Chapter 7: Continuous-time Filters

Problem 7.1

Ideal Bandpass Filter: The transfer function of a bandpass filter is given by

$$H_{bp}(\omega)=\begin{cases}A & \omega_{c1}\le|\omega|\le\omega_{c2}\\ 0 & \omega_{c1}<|\omega| \text{ and } \omega_{c2}<|\omega|<\infty,\end{cases}$$

which can be expressed in terms of the transfer functions of two lowpass filters as

$$H_{bp}(\omega)=\underbrace{\begin{cases}A & |\omega|\le\omega_{c2}\\ 0 & |\omega|>\omega_{c2}\end{cases}}_{H_{lp2}(\omega)}-\underbrace{\begin{cases}A & |\omega|\le\omega_{c1}\\ 0 & |\omega|>\omega_{c1}\end{cases}}_{H_{lp1}(\omega)}.$$

Taking the inverse CTFT and using the result (7.8) in Example 7.1, the impulse response of the bandpass filter is given by

$$h_{bp}(t)=\tfrac{\omega_{c2}A}{\pi}\,\mathrm{sinc}\left(\tfrac{\omega_{c2}t}{\pi}\right)-\tfrac{\omega_{c1}A}{\pi}\,\mathrm{sinc}\left(\tfrac{\omega_{c1}t}{\pi}\right).$$

Ideal Bandstop filter: The transfer function of a bandpstop filter can be expressed in terms of the transfer function of the bandpass filter as

$$H_{bs}(\omega)=A-\underbrace{\begin{cases}A & \omega_{c1}\le|\omega|\le\omega_{c2}\\ 0 & \omega_{c1}<|\omega| \text{ and } \omega_{c2}<|\omega|<\infty\end{cases}}_{H_{bp}(\omega)}$$

Taking the inverse CTFT, we get

$$h_{bs}(t)=A-\tfrac{\omega_{c2}A}{\pi}\,\mathrm{sinc}\left(\tfrac{\omega_{c2}t}{\pi}\right)+\tfrac{\omega_{c1}A}{\pi}\,\mathrm{sinc}\left(\tfrac{\omega_{c1}t}{\pi}\right)$$

where we replaced the result for the impulse response of the bandpass filter from the earlier derivation. ▌

Problem 7.2

(a) Butterworth filter of order $N = 12$: Using Eq. (7.20), the poles of $H(s)H(-s)$ are given by

$$s=\exp\left[j\,\tfrac{\pi}{2}+j\,\tfrac{(2n-1)\pi}{24}\right]$$

for ($0 \le n \le 23$). Substituting different values of n, the locations of the poles are specified in Table S7.2(a). Note that only the shaded cells corresponds to poles lying in the left half of the complex s-plane and are included in the lowpass Butterworth filter.

(b) Butterworth filter of order $N = 13$: Using Eq. (7.20), the poles of $H(s)H(-s)$ are given by

$$s = \exp\left[j\frac{\pi}{2} + j\frac{(2n-1)\pi}{26}\right]$$

for ($0 \le n \le 25$). Substituting different values of n, the locations of the poles are specified in Table S7.2(b). ▮

Table S7.2(a): Location of 24 poles for $H(s)H(-s)$ in Problem 7.2(a) for $N = 12$.

n	0	1	2	3	4	5	6	7	8	9	10	11
p_n	$e^{j11\pi/24}$	$e^{j13\pi/24}$	$e^{j15\pi/24}$	$e^{j17\pi/24}$	$e^{j19\pi/24}$	$e^{j7\pi/8}$	$e^{j23\pi/24}$	$e^{-j23\pi/24}$	$e^{-j7\pi/8}$	$e^{-j19\pi/24}$	$e^{-j17\pi/24}$	$e^{-j5\pi/8}$
n	12	13	14	15	16	17	18	19	20	21	22	23
p_n	$e^{-j13\pi/24}$	$e^{-j11\pi/24}$	$e^{-j3\pi/8}$	$e^{-j7\pi/24}$	$e^{-j5\pi/24}$	$e^{-j\pi/8}$	$e^{-j\pi/24}$	$e^{j\pi/24}$	$e^{j\pi/8}$	$e^{j5\pi/24}$	$e^{j7\pi/24}$	$e^{j3\pi/8}$

Table S7.2(b): Location of 26 poles for $H(s)H(-s)$ in Problem 7.2(b) for $N = 13$.

n	0	1	2	3	4	5	6	7	8	9	10	11	12
p_n	$e^{j6\pi/13}$	$e^{j7\pi/13}$	$e^{j8\pi/13}$	$e^{j9\pi/13}$	$e^{j10\pi/13}$	$e^{j11\pi/13}$	$e^{j12\pi/13}$	-1	$e^{-j12\pi/13}$	$e^{-j11\pi/13}$	$e^{-j10\pi/13}$	$e^{-j9\pi/13}$	$e^{-j8\pi/13}$
n	13	14	15	16	17	18	19	20	21	22	23	24	25
p_n	$e^{-j7\pi/13}$	$e^{-j6\pi/13}$	$e^{-j5\pi/13}$	$e^{-j4\pi/13}$	$e^{-j3\pi/13}$	$e^{-j2\pi/13}$	$e^{-j\pi/13}$	1	$e^{j\pi/13}$	$e^{j2\pi/13}$	$e^{j3\pi/13}$	$e^{j4\pi/13}$	$e^{j5\pi/13}$

Problem 7.3

From Eq. (7.20), the locations of the poles of the lowpass Butterworth filter of order N is given by

$$s = \omega_c \exp\left[j\frac{\pi}{2} + j\frac{(2n-1)\pi}{2N}\right] = \omega_c \exp\left[j\frac{(N+2n-1)\pi}{2N}\right]$$

for ($1 \le n \le N$), where the poles in the left half of the complex s-plane are selected. For at least one pole to lie on the real axis, the argument of the exponent must be equal to π, i.e.,

$$\frac{(N+2n-1)}{2N} = 1\text{, or, } n = (N+1)/2\,,$$

which lies within the range ($1 \le n \le N$). Since $n = (N + 1)/2$ is a whole number for odd values of N, a lowpass Butterworth filter with an odd value of order N has at least one pole on the real axis in the complex s-plane.

If N is even, $n = (N + 1)/2$ does not result in a whole value for n. Therefore, in such a case no pole exists on the real axis in the complex s-plane. ▮

Problem 7.4

In this problem, we have to prove that if

$$s_0 = \omega_c \exp\left[j\tfrac{\pi}{2} + j\tfrac{(2n_0-1)\pi}{2N}\right]$$

is a pole of the lowpass Butterworth filter of order N then its conjugate

$$s_1 = s_0^* = \omega_c \exp\left[-j\tfrac{\pi}{2} - j\tfrac{(2n_0-1)\pi}{2N}\right]$$

is also a pole of the same lowpass Butterworth filter.

Note that

$$\begin{aligned}(\tfrac{s_1}{j\omega_c})^{2N} &= \exp\left[\left(-j\tfrac{\pi}{2} - j\tfrac{(2n_0-1)\pi}{2N} - j\tfrac{\pi}{2}\right)\times 2N\right] \\ &= \exp[-j2N\pi - j(2n_0-1)\pi] = \underbrace{\exp[-j2N\pi]}_{=1} \times \underbrace{\exp[-j(2n_0-1)\pi]}_{=-1} = -1.\end{aligned}$$

In other words, the pole $s = (s_0)^*$ also satisfies Eq. (7.19), and hence all complex poles of the lowpass Butterworth filter occur in conjugate pairs. ▮

Problem 7.5

For the interval $(-1 \le \omega \le 1)$, the N'th order Type I Chebyshev polynomial $T_N(\omega)$ is given by

$$T_N(\omega) = \cos\left(N\cos^{-1}(\omega)\right).$$

The roots of the polynomial are $$\cos\left(N\cos^{-1}(\omega)\right) = 0,$$

or, $$N\cos^{-1}(\omega) = \frac{(2n+1)\pi}{2},$$

for $(0 \le n \le N-1)$. Rearranging terms, the roots are given by

$$\omega_n = \cos\left[\frac{(2n+1)\pi}{2N}\right], \; 0 \le n \le N-1.$$ ▮

Problem 7.6

Consider the function $$1 + \varepsilon^2 T_N^2(\theta) = 0.$$

Case I: For $(\theta \le 1)$, we get $$1 + \varepsilon^2 \cos(N\cos^{-1}(\theta)) = 0,$$

which has roots at $\theta = \cos\left[\tfrac{1}{N}\cos^{-1}(\pm j/\varepsilon) + \tfrac{m\pi}{N}\right]$, for $(0 \le m \le (2N-1)$.

