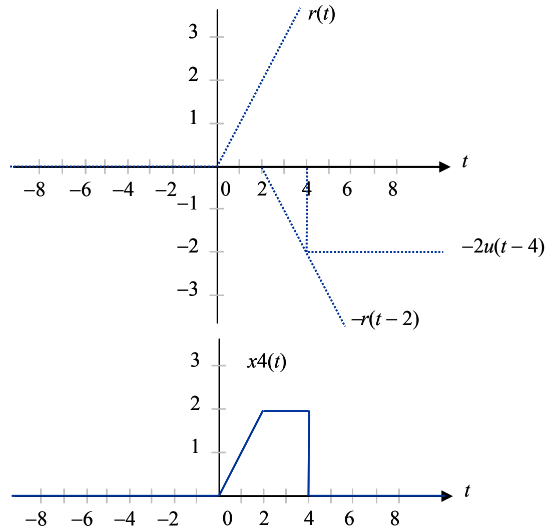


(ii) $x2(t) = u(\sin(\pi t))$

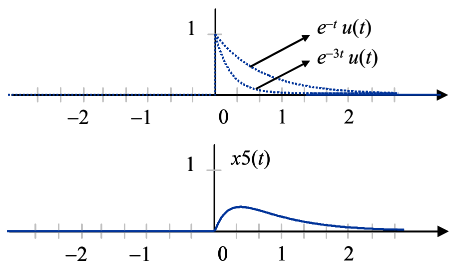


(iii) $x3(t) = \mathrm{rect}(t/6) + \mathrm{rect}(t/4) + \mathrm{rect}(t/2)$

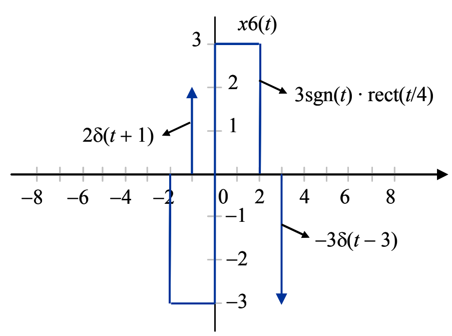
Figure S1.18: Waveforms for CT signals specified in Problem 1.18 (i) – (iii).



(iv) $x4(t) = r(t) - r(t-2) - 2u(t-4)$



(v) $x5(t) = \left(\exp(-t) - \exp(-3t)\right)u(t)$



(vi) $x6(t) = 3\,\mathrm{sgn}(t) \cdot rect(t/4) + 2\delta(t+1) - 3\delta(t-3)$

Figure S1.18 (contd.): Waveforms for CT signals specified in Problem 1.18 (iv) – (vi).

Problem 1.19:

(i) Expressing $$e^{j2\pi t+3} = e^{3}\left(\cos(2\pi t) + j\sin(2\pi t)\right)$$

gives the real and imaginary components as

$$x1_{\text{real}}(t) = e^3 \cos(2\pi t) \quad \text{and} \quad x1_{\text{imag}}(t) = e^3 \sin(2\pi t)\,.$$

The real and imaginary components are plotted separately in Fig. S1.19.1, where we note that the fundamental period is 1 s. The fundamental frequency is, therefore, given by $f_0 = 1$ Hz.

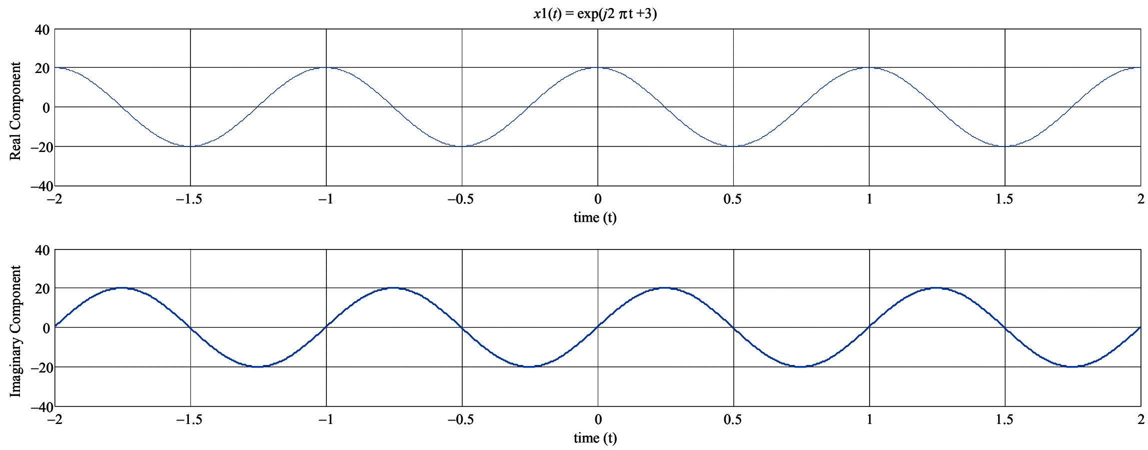


Fig. S1.19.1: Real and imaginary components of $x1(t) = e^{j2\pi t+3}$.

(ii) Expressing $$e^{j2\pi t+3t} = e^{3t}\left(\cos(2\pi t) + j\sin(2\pi t)\right)$$

gives the real and imaginary components as

$$x2_{\text{real}}(t) = e^{3t} \cos(2\pi t) \quad \text{and} \quad x2_{\text{imag}}(t) = e^{3t} \sin(2\pi t)\,.$$

The real and imaginary components are plotted separately in Fig. S1.19.2, where we note that $x2(t)$ is not periodic but is instead a rising exponential modulated with a sine wave.

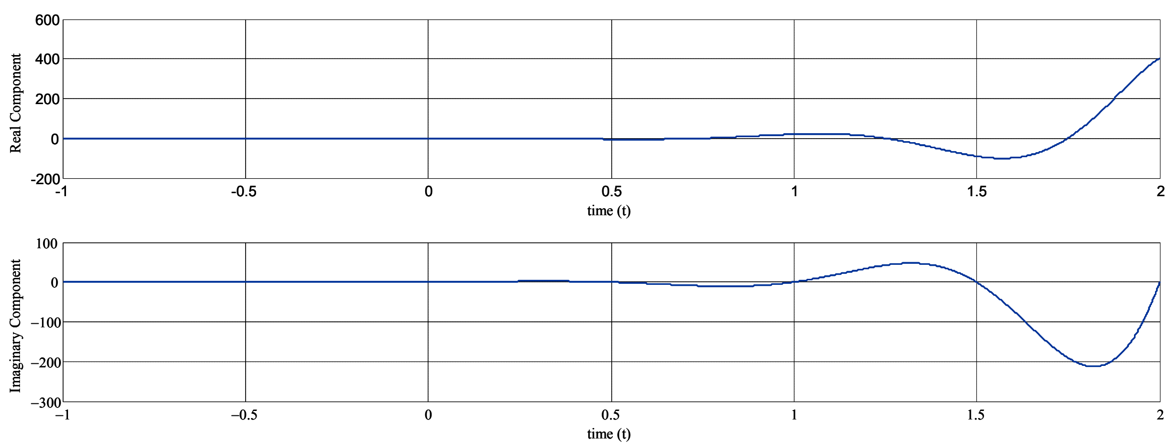


Fig. S1.19.2: Real and imaginary components of $x2(t) = e^{j2\pi t+3t}$.

(iii) Expressing $$e^{-j2\pi t+j3t} = \cos(3t - 2\pi t) + j\sin(3t - 2\pi t)$$

gives the real and imaginary components as

$$x3_{\text{real}}(t) = \cos(3t - 2\pi t) \quad \text{and} \quad x3_{\text{imag}}(t) = \sin(3t - 2\pi t)\,.$$

The real and imaginary components are plotted separately in Fig. S1.19.1.The fundamental frequency is, therefore, given by $f_0 = 1 - 3/(2\pi)$ Hz.