Substituting $\theta = s/j$ for Type I Chebyshev filter results in the roots

$$s_1 = j\cos\left[\tfrac{1}{N}\cos^{-1}(\pm j/\varepsilon) + \tfrac{m\pi}{N}\right]. \tag{P7.6.1}$$

Substituting $\theta = j/s$ for Type II Chebyshev filter results in the roots

$$s_2 = j \Big/ \cos\left[\tfrac{1}{N}\cos^{-1}(\pm j/\varepsilon) + \tfrac{m\pi}{N}\right]. \tag{P7.6.2}$$

Knowing that the roots occur in conjugate pairs ($\pm\alpha \pm j\beta$) that are symmetric about the origin, Eqs. (P7.6.1) and (P7.6.2) prove that roots of the characteristic equation of Type I Chebyshev filter are inverse of the roots of the characteristic equation of Type II Chebyshev filter. $\blacksquare$

Problem 7.7

Using Step 1 of Algorithm 7.3.1.1, the gain terms G_p and G_s are given by

$$G_p = \frac{1}{(1-\delta_p)^2} - 1 = \frac{1}{0.9^2} - 1 = 0.2346 \text{ and } G_s = \frac{1}{(\delta_s)^2} - 1 = \frac{1}{0.1^2} - 1 = 99.$$

Using Eq. (7.29), the order of the Butterworth filter is given by

$$N = \frac{1}{2} \times \frac{\ln(G_p/G_s)}{\ln(\omega_p/\omega_s)} = \frac{1}{2} \times \frac{\ln(0.2346/99)}{\ln(10/20)} = 4.3605.$$

We round off the order of the filter to the higher integer value as $N = 5$.

Using Step 2 of Algorithm 7.3.1.1, the transfer function $H(S)$ of the normalized Butterworth filter with a cut off frequency of 1 radians/s from Table 7.2 is given by

$$\begin{aligned} H(S) &= \frac{1}{(S+1)(S^2+0.6180S+1)(S^2+1.6180S+1)} \\ &= \frac{1}{S^5+3.2360S^4+5.2359S^3+5.2359S^2+3.2360S+1}. \end{aligned}$$

Using the stop band constraint, Eq. (7.32), in Step 3 of Algorithm 7.3.1.1, the cut off frequency of the required Butterworth filter is given by

$$\omega_c = \frac{\omega_s}{(G_s)^{\frac{1}{2N}}} = \frac{20}{(99)^{\frac{1}{10}}} = 12.6318 \text{ radians/s.}$$

Using Step 4 of Algorithm 7.3.1.1, the transfer function $H(s)$ of the required Butterworth filter is obtained by the transformation and simplification.

$$\begin{aligned} H(s) = H(S)\big|_{S=s/\omega_c} &= \left.\frac{1}{S^5+3.2360S^4+5.2359S^3+5.2359S^2+3.2360S+1}\right|_{S=s/12.6318} \\ &= \frac{12.6318^5}{s^5+12.6318\times3.2360s^4+12.6318^2\times5.2359s^3+12.6318^3\times5.2359s^2+12.6318^4\times3.2360s+12.6318^5} \\ &= \frac{3.2161\times10^5}{s^5+40.8765s^4+835.4526s^3+1.0553\times10^4s^2+8.2389\times10^4s+3.2161\times10^5} \end{aligned}$$

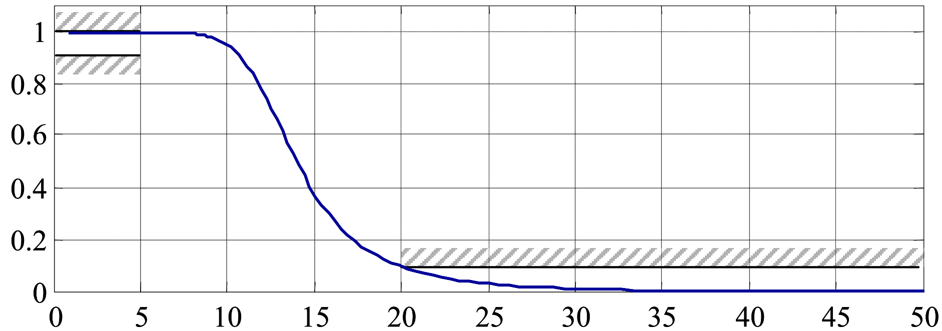

Figure S7.7: Magnitude spectrum of the Butterworth low pass filter designed in Problem 7.7.

Step 5 plots the magnitude spectrum of the Butterworth filter. The CTFT transfer function of the Butterworth filter is given by

$$H(\omega) = H(s)\big|_{s=j\omega} = \frac{3.2161\times10^5}{(j\omega)^5 + 40.8765(j\omega)^4 + 835.4526(j\omega)^3 + 1.0553\times10^4(j\omega)^2 + 8.2389\times10^4(j\omega) + 3.2161\times10^5}$$

The magnitude spectrum $|H(\omega)|$ is plotted in Fig. S7.7 with the specifications shown with the shaded lines. We observe that the design specifications are indeed satisfied by the magnitude spectrum. ∎

Problem 7.8

Expressed on a linear scale, the pass band and stop band gains are given by

$$(1-\delta_p) = 10^{-1/20} = 0.8913 \quad \text{and} \quad \delta_s = 10^{-25/20} = 0.0562 .$$

Using Step 1 of Algorithm 7.3.1.1, the gain terms G_p and G_s are given by

$$G_p = \frac{1}{(1-\delta_p)^2} - 1 = \frac{1}{0.8913^2} - 1 = 0.2588$$

and

$$G_s = \frac{1}{(\delta_s)^2} - 1 = \frac{1}{0.0562^2} - 1 = 315.6120 .$$

Using Eq. (7.29), the order of the Butterworth filter is given by

$$N = \frac{1}{2} \times \frac{\ln(G_p/G_s)}{\ln(\omega_p/\omega_s)} = \frac{1}{2} \times \frac{\ln(0.2588/315.6120)}{\ln(50/65)} = 13.5426 .$$

We round off the order of the filter to the higher integer value as $N = 14$.

Using Eq. (7.20), the poles of $H(s)$ are given by

$$s = \exp\left[j\tfrac{\pi}{2} + j\tfrac{(2n-1)\pi}{28}\right]$$

for $(1 \le n \le 14)$. Substituting different values of n, the locations of the poles are specified in Table S7.8.

Using Step 2 of Algorithm 7.3.1.1, the transfer function $H(S)$ of the normalized Butterworth filter with a cut off frequency of 1 radians/s is given by

$$H(S)=\prod_{n=1}^{14}\frac{1}{(S-p_n)},$$

which simplifies to

$$H(S)=\frac{1}{(S^{14}+8.9314S^{13}+39.8850S^{12}+117.7337S^{11}+256.1214S^{10}+433.7284S^{9}+589.0206S^{8}+\ldots}$$

$$\overline{\ldots+651.2664S^{7}+589.0206S^{6}+433.7284S^{5}+256.1214S^{4}+117.7337S^{3}+39.8850S^{2}+8.9314S+1)}$$

Table S7.8: Location of 14 poles for $H(s)$ in Problem 7.8 for $N = 14$.

n	1	2	3	4	5	6	7
p_n	$e^{j15\pi/28}$	$e^{j17\pi/28}$	$e^{j19\pi/28}$	$e^{j3\pi/4}$	$e^{j23\pi/28}$	$e^{j25\pi/28}$	$e^{j27\pi/28}$
n	8	9	10	11	12	13	14
p_n	$e^{-j27\pi/28}$	$e^{-j25\pi/28}$	$e^{-j23\pi/28}$	$e^{-j3\pi/4}$	$e^{-j19\pi/28}$	$e^{-j17\pi/28}$	$e^{-j15\pi/28}$

Using the stop band constraint, Eq. (7.32), in Step 3 of Algorithm 7.3.1.1, the cut off frequency of the required Butterworth filter is given by

$$\omega_c=\frac{\omega_s}{(G_s)^{\frac{1}{2N}}}=\frac{65}{(315.6120)^{\frac{1}{28}}}=52.9246\ \text{radians/s.}$$