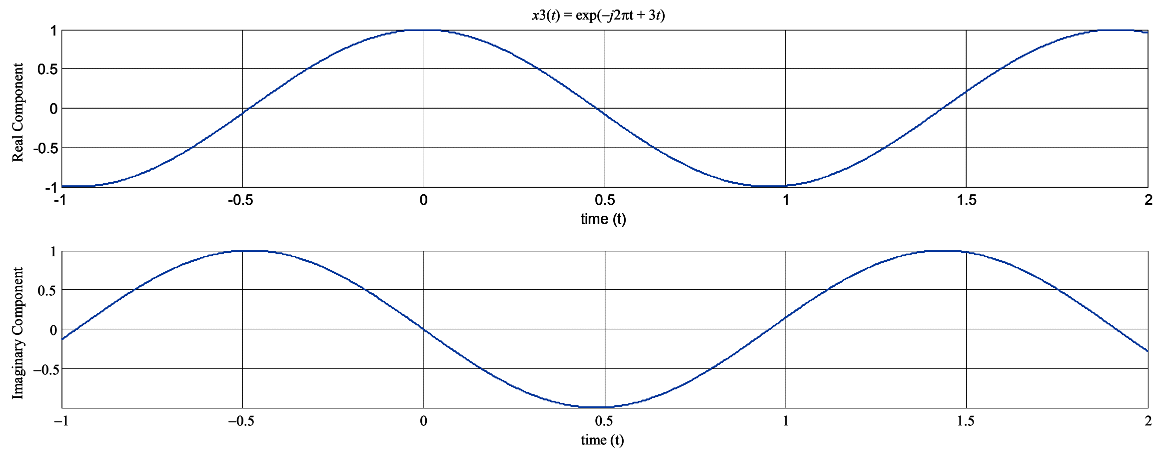


Fig. S1.19.3: Real and imaginary components of $x3(t) = e^{-j2\pi t + 3t}$.

(iv) – (vi) The remaining three signals are all sinusoidal signals. $x4(t)$ has the fundamental period of 1s, $x5(t)$ has the fundamental period of 2 s, and $x6(t)$ has the fundamental period of 2 s. The fundamental frequencies are 1, 1/2, and 1/2 Hz for $x4(t)$, $x5(t)$, and $x6(t)$, respectively. The three waveforms are plotted in Fig. S1.19.4. ∎

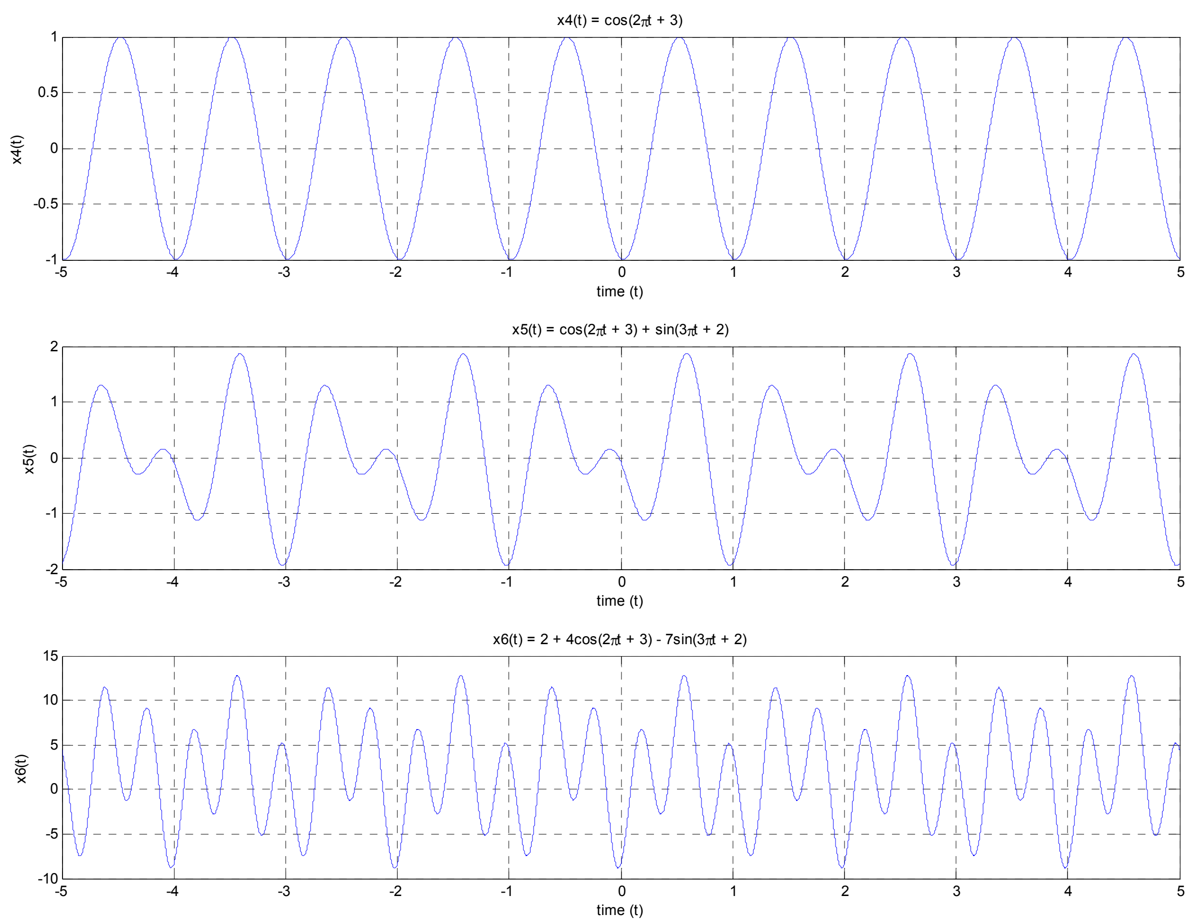


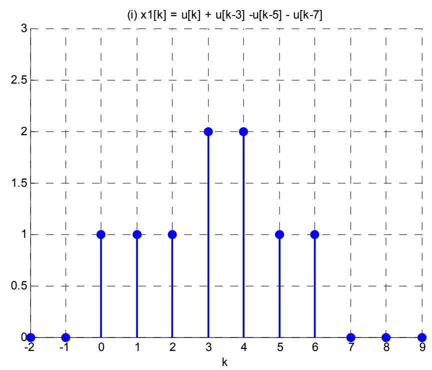
Fig. S1.19.4: Signals $x4(t)$, $x5(t)$, and $x6(t)$ for Problem 1.19.

Problem 1.20:

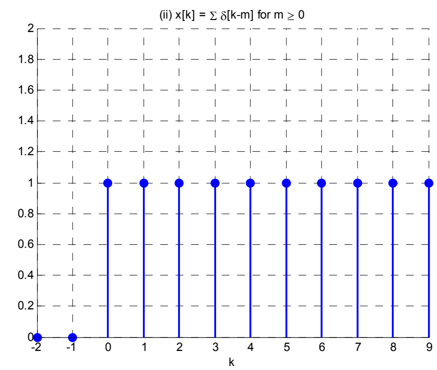
The value of $x1$[k] and $x3$[k] for $-3 \le k \le 8$ is shown in Table. The corresponding waveforms for the above signals are shown in Fig. S1.20. The waveforms for the remaining signals are plotted in a similar way, and are shown in Fig. S1.20.

Table S1.20: Values of $x1$[k] and $x3$[k] for $-3 \le k \le 8$ in Problem 1.20

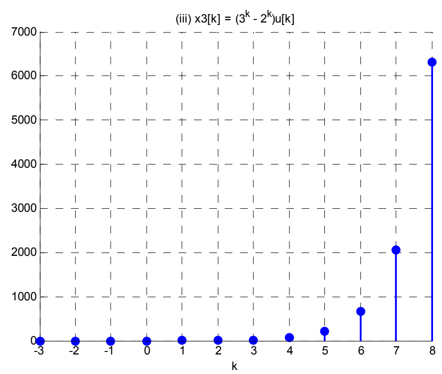
k	−3	−2	−1	0	1	2	3	4	5	6	7	8
$x1[k]$	0	0	0	1	1	1	2	2	1	1	0	0
$x3[k]$	0	0	0	0	1	5	19	65	211	665	2059	6305



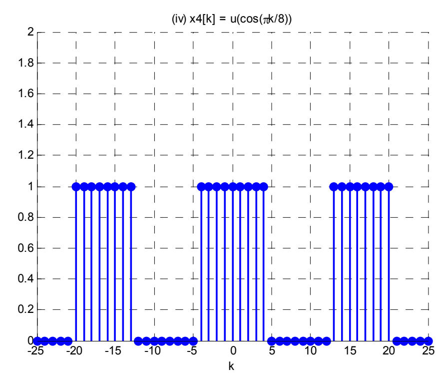
(i)



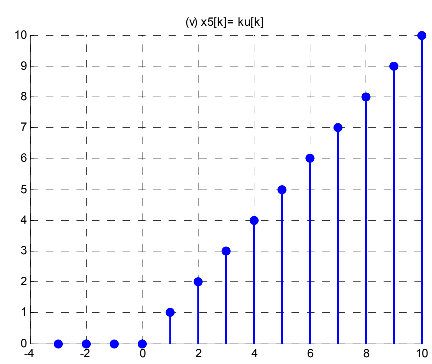
(ii)



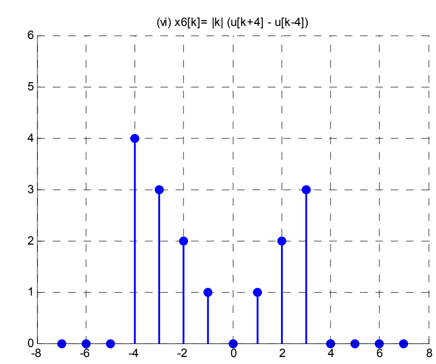
(iii)



(iv)



(v)



(vi)

Figure S1.20: Waveforms for DT signals specified in Problem 1.20.

Program 1.20. MATLAB Program for generating subplots (i) and (iii)

```
% MATLAB code for Problem 1.20 (i) and (iii)
% clear figure
clf
% signal defined in part (i)
k1 =-2:8  ;
x1 = [0 0 1 1 1 2 2 1 1 0 0];
subplot(2,1,1), stem(k1, x1, 'filled'), grid on
xlabel('k')                           % Label of X-axis
ylabel('x1[k]')                       % Label of Y-axis
axis([-2, 8, 0, 3]) ;


% signal defined in part (iii)

k3 = -2:8 ;
x3 = (3.^k3-2.^k3).*(k3>=0) ;
subplot(2,1,2), stem(k3, x3, 'filled'), grid on
xlabel('k')                           % Label of X-axis
ylabel('x3[k]')                       % Label of Y-axis
axis([-2, 8, 0, 7000]) ;

print -dtiff plot.tiff ;              % Save the figure as a TIFF file
```

Problem 1.21:

(i) Using the impulse function property $f(t)\,\delta(t-t_0) = f(t_0)\,\delta(t-t_0)$, we obtain

$$\frac{5+2t+t^2}{7+t^2+t^4}\delta(t-1) = \left.\frac{5+2t+t^2}{7+t^2+t^4}\right|_{t=1}\delta(t-1) = \frac{5+2(1)+1^2}{7+1^2+1^4}\delta(t-1) = \frac{8}{9}\delta(t-1).$$

(ii) Using the impulse function property $f(t)\,\delta(t-t_0) = f(t_0)\,\delta(t-t_0)$, we obtain

$$\frac{\sin(t)}{2t}\delta(t) = \frac{1}{2}\cdot\left.\frac{\sin(t)}{t}\right|_{t=0}\delta(t) = \frac{1}{2}\left[\underbrace{\lim_{t\to 0}\frac{\sin(t)}{t}}_{=1}\right]\delta(t) = \tfrac{1}{2}\delta(t)$$

where the L'Hopital's rule is applied to evaluate the value of $\sin(t)/t$ at $t = 0$.

(iii) Using the impulse function property $f(t)\,\delta(t-t_0) = f(t_0)\,\delta(t-t_0)$, we obtain

$$\tfrac{\omega^3-1}{\omega^2+2}\delta(\omega-5) = \left.\tfrac{\omega^3-1}{\omega^2+2}\right|_{\omega=5}\delta(\omega-5) = \frac{125-1}{25+2}\delta(\omega-5) = \frac{124}{27}\delta(\omega-5).$$

Problem 1.22:

(i) $$\int_{-\infty}^{\infty}(t-1)\delta(t-5)\,dt = \int_{-\infty}^{\infty}4\delta(t-5)\,dt = 4\int_{-\infty}^{\infty}\delta(t-5)\,dt = 4.$$

(ii) $$\int_{-\infty}^{6} (t-1)\delta(t-5)\,dt = \int_{-\infty}^{6} 4\,\delta(t-5)\,dt = 4\int_{-\infty}^{6} \delta(t-5)\,dt = 4\,.$$

(iii) $$\int_{6}^{\infty} (t-1)\delta(t-5)\,dt = \int_{6}^{\infty} 4\,\delta(t-5)\,dt = 4\int_{6}^{\infty} \delta(t-5)\,dt = 0\,.$$

(iv) $$\int_{-\infty}^{\infty} (2t/3-5)\delta(3t/4-5/6)\,dt = \int_{-\infty}^{\infty} \left(\tfrac{2}{3}t-5\right)\delta\left(\tfrac{3}{4}(t-\tfrac{10}{9})\right)dt = \tfrac{4}{3}\int_{-\infty}^{\infty} \left(\tfrac{2}{3}t-5\right)\delta\left(t-\tfrac{10}{9}\right)dt$$

which simplifies to

$$= \tfrac{4}{3}\int_{-\infty}^{\infty} \left(\underbrace{\tfrac{2}{3}\times\tfrac{10}{9}-5}_{\approx -115/27}\right)\delta\left(t-\tfrac{10}{9}\right)dt = \tfrac{-460}{81}\int_{-\infty}^{\infty} \delta\left(t-\tfrac{10}{9}\right)dt = \tfrac{-460}{81}\,.$$

(v) $$\int_{-\infty}^{\infty} \exp(t-1)\sin\left(\pi(t+5)/4\right)\delta(1-t)dt = \int_{-\infty}^{\infty} \exp(t-1)\sin\left(\pi(t+5)/4\right)\delta(t-1)dt$$

which simplifies to

$$= \int_{-\infty}^{\infty} \exp(0)\sin(\pi 6/4)\delta(t-1)dt = \sin(\pi 6/4)\int_{-\infty}^{\infty} \delta(t-1)dt = \sin(3\pi/2) = -1\,.$$

(vi) $$\int_{-\infty}^{\infty} \left[\sin(3\pi t/4)+e^{-2t+1}\right]\delta(-(t+1))\,dt = \int_{-\infty}^{\infty} \left[\sin(3\pi t/4)+e^{-2t+1}\right]\delta(t+1)\,dt = \left[\sin(3\pi t/4)+e^{-2t+1}\right]\Big|_{t=-1}$$

which simplifies to

$$= \sin(-3\pi/4)+e^{3} = e^{3}-\sin(3\pi/4) = e^{3}-\tfrac{1}{\sqrt{2}}\,.$$

(vii) $$\int_{-\infty}^{\infty} \left[u(t-6)-u(t-10)\right]\sin(3\pi t/4)\delta(t-5)\,dt = \left[u(t-6)-u(t-10)\right]\sin(3\pi t/4)\Big|_{t=5}$$

which simplifies to

$$= \left[u(5-6)-u(5-10)\right]\sin(3\pi 5/4) = [0-0]\sin(15\pi/4) = 0\,.$$

(viii) By noting that only the impulses located at $t = -20$ ($m = -4$), $t = -15$ ($m = -3$), $t = -10$ ($m = -2$), $t = -5$ ($m = -1$), $t = 0$ ($m = 0$), $t = 5$ ($m = 1$), $t = 10$ ($m = 2$), $t = 15$ ($m = 3$), and $t = 20$ ($m = 4$) lie within the integration range of ($-21 \le t \le 21$), the integral reduces to

$$I = \int_{-21}^{21} \left(\sum_{m=-\infty}^{\infty} t\delta(t-5m)\right)dt = \int_{-21}^{21} \left(\sum_{m=-4}^{4} t\delta(t-5m)\right)dt\,.$$

Changing the order of summation and integration, we obtain

$$I = \sum_{m=-4}^{4} \int_{-21}^{21} t\delta(t-5m)dt = \sum_{m=-4}^{4} 5m = 5(-4-3-2-1+0+1+2+3+4) = 0.$$ ▌

Problem 1.23:

(i) Equation 1.43(a) is satisfied as

$$\lim_{\varepsilon\to 0}\frac{\varepsilon}{\pi(t^2+\varepsilon^2)}=\lim_{\varepsilon\to 0}\frac{\varepsilon}{\pi t^2}=0 \text{ provided } t\neq 0.$$

Integrating $$\int_{-\infty}^{\infty}\lim_{\varepsilon\to 0}\frac{\varepsilon}{\pi(t^2+\varepsilon^2)}dt=\lim_{\varepsilon\to 0}\int_{-\infty}^{\infty}\frac{\varepsilon}{\pi(t^2+\varepsilon^2)}dt=\frac{1}{\pi}\left[\tan^{-1}(\varepsilon)\right]_{-\infty}^{\infty}=1,$$

confirming that Equation (1.43b) is also satisfied.