Using Step 4 of Algorithm 7.3.1.1, the transfer function $H(s)$ of the required Butterworth filter is obtained by the transformation

$$H(s)=H(S)\big|_{S=s/\omega_c}=H(S)\big|_{S=s/52.9246},$$

which simplifies to

$$H(s)=\frac{1.3536\times10^{24}}{(s^{14}+472.7119s^{13}+1.1173\times10^{5}s^{12}+1.7455\times10^{7}s^{11}+2.0098\times10^{9}s^{10}+1.8014\times10^{11}s^{9}+\ldots}$$

$$\overline{\ldots+1.2948\times10^{13}s^{8}+7.5770\times10^{14}s^{7}+3.6270\times10^{16}s^{6}+1.4135\times10^{18}s^{5}+4.4179\times10^{19}s^{4}+\ldots}$$

$$\overline{\ldots+1.0748\times10^{21}s^{3}+1.9272\times10^{22}s^{2}+2.2841\times10^{23}s+1.3536\times10^{24})}$$

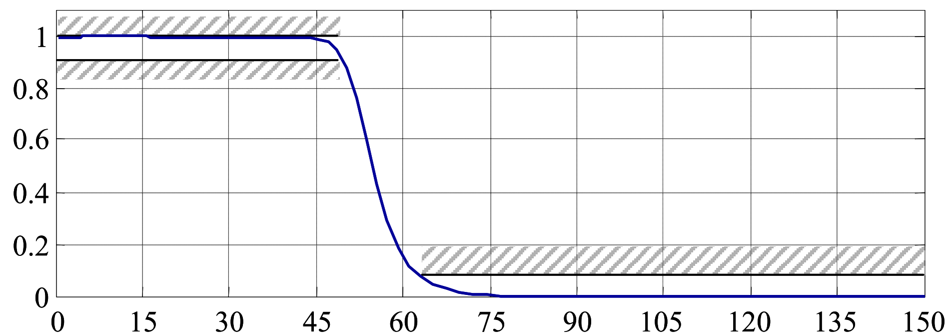

Figure S7.8: Magnitude spectra of the Butterworth low pass filters, designed in Problem 7.8.

Step 5 plots the magnitude spectrum of the Butterworth filter. The magnitude spectrum $|H(\omega)|$ is plotted in Fig. S7.8 with the specifications shown with the shaded lines. We observe that the design specifications are indeed satisfied by the magnitude spectrum. ▮

Problem 7.9

(a) In Problem 7.7, the gain terms are given by $G_p = 0.2346$ and $G_s = 99$.

Step 1 determines the value of the ripple control factor ε as

$$\varepsilon = \sqrt{G_p} = \sqrt{0.2346} = 0.4844 .$$

Step 2 determines the order N of the Chebyshev polynomial as:

$$N = \frac{\cosh^{-1}\left[(99/0.2346)^{0.5}\right]}{\cosh^{-1}\left[20/10\right]} = 2.8209 .$$

We round off N to the closest higher integer as 3.

Step 3 determines the location of the six poles of $H(S)H(-S)$ as

$$[-0.2553 + j0.9724, \quad 0.2553 + j0.9724, \quad 0.2553 - j0.972, \quad -0.2553 - j0.972, \quad -0.5106, \quad 0.5106].$$

The 3 poles lying in the left half s-plane are included in the transfer function $H(S)$ of the normalized Type I Chebyshev filter. These poles are located at

$$[-0.2553 + j0.9724, \quad -0.2553 - j0.972, \quad -0.5106 .$$

The transfer function for the normalized Type-I Chebshev filter is, therefore, given by

$$H(S) = \frac{K}{(S + 0.2553 + j0.9724)(S + 0.2553 - j0.9724)(S + 0.5106)},$$

which simplifies to $$H(S) = \frac{K}{S^3 + 1.0213S^2 + 1.2715S + 0.5162}.$$

Since $|H(\omega)|$ at $\omega = 0$ is $K/0.5162$, therefore, K is set to 0.5162 to make the dc gain equal to 1. The new transfer function with unity gain at $\omega = 0$ is given by

$$H(S) = \frac{0.5162}{S^3 + 1.0213S^2 + 1.2715S + 0.5162}.$$

Step 4 transforms the normalized Type-I Chebyshev filter using the relationship

$$H(s) = H(S)\big|_{S=s/10} = \frac{0.5162}{(s/10)^3 + 1.0213(s/10)^2 + 1.2715(s/10) + 0.5162},$$

or,
$$H(s) = \frac{516.2}{s^3 + 10.213s^2 + 127.15\times 10^3 s + 516.2},$$

which is the transfer function of the required low pass filter.

The magnitude spectrum of the Type-I Chebyshev filter is plotted in Fig. S7.9(a). It is observed that Fig. S7.9(a) satisfies the initial design specifications.

(b) In Problem 7.8, the gain terms are given by $G_p = 0.2588$ and $G_s = 315.6120$.

Step 1 determines the value of the ripple control factor ε as

$$\varepsilon = \sqrt{G_p} = \sqrt{0.2588} = 0.5087.$$

Step 2 determines the order N of the Chebyshev polynomial as.

$$N = \frac{\cosh^{-1}\left[(315.6120/0.2588)^{0.5}\right]}{\cosh^{-1}\left[65/50\right]} = 5.6133.$$

We round off N to the closest higher integer as 6.

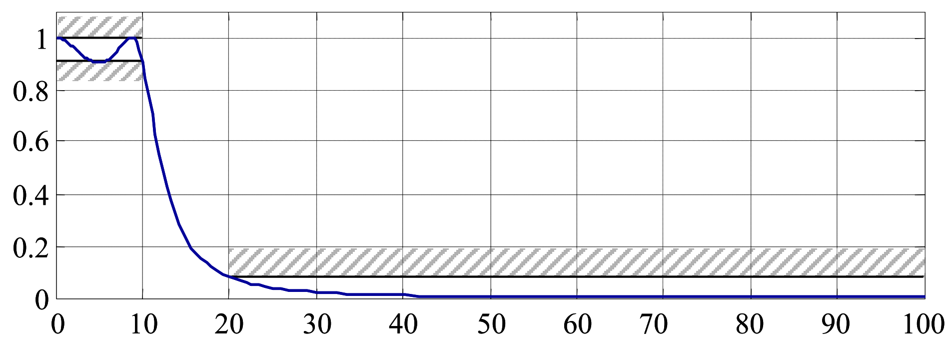

Figure S7.9(a): Magnitude spectrum of the Type-I Chebyshev lowpass filter designed in Problem 7.9(a).

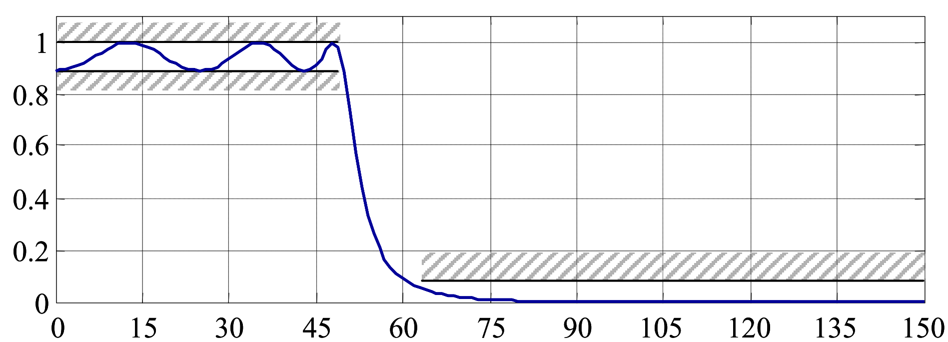

Figure S7.9(b): Magnitude spectrum of the Type-I Chebyshev lowpass filter designed in Problem 7.9(b).