(ii) Equation 1.43(a) is satisfied as

$$\lim_{\varepsilon\to 0}\frac{2\varepsilon}{4\pi^2t^2+\varepsilon^2}=\lim_{\varepsilon\to 0}\frac{2\varepsilon}{4\pi^2t^2}=0 \text{ provided } t\neq 0.$$

Integrating $$I=\int_{-\infty}^{\infty}\lim_{\varepsilon\to 0}\frac{2\varepsilon}{4\pi^2t^2+\varepsilon^2}dt=\lim_{\varepsilon\to 0}\int_{-\infty}^{\infty}\frac{2\varepsilon}{4\pi^2t^2+\varepsilon^2}dt.$$

Substituting $x=2\pi t$ gives $I=\lim_{\varepsilon\to 0}\int_{-\infty}^{\infty}\frac{2\varepsilon}{x^2+\varepsilon^2}\frac{dx}{2\pi}=\frac{1}{\pi}\lim_{\varepsilon\to 0}\int_{-\infty}^{\infty}\frac{2\varepsilon}{x^2+\varepsilon^2}dx=1$

confirming that Equation (1.43b) is also satisfied.

(iii) Equation 1.43(a) is satisfied as

$$\lim_{\varepsilon\to 0}\frac{1}{\pi t}\sin(\varepsilon t)=0 \text{ provided } t\neq 0.$$

Integrating $$I=\int_{-\infty}^{\infty}\lim_{\varepsilon\to 0}\frac{1}{\pi t}\sin(\varepsilon t)dt=\lim_{\varepsilon\to 0}\int_{-\infty}^{\infty}\frac{\sin(\varepsilon t)}{\pi t}dt=\lim_{\varepsilon\to 0}\frac{\varepsilon}{\pi}\int_{-\infty}^{\infty}\mathrm{sinc}\left(\frac{\varepsilon t}{\pi}\right)dt.$$

Using the CTFT pairs discussed in Chapter 5, it can be shown that (see below)

$$\int_{-\infty}^{\infty}\sin c(\sigma\, t)\,dt=\frac{1}{\sigma}.$$

From Table 5.2, we know: $\mathrm{rect}\left(\frac{t}{\tau}\right)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tau\sin c\left(\frac{\omega\tau}{2\pi}\right)e^{j\omega t}d\omega.$

Substituting $t=0$ in both side, we obtain $\frac{1}{2\pi}\int_{-\infty}^{\infty}\tau\sin c\left(\frac{\omega\tau}{2\pi}\right)d\omega=1,$

which implies that $\int_{-\infty}^{\infty}\sin c\left(\frac{\omega\tau}{2\pi}\right)d\omega=\frac{2\pi}{\tau}$. By changing variables, we obtain:

$$\int_{-\infty}^{\infty}\sin c(\sigma\, t)\,dt=\frac{1}{\sigma}$$

Applying the above identity, the integral is simplified as:

$$I=\lim_{\varepsilon\to 0}\frac{\varepsilon}{\pi}\int_{-\infty}^{\infty}\mathrm{sinc}\left(\frac{t\varepsilon}{\pi}\right)dt=\lim_{\varepsilon\to 0}\frac{\varepsilon}{\pi}\times\frac{\pi}{\varepsilon}=1$$

confirming that Equation (1.43b) is also satisfied.

(iv) Equation 1.43(a) is satisfied as

$$\lim_{\varepsilon\to 0}\frac{1}{\varepsilon\sqrt{2\pi}}\exp\left(-\frac{t^2}{2\varepsilon^2}\right)=\lim_{\varepsilon\to 0}\frac{\exp\left(-\frac{t^2}{2\varepsilon^2}\right)}{\varepsilon\sqrt{2\pi}}=0 \text{ provided } t\neq 0.$$

Integrating $$I=\int_{-\infty}^{\infty}\lim_{\varepsilon\to 0}\frac{1}{\varepsilon\sqrt{2\pi}}\exp\left(-\frac{t^2}{2\varepsilon^2}\right)=\lim_{\varepsilon\to 0}\int_{-\infty}^{\infty}\frac{1}{\varepsilon\sqrt{2\pi}}\exp\left(-\frac{t^2}{2\varepsilon^2}\right)dt=1,$$

confirming that Equation (1.43b) is also satisfied. The last result is observed by noting that a normal distribution is being integrated, which must equal 1. ▮

Problem 1.24:

(a) The waveforms for signals $x(t-3)$, $x(-2t-3)$, and $x(-0.75t-3)$ are shown in Fig. S1.24.

(b) The analytical expressions, directly from the $x(t)$ definition, are obtained below.

$$x(t-3)=\begin{cases}(t-3)+2 & -2\le t-3\le -1\\ 1 & -1\le t-3\le 1\\ -(t-3)+2 & 1\le t-3\le 2\\ 0 & \text{elsewhere}\end{cases}=\begin{cases}t-1 & 1\le t\le 2\\ 1 & 2\le t\le 4\\ -t+5 & 4\le t\le 5\\ 0 & \text{elsewhere.}\end{cases}$$

$$x(2t-3)=\begin{cases}(2t-3)+2 & -2\le 2t-3\le -1\\ 1 & -1\le 2t-3\le 1\\ -(2t-3)+2 & 1\le 2t-3\le 2\\ 0 & \text{elsewhere}\end{cases}=\begin{cases}2t-1 & 1\le 2t\le 2\\ 1 & 2\le 2t\le 4\\ -2t+5 & 4\le 2t\le 5\\ 0 & \text{elsewhere}\end{cases}=\begin{cases}2t-1 & 1/2\le t\le 1\\ 1 & 1\le t\le 2\\ -2t+5 & 2\le t\le 5/2\\ 0 & \text{elsewhere.}\end{cases}$$

$$x(-2t-3)=\begin{cases}(-2t-3)+2 & -2\le -2t-3\le -1\\ 1 & -1\le -2t-3\le 1\\ -(-2t-3)+2 & 1\le -2t-3\le 2\\ 0 & \text{elsewhere}\end{cases}=\begin{cases}-2t-1 & 1\le -2t\le 2\\ 1 & 2\le -2t\le 4\\ 2t+5 & 4\le -2t\le 5\\ 0 & \text{elsewhere}\end{cases}$$

$$=\begin{cases}-2t-1 & -1\le t\le -1/2\\ 1 & -2\le t\le -1\\ 2t+5 & -5/2\le t\le -2\\ 0 & \text{elsewhere.}\end{cases}$$

$$x(-0.75t-3)=\begin{cases}(-0.75t-3)+2 & -2\le -0.75t-3\le -1\\ 1 & -1\le -0.75t-3\le 1\\ -(-0.75t-3)+2 & 1\le -0.75t-3\le 2\\ 0 & \text{elsewhere}\end{cases}=\begin{cases}-0.75t-1 & 1\le -0.75t\le 2\\ 1 & 2\le -0.75t\le 4\\ 0.75t+5 & 4\le -0.75t\le 5\\ 0 & \text{elsewhere}\end{cases}$$

$$=\begin{cases}-0.75t-1 & -8/3\le t\le -4/3\\ 1 & -16/3\le t\le -8/3\\ 0.75t+5 & -20/3\le t\le -16/3\\ 0 & \text{elsewhere.}\end{cases}$$

It is observed that the plots in Fig. S1.24 match with the analytical expressions obtained. ▮

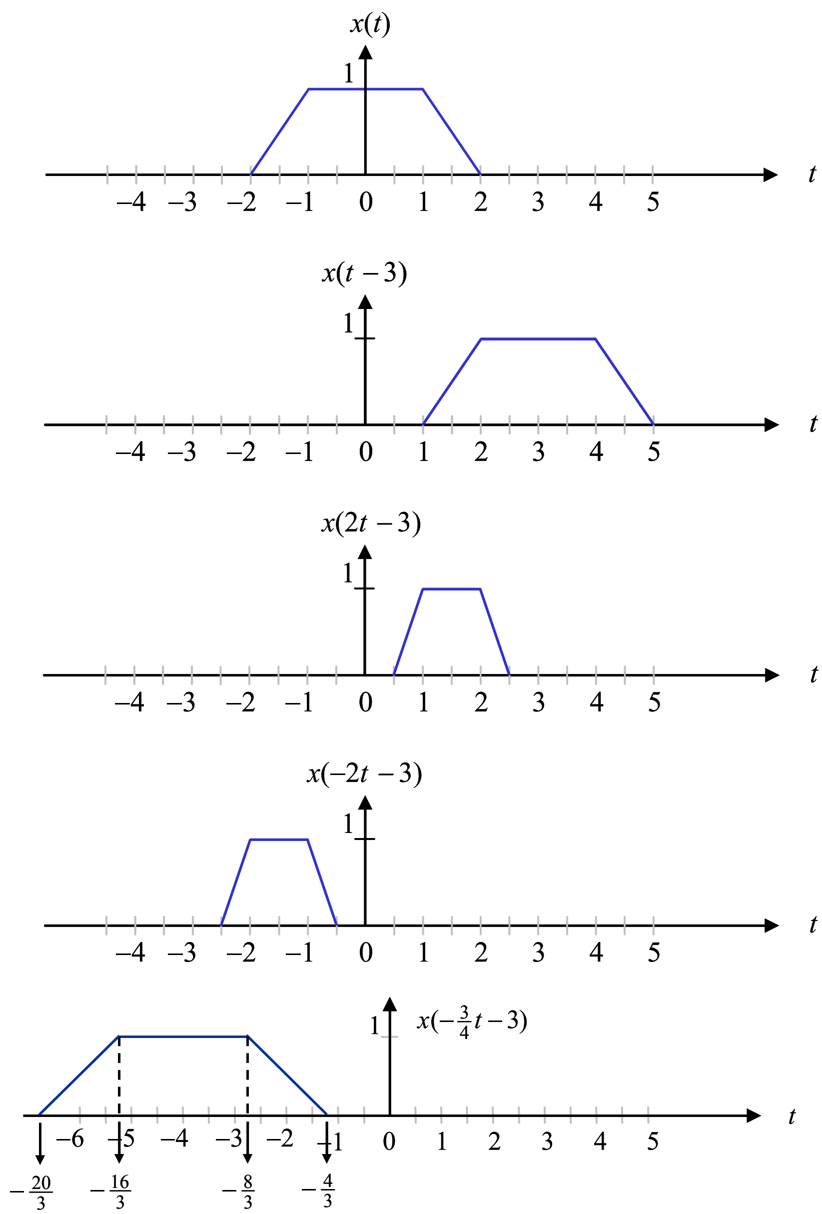


Figure S1.24: Waveforms for the shifted and scaled signals specified in Problem 1.24.

Problem 1.25:

(i) To obtain the waveform for $g(t)$ from $f(t)$, one possible order of transformations is:

$$f(t) \xrightarrow{\text{reflect about } y\text{-axis}} f(-t) \xrightarrow{\text{shift to the left by 9}} f(-(t-9)) = f(9-t) \xrightarrow{\text{scale by a factor of 3}} f(9-3t).$$

The final waveform for $g(t) = f(-3t+9)$ is sketched in Fig. S1.25.

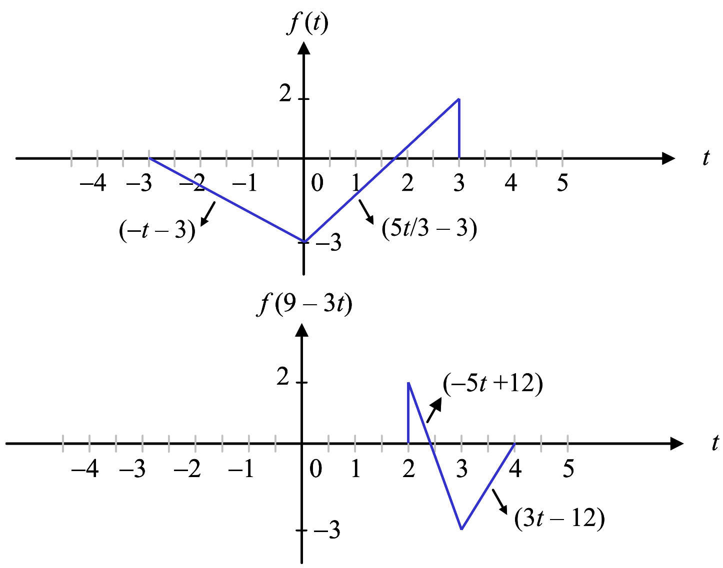


Figure S1.25: Waveform for Problem 1.25.

(ii) Since $f(t)$ is a finite duration signal, it is an energy signal. The average power in $f(t)$ is 0, while its total energy is given by

$$E_f = \int_{-\infty}^{\infty} f^2(t)dt = \int_{-3}^{0} (t+3)^2 dt + \int_{0}^{3} (\tfrac{5}{3}t-3)^2 dt = \int_{-3}^{0} (t^2+6t+9)dt + \int_{0}^{3} (\tfrac{25}{9}t^2-10t+9)dt$$

$$= \left[\tfrac{1}{3}t^3+3t^2+9t\right]_{-3}^{0} + \left[\tfrac{25}{27}t^3-5t^2+9t\right]_{0}^{3} = -(-9+27-27)+(25-45+27) = 9+7$$

$$= 16.$$

(iii) The function $g(t)$ can be represented as $g(t) = \begin{cases} -5t+12 & 2 \le t \le 3 \\ 3t-12 & 3 \le t \le 4 \end{cases}$

Since $g(t)$ is a finite duration signal, it is an energy signal. The average power in $g(t)$ is 0, while its total energy is given by

$$E_g = \int_{-\infty}^{\infty} g^2(t)dt = \int_{2}^{3} (-5t+12)^2 dt + \int_{3}^{4} (3t-12)^2 dt = \int_{2}^{3} (25t^2-120t+144)dt + \int_{3}^{4} (9t^2-72t+144)dt$$

$$= \left[\tfrac{25}{3}t^3-60t^2+144t\right]_{2}^{3} + \left[3t^3-36t^2+144t\right]_{3}^{4} = \frac{16}{3}$$

Problem 1.26:

(i) The function $g(t) = f(-2t+6)$ is shown in Fig. S1.26.

(ii) The end and odd components of $f(t)$ are also shown in Fig. S1.26.