Step 3 determines the location of the six poles of $H(S)H(-S)$ as

$$\begin{matrix}[-0.0622-j0.9934, & 0.0622-j0.9934, & 0.1699-j0.7272, & 0.2321-j0.2662\\ 0.2321+j0.2662, & 0.1699+j0.7272, & 0.0622+j0.9934, & -0.0622+j0.9934,\\ -0.1699+j0.7272, & -0.2321+j0.2662, & -0.2321-j0.2662, & -0.1699-j0.7272]\end{matrix}.$$

The 6 poles lying in the left half s-plane are included in the transfer function $H(S)$ of the normalized Type I Chebyshev filter. These poles are located at

$$\begin{matrix}[-0.0622-j0.9934, & -0.0622+j0.9934, & -0.1699+j0.7272, & -0.2321+j0.2662\\ & & -0.2321-j0.2662, & -0.1699-j0.7272,]\end{matrix}$$

The transfer function for the normalized Type-I Chebshev filter is, therefore, given by

$$H(S)=\frac{K}{(S+0.0622+j0.9934)(S+0.0622-j0.9934)(S+0.1699-j0.7272)}\times \frac{1}{(S+0.1699-j0.7272)(S+0.2321+j0.2662)(S+0.2321+j0.2662)}$$

which simplifies to

$$H(S)=\frac{K}{S^6+0.9284S^5+1.9310S^4+1.2024S^3+0.9395S^2+0.3072S+0.0689}.$$

Since $|H(\omega)|$ at $\omega = 0$ is $K/0.0689$, therefore, K is set to 0.0689 to make the dc gain equal to 1. The new transfer function with unity gain at $\omega = 0$ is given by

$$H(S)=\frac{0.0689}{S^6+0.9284S^5+1.9310S^4+1.2024S^3+0.9395S^2+0.3072S+0.0689}.$$

Step 4 transforms the normalized Type-I Chebyshev filter using the relationship

$$H(s)=H(S)\big|_{S=s/50},$$

we get

$$H(S)=\frac{1.0767\times10^9}{S^6+46.42S^5+4827.5S^4+1.503\times10^5S^3+5.871\times10^6S^2+9.6\times10^7S+1.0767\times10^9},$$

which is the transfer function of the required lowpass filter.

The magnitude spectrum of the Type-I Chebyshev filter is plotted in Fig. S7.9b, which satisfies the initial design specifications. ▌

Problem 7.10

(a) In Problem 7.7, the gain terms are given by $G_p = 0.2346$ and $G_s = 99$.

Step 1 determines the value of the ripple control factor ε as

$$\varepsilon=\frac{1}{\sqrt{G_s}}=\frac{1}{\sqrt{99}}=0.1005.$$

Step 2 determines the order N of the Chebyshev polynomial as.

$$N = \frac{\cosh^{-1}\left[(99/0.2346)^{0.5}\right]}{\cosh^{-1}\left[20/10\right]} = 2.8209,$$

which is the same as in Type I Chebyshev filter. We round off N to the closest higher integer as 3.

Step 3 determines the location of the poles and zeros of $H(S)H(-S)$.

We first determine the location of poles for the Type-I Chebshev filter with $\varepsilon = 0.1005$ and $N = 3$. Using Eq. (7.47), the location of poles for $H(s)H(-s)$ of the Type I Chebyshev filter are given by

$$[-0.5859 - j1.3341, \quad 0.5859 - j1.3341, \quad 0.5859 + j1.3341, \quad -0.5859 + j1.3341, \quad 1.1717, \quad -1.1717]$$

Selecting the poles located in the left half s-plane, we get

$$[-0.5859 + j1.3341, \quad -0.5859 - j1.3341, \quad -1.1717].$$

The poles of the normalized Type II Chebyshev filter are located at the inverse of the above locations and are given by

$$[-0.2760 - j0.6284, \quad -0.2760 + j0.6284, \quad -0.8534].$$

The zeros of the normalized Chebyshev Type II filter are computed using Eq. (7.60) and are given by

$$[-j1.1547, \quad j1.1547, \quad \infty].$$

The zero at $s = \infty$ is ignored. The transfer function for the normalized Type II Chebyshev filter is given by

$$H(S) = \frac{K(S + j1.1547)(S - j1.1547)}{(S + 0.2760 + j0.0.6284)(S + 0.2760 - j0.0.6284)(S + 0.8534)},$$

which simplifies to $$H(S) = \frac{K(S^2 + 1.3333)}{S^3 + 1.4054S^2 + 0.9421S + 0.4020}.$$

Since $|H(\omega)|$ at $\omega = 0$ is $1.3333/0.4020 = 3.3167$, therefore, K is set to $1/3.3167 = 0.3015$ to make the dc gain equal to 1. The new transfer function with unity gain at $\omega = 0$ is given by

$$H(S) = \frac{0.3015(S^2 + 1.3333)}{S^3 + 1.4054S^2 + 0.9421S + 0.4020}.$$

Step 4 normalizes $H(S)$ based on the transformation

$$H(s) = H(S)\big|_{S=s/20} = \frac{0.3015((s/20)^2 + 1.3333)}{(s/20)^3 + 1.4054(s/20)^2 + 0.9421(s/20) + 0.4020}$$

which simplifies to

$$H(s) = \frac{6.03(s^2 + 533.32)}{s^3 + 28.108s^2 + 376.84s + 3216}.$$

Step 5 plots the magnitude spectrum, which is shown in Fig. 7.10(a).

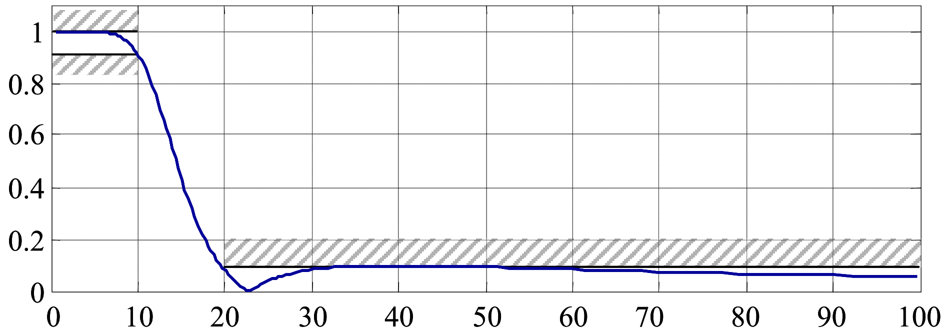

Figure S7.10(a): Magnitude spectrum of the Type-II Chebyshev filter designed in Problem 7.10(a).

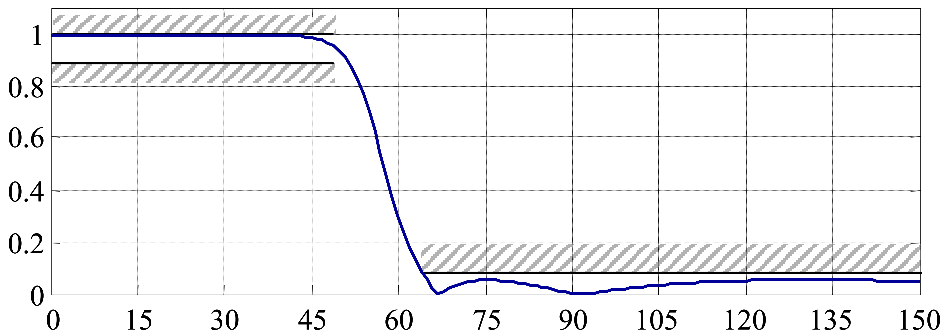

Figure S7.10(b): Magnitude spectrum of the Type-II Chebyshev filter designed in Problem 7.10(b).

(b) In Problem 7.8, the gain terms are given by $G_p = 0.2588$ and $G_s = 315.6120$.

Step 1 determines the value of the ripple control factor ε as

$$\varepsilon = \frac{1}{\sqrt{G_s}} = \frac{1}{\sqrt{315.6120}} = 0.0563 .$$

Step 2 determines the order N of the Chebyshev polynomial as.

$$N = \frac{\cosh^{-1}\left[(315.6120/0.2588)^{0.5}\right]}{\cosh^{-1}\left[65/50\right]} = 5.6133,$$

which is the same as in Type I Chebyshev filter. We round off N to the closest higher integer as 6.

Step 3 determines the location of the poles and zeros of $H(S)H(-S)$.