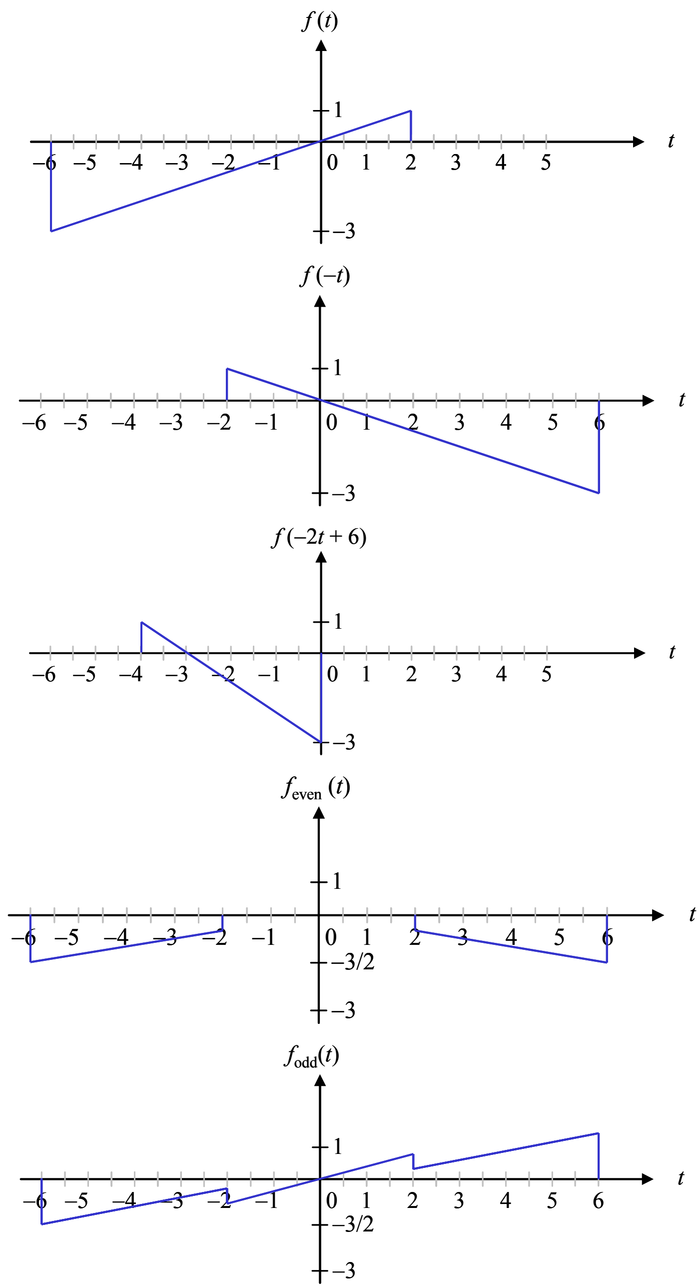


Figure S1.26: Waveforms for Problem 1.26.

Problem 1.27:

The waveforms for $g(t)$ and $g(2t)$ are plotted in Fig. S1.27.

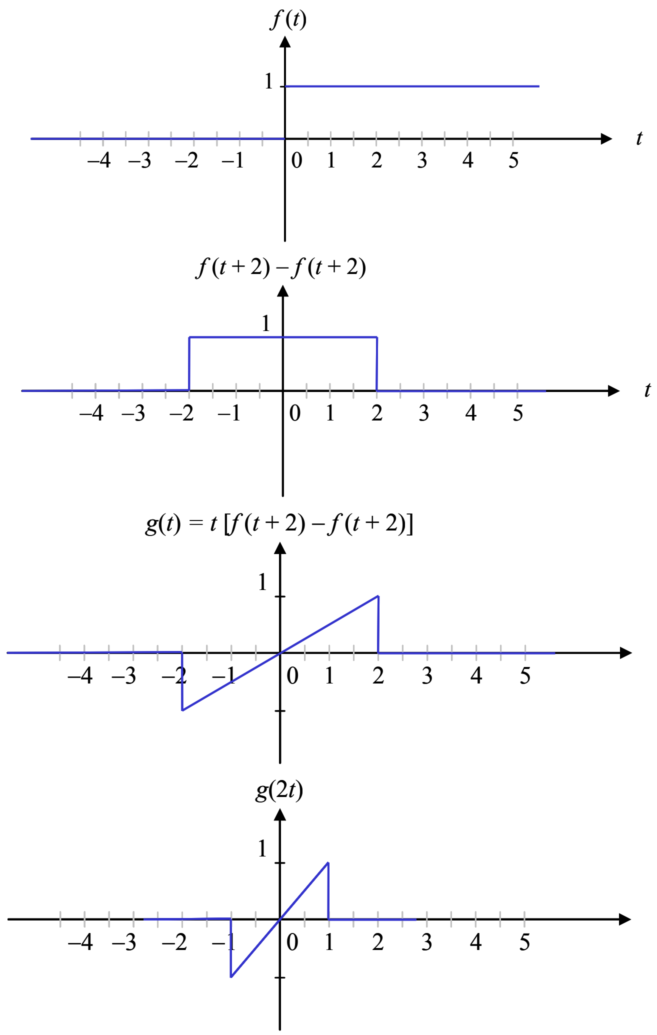


Fig. P1.27: Waveforms for Problem 1.27.

Problem 1.28:

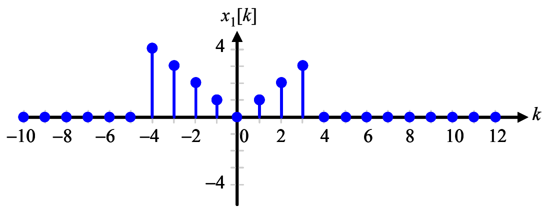
The values for $x1[k]$ and $x2[k]$ for $(-6 \le k \le 5)$ are shown in Table S1.28.

Table S1.28: Values of $x1$[k] and $x2$[k] in Problem 1.28.

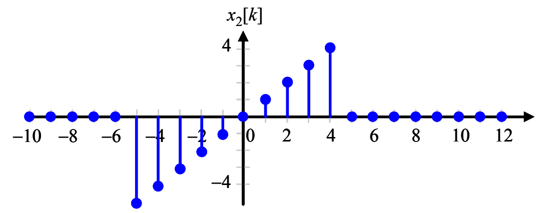
k	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5
$x1$	0	0	4	3	2	1	0	1	2	3	0	0
$x2$	0	1	1	1	1	1	1	1	1	1	1	0

The sketch of $x1$[k] and $x2$[k] is shown in Fig. S1.28. The remaining figures are obtained by applying translation, inversion and scaling procedures, and are also shown in Fig. S1.28. Note that all functions, except $x1[k/2]$ are uniquely defined. The function $x1[k/2]$ is not uniquely defined when k is odd. Here, we have used linear interpolation, defined as follows, to calculate the odd samples.

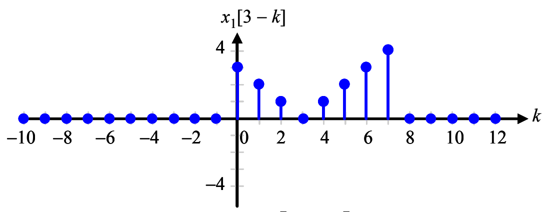
$$x1\left[\tfrac{k}{2}\right]=\tfrac{1}{2}\left\{x1\left[\tfrac{k-1}{2}\right]+x1\left[\tfrac{k+1}{2}\right]\right\} \quad \text{when } k=\pm1,\pm3,\pm5,.... \qquad \blacksquare$$