We first determine the location of poles for the Type-I Chebshev filter with $\varepsilon = 0.0563$ and $N = 6$. Using Eq. (7.47), the location of poles for $H(s)H(-s)$ of the Type I Chebyshev filter are given by

$$\begin{matrix} [-0.1633 - j1.1421, & 0.1633 - j1.1421, & 0.4462 - j0.8361, & 0.6095 - j0.3060, \\ 0.6095 + j0.3060, & 0.4462 + j0.8361, & 0.1633 + j1.1421, & -0.1633 + j1.1421, \\ -0.4462 + j0.8361, & -0.6095 + j0.3060, & -0.6095 - j0.3060, & -0.4462 - j0.8361] \end{matrix}$$

Selecting the poles located in the left half s-plane, we get

$$\begin{matrix} [-0.1633 - j1.1421, & -0.1633 + j1.1421, & -0.4462 + j0.8361, & -0.6095 + j0.3060, \\ & & -0.6095 - j0.3060, & -0.4462 - j0.8361] \end{matrix}.$$

The poles of the normalized Type II Chebyshev filter are located at the inverse of the above locations and are given by

$$[-0.1227 + j0.8580,\quad -0.1227 - j0.8580,\quad -0.4968 - j0.9309,\quad -1.3104 - j0.6580,\quad -1.3104 + j0.6580,\quad -0.4968 + j0.9309].$$

The zeros of the normalized Chebyshev Type II filter are computed using Eq. (7.60) and are given by

$$[-j1.0353,\quad -j1.4142,\quad -j3.8637,\quad j3.8637,\quad 1.4142,\quad j1.0353].$$

The zero at $s = \infty$ is ignored. The transfer function for the normalized Type II Chebyshev filter is given by

$$H(S) = \frac{K(S + j1.0353)(S - j1.0353)(S + j3.8637)(S - j3.8637)}{(S + 0.1227 - j0.8580)(S + 0.1227 + j0.8580)(S + 0.4968 + j0.9309)} \times \frac{(S + j1.4142)(S - j1.4142)}{(S + 0.4968 + j0.9309)(S + 1.3104 - j0.6580)(S + 1.3104 - j0.6580)},$$

which simplifies to

$$H(S) = K\frac{S^6 + 18S^4 + 48S^2 + 32}{S^6 + 3.8597S^5 + 7.5054S^4 + 9.2090S^3 + 8.0418S^2 + 4.3843S + 1.7984}$$

Since $|H(\omega)|$ at $\omega = 0$ is 32/1.7984 = 17.7936, therefore, K is set to 1/17.7936 = 0.0562 to make the dc gain equal to 1. The new transfer function with unity gain at $\omega = 0$ is given by

$$H(S) = \frac{0.0562(S^6 + 18S^4 + 48S^2 + 32)}{S^6 + 3.8597S^5 + 7.5054S^4 + 9.2090S^3 + 8.0418S^2 + 4.3843S + 1.7984}$$

Step 4 normalizes $H(S)$ based on the transformation

$$H(s) = H(S)\big|_{S=s/65}$$

which simplifies to

$$H(s) = \frac{0.0562(s^6 + 7.605\times10^4 s^4 + 8.568\times10^8 s^2 + 2.4134\times10^{12})}{s^6 + 250.8783s^5 + 3.171\times10^4 s^4 + 2.529\times10^6 s^3 + 1.436\times10^8 s^2 + 5.087\times10^9 s + 1.356\times10^{11}}.$$

Step 5 plots the magnitude spectrum, which is shown in Fig. S7.10(b). ▌

Problem 7.11

(a) In Problem 7.7, the gain terms are given by G_p = 0.2346 and G_s = 99.. The pass band and stop band corner frequencies are specified as ω_p = 10 radians/s and ω_s = 20 radians/s.

Using Eq. (7.64), the ripple control factor is given by

$$\varepsilon = \sqrt{G_p} = \sqrt{0.2346} = 0.4844.$$

Using Eq. (7.65) with ω_p/ω_s = 0.5 and G_p/G_s = 0.0024, the order N of the elliptic filter is given by

$$N = \frac{\psi\left[(\omega_p/\omega_s)^2\right]\psi\left[\sqrt{1 - G_p/G_s}\right]}{\psi\left[G_p/G_s\right]\psi\left[\sqrt{1-(\omega_p/\omega_s)^2}\right]} = \frac{\psi[0.25]\ \psi[0.9988]}{\psi[0.0024]\ \ \psi[0.8660]}.$$

Using MATLAB, $\psi[0.25] = 1.5962$, $\psi[0.9988] = 4.4048$, $\psi[0.0024] = 1.5708$, and $\psi[0.8660] = 2.1564$. The value of N is

$$N = \frac{1.5962 \times 4.4048}{1.5708 \times 2.1564} = 2.0757 .$$

Rounding off to the nearest higher integer, the order N of the filter equals 3.

(b) In Problem 7.8, the gain terms are given by $G_p = 0.2588$ and $G_s = 315.6120$. The pass band and stop band corner frequencies are specified as $\omega_p = 50$ radians/s and $\omega_s = 60$ radians/s.

Using Eq. (7.64), the ripple control factor is given by

$$\varepsilon = \sqrt{G_p} = \sqrt{0.2588} = 0.5087 .$$

Using Eq. (7.65) with $\omega_p/\omega_s = 0.8333$ and $G_p/G_s = 0.00082$, the order N of the elliptic filter is

$$N = \frac{\psi\left[(\omega_p/\omega_s)^2\right]\psi\left[\sqrt{1-G_p/G_s}\right]}{\psi\left[G_p/G_s\right]\psi\left[\sqrt{1-(\omega_p/\omega_s)^2}\right]} = \frac{\psi[0.8333]\,\psi[0.9996]}{\psi[0.00082]\ \ \psi[0.5528]} .$$

Using MATLAB, $\psi[0.8333] = 2.0672$, $\psi[0.9996] = 4.9526$, $\psi[0.00082] = 1.5708$, and $\psi[0.5528] = 1.7172$. The value of N is

$$N = \frac{2.0672 \times 4.9526}{1.5708 \times 1.7172} = 3.7955 .$$

Rounding off to the nearest higher integer, the order N of the filter equals 4. ▌

Problem 7.12

The computational complexity of implementing the filter is directly related to the order of the filter. For Problem 7.7, the orders N of the four types of the low pass filter are:

Butterworth:	$N = 4$
Type I Chebyshev:	$N = 3$
Type II Chebyshev:	$N = 3$
Elliptic:	$N = 3$

The Butterworth filter has the highest order (hence, the highest computational complexity), while Chebyshev and elliptic have the same order. For Problem 7.8, the orders N of the filters are:

Butterworth:	$N = 14$
Type I Chebyshev:	$N = 6$
Type II Chebyshev:	$N = 6$
Elliptic:	$N = 4$

By introducing permissible ripples in both pass and stop bands, the elliptic filter has the lowest order. The Chebyshev filters have the same order, while the Butterworth filter has the highest order. The amount of ripple is smallest in the Butterworth filter. ▌

Problem 7.13

Assume $S = (\sigma + j\omega)$ corresponds to the lowpass domain, while $s = (\gamma + j\xi)$ corresponds to the bandstop domain. In the frequency domain, the transformation is given by

$$\omega = \frac{\xi(\xi_{p2} - \xi_{p1})}{-\xi^2 + \xi_{p1}\xi_{p2}}.$$

Case I: Consider pass band I ($-\xi_{p1} \le \xi \le \xi_{p1}$) of the bandstop filter.

Frequency $\xi = \xi_{p1}$ maps to frequency $\omega = 1$ in the transformed domain.

Frequency $\xi = 0$ maps to frequency $\omega = 0$ in the transformed domain.

Frequency $\xi = -\xi_{p1}$ maps to frequency $\omega = -1$ in the transformed domain.

Case II: Consider pass band II ($\xi_{p2} \le \xi < \infty$) of the bandstop filter.

Frequency $\xi = \xi_{p2}$ maps to frequency $\omega = -1$ in the transformed domain.

Frequency $\xi = \infty$ maps to frequency $\omega = 0$ in the transformed domain.

Case III: Consider pass band II ($-\infty < \xi \le -\xi_{p2}$) of the bandstop filter.

Frequency $\xi = -\xi_{p2}$ maps to frequency $\omega = 1$ in the transformed domain.

Frequency $\xi = -\infty$ maps to frequency $\omega = 0$ in the transformed domain.

Case IV: Any frequency in the stop band I ($\xi_{p1} < \xi < \xi_{p2}$) of the bandstop filter is mapped in the range $|\omega| > 1$ in the transformed domain.