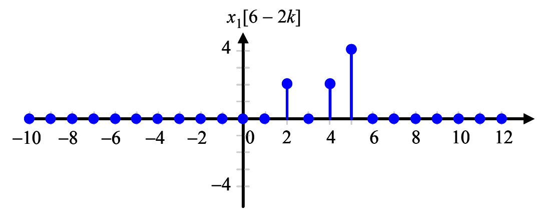
(i) $x_1[k]$



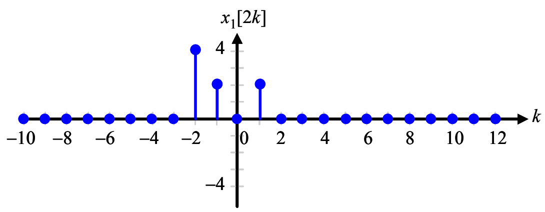
(ii) $x_2[k]$



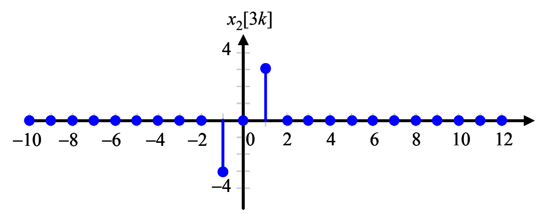
(iii) $x_1[3-k]$



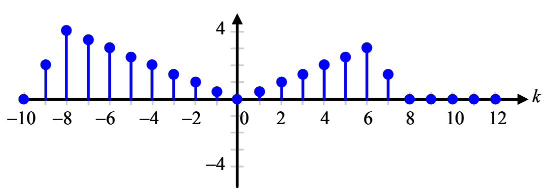
(iv) $x_1[6-2k]$



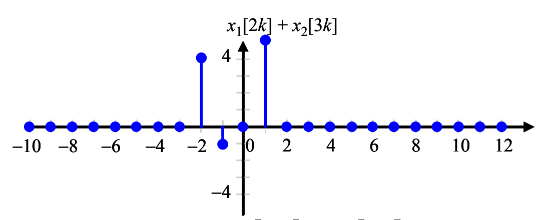
(v) $x_1[2k]$



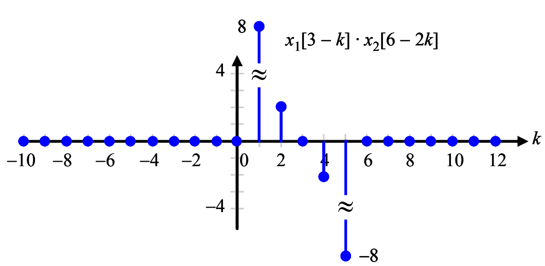
(vi) $x_2[3k]$

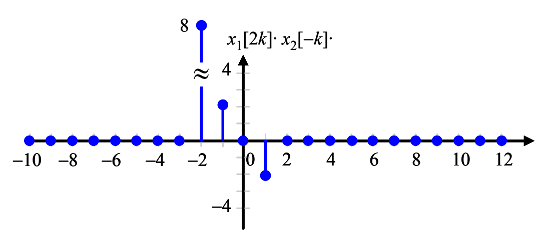


(vii) $x_1[k/2]$



(viii) $x_1[2k]+x_2[3k]$





(ix) $x_1[3-k]x_2[6-2k]$ (x) $x_1[2k]x_2[-k]$

Fig. S1.28: Waveforms for Problem 1.28.

Program 1.28: MATLAB Program

```
% clear figure
clf
% signal defined in part (i)
k1 =-6:6  ;
x1 = [0 0 4 3 2 1 0 1 2 3 0 0 0];
subplot(2,2,1), stem(k1, x1, 'filled'), grid on
xlabel('k')                                % Label of X-axis
ylabel('x1[k]')                        % Label of Y-axis
axis([-6, 6, 0, 5]) ;

% signal defined in part (iii)
x1flip = fliplr(x1) ;          % inverted x1
subplot(2,2,2), stem(k1+5, x1flip, 'filled'), grid on
xlabel('k')                                % Label of X-axis
ylabel('x1[3-k]')                      % Label of Y-axis
axis([-1, 11, 0, 5]) ;

% signal defined in part (v)
x1_compress = x1(1:2:length(x1));                % decimated by 2
subplot(2,2,3), stem([-3:3], x1_compress, 'filled'), grid on
xlabel('k')                            % Label of X-axis
ylabel('x1[2k]')                       % Label of Y-axis
axis([-3, 3, 0, 5]) ;

% signal defined in part (vii)
k4 = [-12:12] ;
x1_expand = [0  0  0  2  4  3.5  3  2.5  2
subplot(2,2,4), stem(k4, x1_expand, 'filled'), grid on
xlabel('k')                            % Label of X-axis
ylabel('x1[2k]')                       % Label of Y-axis
axis([-12, 10, 0, 5]) ;
print -dtiff plot.tiff                 % Save the figure as a TIFF file
```

Problem 1.29

The classification of the ECG signal is explained below.

Continuous-time vs discrete-time: The signal generated by heart is continuous-time in nature. However, the ECG signal produced by the ECG instrument can be CT or DT, depending on the instrument type. In the older days, the signals were typically CT. However, with advances in digital technology, the modern ECG instruments are generally discrete-time. However, when a discrete time signal is generated with a high sampling rate, and plotted, the plot looks continuous-time (your eyes are fooled).

Analog vs. Digital: The signal can be CT or DT depending on the instrument type.

Deterministic vs Random: The heartbeat of a person is generally random in nature (otherwise you could predict heart attack).

Periodic vs. Aperiodic: The ECG signals looks like a periodic signal where the pattern repeats itself roughly every 0.4-1 second (i.e., once in every heart beat). However, the heart beat rate is not constant. During sleep, it is the lowest, and during exercise, it is the highest. Therefore, it is not periodic in strict mathematical sense.

Power vs. Energy signal: The ECG signal corresponding to a person is a bounded (the amplitude does not exceed a few milli-volt) and time-limited. Therefore, it is an energy signal.

Even or Odd: A random signal is generally neither even nor odd. Also, how do you define t=0 point for an ECG signal? Even if you look at just one pattern, it does not look like an even or odd function. Therefore, the ECG signal is neither even nor odd. ▌

Problem 1.30:

Recall that the ramp function $$r(t) = tu(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Therefore, *f*(t) can be expressed as

$$f(t) = \tfrac{1}{2} r(t) \times [u(t) - u(t-2)] - \tfrac{1}{2} r(-t) \times [u(t+6) - u(t)].$$ ▌

Problem 1.31:

The MATLAB code is given in Program S1.31. The plots are shown in Fig. S1.31. ▌

Program S1.31: MATLAB code for Problem 1.31.

```
% Problem 1.31 from Mandal and Asif text
% part (i)
t = -1:0.001:1;
x = exp(-2*t).*sin(10*pi*t);
subplot(5,1,1)
plot(t,x);
xlabel('t');
title('(i) exp(-2t) sin(10\pit)');
grid on
axis tight
%
% part (ii)
t = -10:0.001:15;
x = sawtooth(2*pi*t/5);
subplot(5,1,2)
plot(t,x);
xlabel('t');
title('(ii) Sawtooth wave with a period of 5s');
grid on
axis tight
%
% part (iii)
t = -10:0.001:10;
x = 0.5*(1 + sign(t));
subplot(5,1,3)
```

```
plot(t,x);
xlabel('t');
title('(iii) u(t)');
grid on
axis([-10 10 -0.1 1.1]);
%
% part (iv)
t = -10:0.001:10;
unit_step1 = 0.5*(1 + sign(t + 5));
unit_step2 = 0.5*(1 + sign(t - 5));
x = unit_step1 - unit_step2;
subplot(5,1,4)
plot(t,x);
xlabel('t');
title('(iv) rect(t/10)');
grid on
axis([-10 10 -0.1 1.1]);
%
% part (v)
t = -12:0.001:12;
x = 3*square(2*pi*(t+1)/6,100/3);
subplot(5,1,5)
plot(t,x);
xlabel('t');
title('(v) Square wave');
grid on
axis([-10 10 -3.1 3.1]);
```