Consider, for example, $\xi = 0.5(\xi_{p1} + \xi_{p2})$, which is mapped as $\omega = 2 + 4 \times \xi_{p1}/(\xi_{p2} - \xi_{p1})$ and similarly, for all frequencies in the stop band range. $\blacksquare$

Problem 7.14

Using Eq. (7.68) with $\xi_p = 30$ radians/s to transform the specifications from the domain $s = \gamma + j\xi$ of the highpass filter to the domain $S = \sigma + j\omega$ of the lowpass filter, we get

Stop band ($\infty < |\omega| \le 2$ radians/s): $|H(\omega)| \le 0.15$

Pass band ($|\omega| < 1$ radians/s): $0.85 \le |H(\omega)| \le 1$.

The above specifications are used to design a normalized lowpass Butterworth filter.

The gain terms G_p and G_s are given by

$$G_p = \frac{1}{(1-\delta_p)^2} - 1 = \frac{1}{0.85^2} - 1 = 0.3841$$

and

$$G_s = \frac{1}{(\delta_s)^2} - 1 = \frac{1}{0.15^2} - 1 = 43.4444.$$

The order N of the Butterworth filter is obtained using Eq. (7.25) as

$$N = \frac{1}{2} \times \frac{\ln(G_p/G_s)}{\ln(\xi_p/\xi_s)} = \frac{1}{2} \times \frac{\ln(0.3841/43.44444)}{\ln(1/2)} = 3.4108.$$

We round off the order of the filter to the higher integer value as $N = 4$.

Using the stop band constraint, Eq. (7.32), the cut off frequency of the required Butterworth filter is

$$\omega_c = \frac{\omega_s}{(G_s)^{\frac{1}{2N}}} = \frac{2}{(43.4444)^{\frac{1}{8}}} = 1.2482 \text{ radians/s.}$$

The poles of the lowpass filter are located at

$$S = \omega_c \exp\left[j\tfrac{\pi}{2} + j\tfrac{(2n-1)\pi}{8}\right]$$

for $1 \le n \le 4$. Substituting different values of n gives

$$S = [-0.4777+j1.1532 \quad -1.1532+j0.4777 \quad -1.1532-j0.4777 \quad -0.4777-j1.1532].$$

The transfer function of the lowpass filter is given by

$$H(S) = \frac{K}{(S+0.4777+j1.1532)(S+1.1532-j0.4777)(S+1.1532+j0.4777)(S+0.4777-j1.1532)}$$

or,
$$H(S) = \frac{K}{S^4 + 3.2617S^3 + 5.3194S^2 + 5.0817S + 2.4274}.$$

To ensure a dc gain of 1 for the lowpass filter, we set K = 2.4274. The transfer function of unity gain lowpass filter is given by

$$H(S) = \frac{2.4274}{S^4 + 3.2617S^3 + 5.3194S^2 + 5.0817S + 2.4274}$$

To derive the transfer function of the required highpass filter, we use transformation (7.64) with ξ_p = 30 radians/s. The transfer function of the highpass filter is given by

$$H(s) = H(S)\Big|_{S=30/s} = \frac{2.4274}{(30/s)^4 + 3.2617(30/s)^3 + 5.3194(30/s)^2 + 5.0817(30/s) + 2.4274}$$

or,
$$H(s) = \frac{s^4}{s^4 + 62.8042s^3 + 1.9723\times10^3 s^2 + 3.6280\times10^4 s + 3.3369\times10^5}.$$

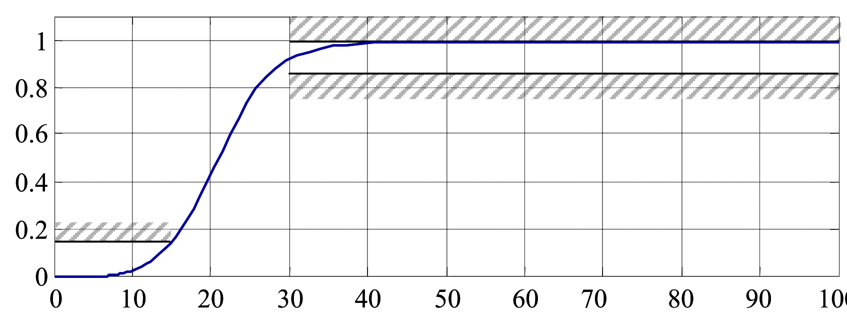

Figure S7.14: Magnitude spectrum of the Butterworth highpass filter designed in Problem 7.14.

The magnitude spectrum of the highpass filter is included in Fig. S7.14, which confirms that the given specifications are satisfied. ▌

Problem 7.15

Repeating the procedure for Problem 7.14, the specifications of the transposed lowpass filter for ξ_p = 30 radians/s are given by

Stop band ($\infty < |\omega| \leq 2$ radians/s): $|H(\omega)| \leq 0.15$

Pass band ($|\omega| < 1$ radians/s): $0.85 \leq |H(\omega)| \leq 1$.

The above specifications are used to design a normalized lowpass Type I Chebyshev filter.

Type I Chebyshev Filter:

The gain terms G_p and G_s are given by 0.3841 and 43.4444.

Step 1 determines the value of the ripple control factor ε as

$$\varepsilon = \sqrt{G_p} = \sqrt{0.3841} = 0.6198.$$

Step 2 determines the order N of the Chebyshev polynomial as

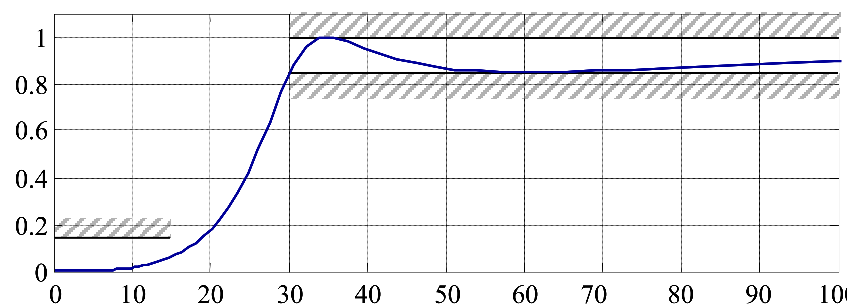

Figure S7.15: Magnitude spectrum of the Type I Chebyshev highpass filter for Problem 7.15.

$$N = \frac{\cosh^{-1}\left[(43.4444/0.3841)^{0.5}\right]}{\cosh^{-1}[2/1]} = 2.3198.$$

We round off N to the closest higher integer as 3.

Step 3 determines the location of the six poles of $H(S)H(-S)$ as

$$[0.2155 + j0.9430, \quad 0.2155 - j0.9430, \quad -0.2155 - j0.9430, \quad -0.2155 + j0.9430, \quad 0.431, \quad -0.431].$$

The 3 poles lying in the left half s-plane are included in the transfer function $H(S)$ of the normalized Type I Chebyshev filter. These poles are located at

$$[-0.2155 - j0.9430, \quad -0.2155 + j0.9430, \quad -0.4310].$$

The transfer function for the normalized Type-I Chebyshev filter is, therefore, given by

$$H(S) = \frac{K}{(S + 0.2155 + j0.9430)(S + 0.2155 - j0.0.9430)(S + 0.4310)},$$

which simplifies to $$H(S) = \frac{K}{S^3 + 0.8621S^2 + 1.1216S + 0.4034}.$$

Since $|H(\omega)|$ at $\omega = 0$ is $K/0.1047$, therefore, K is set to 0.1047 to make the dc gain equal to 1. The new transfer function with unity gain at $\omega = 0$ is given by

$$H(S) = \frac{0.4034}{S^3 + 0.8621S^2 + 1.1216S + 0.4034}.$$

Step 4 is not needed as the passband cutoff frequency is $\omega_p = 1$ radians/s.

This completes the design of Type I Chebyshev filter.

To derive the transfer function of the required highpass filter, we use transformation (7.64) with $\xi_p = 30$ radians/s. The transfer function of the highpass filter is given by

$$H(s) = H(S)\big|_{S=30/s} = \frac{0.4034}{(30/s)^3 + 0.8621(30/s)^2 + 1.1216(30/s) + 0.4034},$$

or,
$$H(s) = \frac{s^3}{s^3 + 83.4110s^2 + 1923.4s + 6.6931 \times 10^4}.$$

The magnitude spectrum of the highpass filter is included in Fig. S7.15, which confirms that the given specifications are satisfied. ▌

Problem 7.16

Repeating the procedure for Problem 7.14, the specifications of the transposed lowpass filter for $\xi_p = 30$ radians/s are given by

Stop band ($\infty < |\omega| \le 2$ radians/s): $|H(\omega)| \le 0.15$

Pass band ($|\omega| < 1$ radians/s): $0.85 \le |H(\omega)| \le 1$.

The above specifications are used to design a normalized lowpass Type II Chebyshev filter.

Type II Chebyshev Filter:

The gain terms G_p and G_s are given by 0.3841 and 43.4444.

Step 1 determines the value of the ripple control factor ε as

$$\varepsilon = \frac{1}{\sqrt{G_s}} = \frac{1}{\sqrt{43.4444}} = 0.1517.$$

Step 2 determines the order N of the Chebyshev polynomial as.

$$N = \frac{\cosh^{-1}\left[(43.4444/0.3841)^{0.5}\right]}{\cosh^{-1}\left[2/1\right]} = 2.3198.$$

We round off order N to the closest higher integer as 3.

Step 3 determines the location of the poles and zeros of $H(S)H(-S)$.

We first determine the location of poles for the Type-I Chebyshev filter with $\varepsilon = 0.1517$ and $N = 3$. Using Eq. (7.46), the location of poles for $H(s)H(-s)$ of the Type I Chebyshev filter are given by

$$[-0.4861 + j1.2078, \quad 0.4861 + j1.2078, \quad 0.4861 - j1.2078, \quad -0.4861 - j1.2078, \quad 0.9722, \quad -0.9722]$$

Selecting the poles located in the left half s-plane, we get

$$[-0.4861 + j1.2078, \quad -0.4861 - j1.2078, \quad -0.9722].$$

The poles of the normalized Type II Chebyshev filter are located at the inverse of the above locations and are given by

$$[-0.2868 - j0.7125, \quad -0.2868 + j0.7125, \quad -1.0286].$$

The zeros of the normalized Chebyshev Type II filter are computed using Eq. (7.60) and are given by

$$[-j1.1547, \quad +j1.1547, \quad \infty].$$

The zero at $s = \infty$ is neglected. The transfer function for the normalized Type II Chebyshev filter is given by

$$H(S) = \frac{K(S + j1.1547)(S - j1.1547)}{(S + 0.2868 + j0.7125)(S + 0.2868 - j0.7125)(S + 1.0286)},$$

which simplifies to
$$H(S) = \frac{K(S^2 + 1.3333)}{S^3 + 1.6021S^2 + 1.1798S + 0.6068}.$$

Since $|H(\omega)|$ at $\omega = 0$ is 1.3333/0.6068 = 2.1973, therefore, K is set to 1/2.1973 = 0.4551 to make the dc gain equal to 1. The new transfer function with unity gain at $\omega = 0$ is given by

$$H(S) = \frac{0.4551(S^2 + 1.3333)}{S^3 + 1.6021S^2 + 1.1798S + 0.6068}.$$

Step 4 normalizes $H(S)$ based on stop band frequency $\omega_s = 2$ radians/s, which gives

$$H(s) = H(S)\big|_{S=s/2} = \frac{0.4551((s/2)^2 + 1.3333)}{(s/2)^3 + 1.6021(s/2)^2 + 1.1798(s/2) + 0.6068}$$

which simplifies to

$$H(s) = \frac{0.9102(s^2 + 5.3333)}{s^3 + 3.2042s^2 + 4.7192s + 4.8544}.$$

This completes the design of Type II Chebyshev lowpass filter.

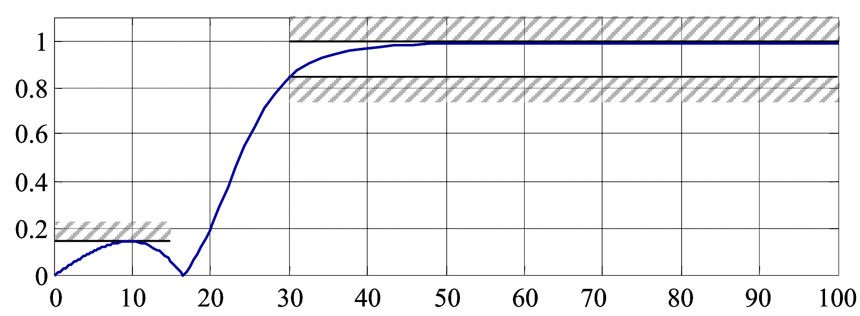

Figure S7.16: Magnitude spectrum of the Type II Chebyshev highpass filter for Problem 7.16.

To derive the transfer function of the required highpass filter, we use transformation (7.64) with $\xi_p = 30$ radians/s. The transfer function of the highpass filter is given by

$$H(s) = H(S)\big|_{S=30/s} = \frac{0.9102((30/s)^2 + 5.3333)}{(30/s)^3 + 3.2042(30/s)^2 + 4.7192(30/s) + 4.8544},$$

or,
$$H(s) = \frac{0.1707s(s^2 + 168.7511)}{s^3 + 29.1645s^2 + 594.0549s + 5.5620\times 10^3}.$$

The magnitude spectrum of the highpass filter is included in Fig. S7.16, which confirms that the given specifications are satisfied. ▌

Problem 7.17

For ξ_{p1} = 100 radians/s and ξ_{p2} = 150 radians/s, Eq. (7.70) is given by

$$\omega = \frac{\xi^2 - 1.5\times 10^4}{50\xi},$$

to transform the specifications from the domain $s = \gamma + j\xi$ of the bandpass filter to the domain $S = \sigma + j\omega$ of the lowpass filter. The specifications for the normalized lowpass filter are given by

Pass band ($0 \le |\omega| < 1$ radians/s): $-1\text{dB} \le 20\log_{10}|H(\omega)| \le 0$.

Stop band ($|\omega| \ge \min(1.7857, 2.5)$ radians/s: $20\log_{10}|H(\omega)| \le -15\text{dB}$.

Lowpass Butterworth filter:

The above specifications are used to design a normalized lowpass Butterworth filter. Here, we use the following MATLAB code to design the Butterworth filter. The same can be derived using the design steps outlined in the text.

```
>> wp=1; ws=1.7857; Rp=1; Rs=15 ;          % specify design parameters
>> [N,wc]=buttord(wp,ws,Rp,Rs,'s');        % determine order and cut-off freq
>> [num,den]=butter(N,wc,'s');             % determine num and denom coeff.
>> Ht = tf(num,den);                       % determine transfer function
```

The transfer function of the lowpass Butterworth filter is given by

$$H(S) = \frac{3.281}{S^5 + 4.104S^4 + 8.422S^3 + 10.68S^2 + 8.372S + 3.281}.$$

To derive the transfer function of the required bandpass filter, we use transformation (7.69) with ξ_{p1} = 100 radians/s and ξ_{p2} = 150 radians/s. The transformation is given by

$$S = \frac{s^2 + 1.5\times 10^4}{50s},$$

from which the transfer function of the bandpass filter is calculated as

$$H(s)=H(S)\Big|_{S=\frac{s^2+1.5\times10^4}{50s}}=\frac{3.281}{\left[\frac{s^2+1.5\times10^4}{50s}\right]^5+4.104\left[\frac{s^2+1.5\times10^4}{50s}\right]^4+8.422\left[\frac{s^2+1.5\times10^4}{50s}\right]^3+10.68\left[\frac{s^2+1.5\times10^4}{50s}\right]^2+8.372\left[\frac{s^2+1.5\times10^4}{50s}\right]+3.281},$$

which reduces to

$$H(s)=\frac{1.0253\times10^9 s^5}{s^{10}+205.2s^9+9.606\times10^4 s^8+1.365\times10^7 s^7+3.25\times10^9 s^6+3.181\times10^{11} s^5+4.875\times10^{13} s^4+3.071\times10^{15} s^3+3.242\times10^{17} s^2+1.039\times10^{19} s+7.5938\times10^{20}}.$$

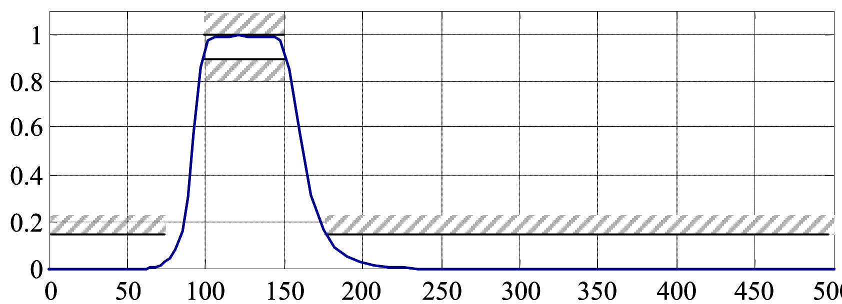

Figure S7.17: Magnitude spectrum of the Butterworth bandpass filter in Problem 7.17.