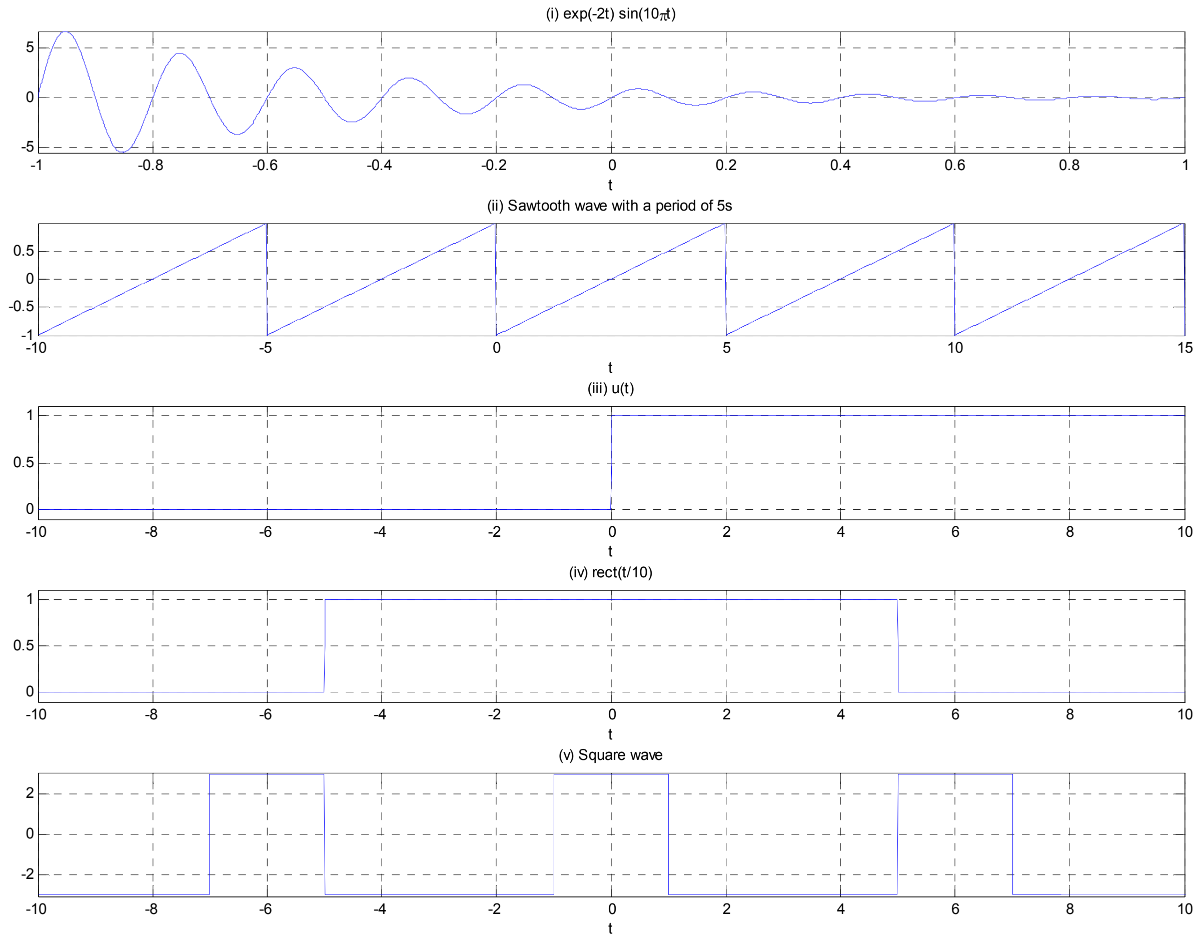


Figure S1.31: Plots for Problem 1.31.

Problem 1.32:

The MATLAB function `mydecimate` is given in Program S1.32.

Program S1.32: MATLAB code for Problem 1.32.

```
function [y] = mydecimate(x, N)
% MYSCALE: computes y[k] = x[k/N]
% where
% x is a column vector containing the DT input signal
% N is the scaling factor greater than 1
% y is a column vector containing the DT output signal time expanded by N

y = x(1:N:length(x));
y = y';
end
```

Problem 1.33:

The MATLAB function `myinterpolate` is given in Program S1.33. ▌

Program S1.33: MATLAB code for Problem 1.33.

```
function [y] = myinterpolate(x, N)
% MYINTERPOLATE: computes y[k] = x[k/N]
% where
% x is a column vector containing the DT input signal
% N is the scaling factor greater than 1
% y is a column vector containing the DT output signal time expanded by N

all_but_last = x(1:length(x)-1);
all_but_first = x(2:length(x));

y = all_but_last;
for i = 2:N,
    y(:,i) = y(:,1) + (i-1)/N * (all_but_first - all_but_last);
    % linear interpolation is used to predict the unknown values.
end

y = y';
y = y(:);
y(length(y)+1) = x(length(x));
end
```

Problem 1.34:

The MATLAB code is given in Program S1.34. The plots are shown in Fig. S1.34.

Program S1.34: MATLAB code for Problem 1.34.

```
% Problem 1.34 from Mandal and Asif text

% Define the signal
k = 0:120;
x = (1 - exp(-0.003*k)).*cos(pi*k/10);
x = x';

% part (i) -- plot the signal
subplot(311);
stem(k,x);
xlabel('k');
ylabel('x[k]');
title('x[k] = (1 - exp(-0.003k)) cos(\pik/20)');

% part (ii)  -- Decimation followed by interpolation
z1 = myinterpolate(mydecimate(x,5),5);
subplot(312);
stem(k,z1);
xlabel('k');
ylabel('z_1[k]');
title('z_1[k] = y[5k] where y[k] = x[k/5]');

% part (iii) -- Interpolation followed by decimation
z2 = mydecimate(myinterpolate(x,5),5);
subplot(313);
stem(k,z2);
xlabel('k');
```

```
ylabel('z_2[k]');
title('z_2[k] = y[k/5] where y[k] = x[5k]');
```

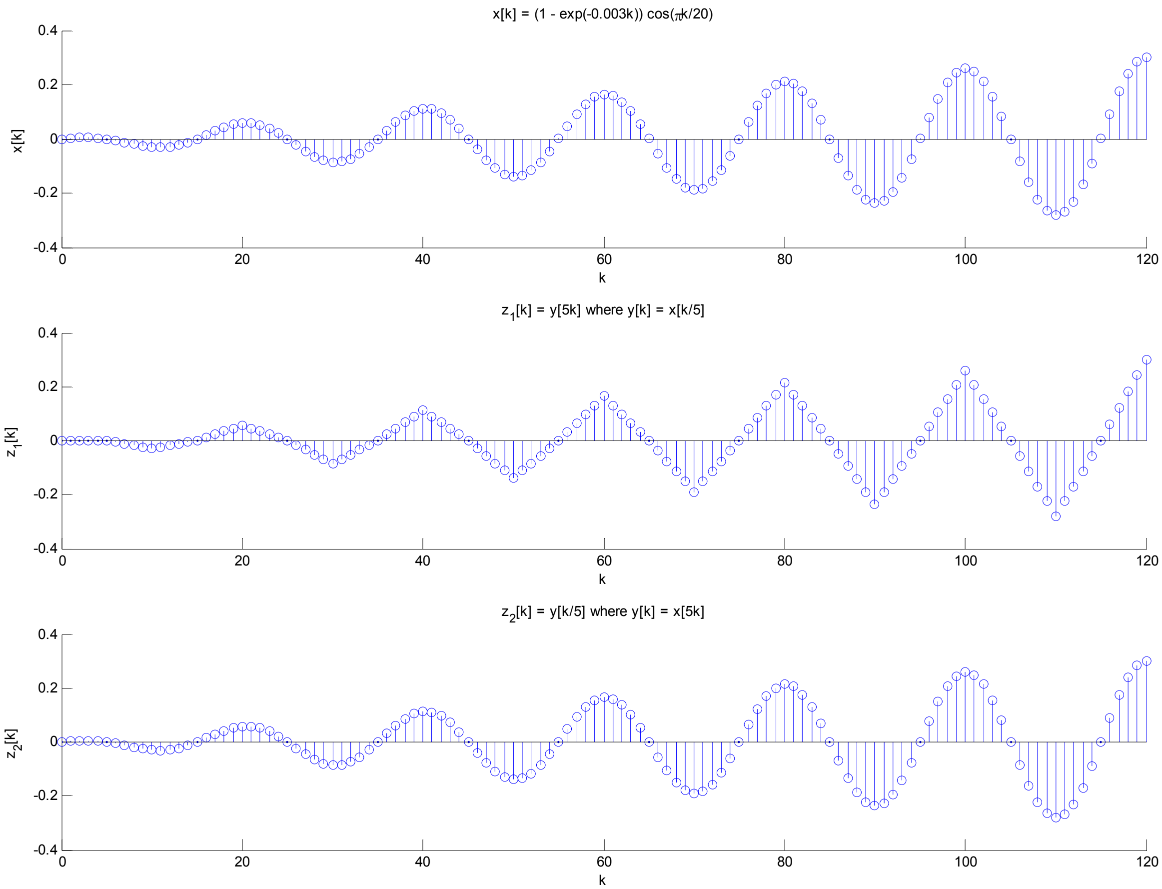


Fig. 1.34: Output for Problem 1.34.

Note that decimation followed by interpolation distorts the signal such that the reconstructed signal is different from the original signal. By doing decimation first, we lose 4 out of every 5 samples. Interpolation can only reconstruct the lost samples approximately.

On the other hand, interpolation followed by decimation reconstructs the signal exactly. Interpolation introduces 5 additional samples in between every two neighboring samples. Decimation removes the interpolated values so the original signal is not affected. ▌