The magnitude spectrum of the bandpass filter is included in Fig. S7.17, which confirms that the given specifications for the bandpass filter are indeed satisfied. ▮

Problem 7.18

For ξ_{p1} = 100 radians/s and ξ_{p2} = 150 radians/s, the transformed specifications for the normalized lowpass filter are given by

Pass band (0 ≤ |ω| < 1 radians/s): $-1\text{dB}\le 20\log_{10}|H(\omega)|\le 0$.

Stop band (|ω| ≥ min(1.7857, 2.5) radians/s: $20\log_{10}|H(\omega)|\le -15\text{dB}$.

Lowpass Type I Chebyshev filter:

The above specifications are used to design a normalized lowpass Type I Chebyshev filter. Here, we use the following MATLAB code to design the Butterworth filter. The same can be derived using the design steps outlined in the text.

```
>> wp=1; ws=1.7857; rp=1; rs=15;          % specify design parameters
>> [N,wn] = cheb1ord(wp,ws,rp,rs,'s');    % determine order and natural freq
>> [num,den] = cheby1(N,rp,wn,'s');       % determine num and denom coeff.
>> Ht = tf(num,den);                      % determine transfer function
```

The transfer function of the Type I Chebyshev filter is given by

$$H(S) = \frac{0.4913}{S^3 + 0.9883S^2 + 1.238S + 0.4913}.$$

To derive the transfer function of the required bandpass filter, we use the following transformation obtained by substituting ξ_{p1} = 100 radians/s and ξ_{p2} = 150 radians/s in Eq. (7.69)

$$S = \frac{s^2 + 1.5\times10^4}{50s}.$$

The resulting transfer function of the bandpass filter is given by

$$H(s) = H(S)\big|_{S=\frac{s^2+1.5\times10^4}{50s}} = \frac{0.4913}{\left[\frac{s^2+1.5\times10^4}{50s}\right]^3 + 0.9883\left[\frac{s^2+1.5\times10^4}{50s}\right]^2 + 1.238\left[\frac{s^2+1.5\times10^4}{50s}\right] + 0.4913}$$

which simplifies to

$$H(s) = \frac{6.141\times10^4 s^3}{s^6 + 49.42s^5 + 4.81\times10^4 s^4 + 1.544\times10^6 s^3 + 7.214\times10^8 s^2 + 1.112\times10^{10} s + 3.375\times10^{12}}.$$

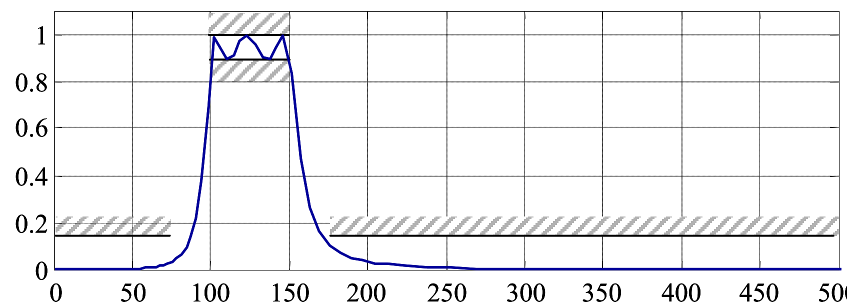

Figure S7.18: Magnitude spectrum of the Type I Chebyshev bandpass filter in Problem 7.18.

The magnitude spectrum of the bandpass filter is included in Fig. S7.18, which confirms that the given specifications for the bandpass filter are indeed satisfied. ▌

Problem 7.19

For ξ_{p1} = 100 radians/s and ξ_{p2} = 150 radians/s, the transformed specifications for the normalized lowpass filter are given by

Pass band ($0 \le |\omega| < 1$ radians/s): $-1\text{dB} \le 20\log_{10}|H(\omega)| \le 0$.

Stop band ($|\omega| \ge \min(1.7857, 2.5)$ radians/s: $20\log_{10}|H(\omega)| \le -15\text{dB}$.

Lowpass Type II Chebyshev filter:

The above specifications are used to design a normalized lowpass Type II Chebyshev filter. Here, we use the following MATLAB code to design the Butterworth filter. The same can be derived using the design steps outlined in the text.

```
>> wp=1; ws=1.7857; rp=1; rs=15;          % specify design parameters
>> [N,wn] = cheb2ord(wp,ws,rp,rs,'s');     % determine order and natural freq
>> [num,den] = cheby2(N,rs,wn,'s');        % determine num and denom coeff.
```

```
>> Ht = tf(num,den);                    % determine transfer function
```

The transfer function of the Type II Chebyshev filter is given by

$$H(S)=\frac{0.8533S^2+2.819}{S^3+2.67S^2+3.2S+2.819}.$$

To derive the transfer function of the required bandpass filter, we use the following transformation obtained by substituting ξ_{p1} = 100 radians/s and ξ_{p2} = 150 radians/s in Eq. (7.69)

$$S=\frac{s^2+1.5\times10^4}{50s}.$$

The resulting transfer function of the bandpass filter is given by

$$H(s)=H(S)\Big|_{S=\frac{s^2+1.5\times10^4}{50s}}=\frac{0.8533\left[\frac{s^2+1.5\times10^4}{50s}\right]^2+2.819}{\left[\frac{s^2+1.5\times10^4}{50s}\right]^3+2.67\left[\frac{s^2+1.5\times10^4}{50s}\right]^2+3.2\left[\frac{s^2+1.5\times10^4}{50s}\right]+2.819}$$

which simplifies to

$$H(s)=\frac{23.56s^5+8.011\times10^5s^3+5.3\times10^9s}{s^6+49.03s^5+4.806\times10^4s^4+1.565\times10^6s^3+7.209\times10^8s^2+1.103\times10^{10}s+3.375\times10^{12}}.$$

The magnitude spectrum of the bandpass filter is included in Fig. S7.19, which confirms that the given specifications for the bandpass filter are indeed satisfied. ▮

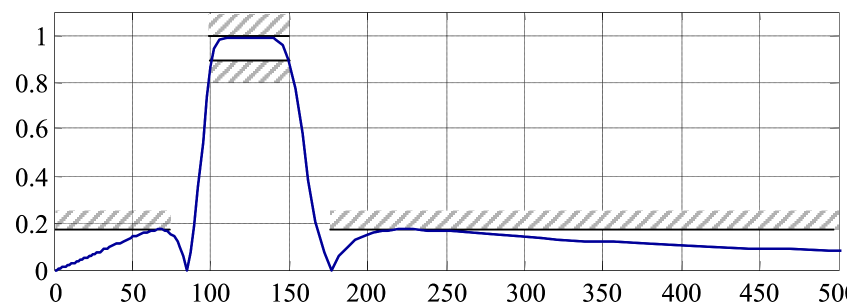

Figure S7.19: Magnitude spectrum of the Type I Chebyshev bandpass filter in Problem 7.19.

Problem 7.20

For ξ_{p1} = 25 radians/s and ξ_{p2} = 325 radians/s, Eq. (7.67) is given by

$$\omega=\frac{300\xi}{8125-\xi^2},$$

to transform the specifications from the domain $s=\gamma+j\xi$ of the bandstop filter to the domain $S=\sigma+j\omega$ of the lowpass filter. The specifications for the normalized lowpass filter are given by

Pass band ($0\le|\omega|<1$ radians/s): $-4\text{dB}\le20\log_{10}|H(\omega)|\le0$.

Stop band ($|\omega|\ge\min(1.3793,16)$ radians/s: $20\log_{10}|H(\omega)|\le-20\text{dB}$.

The above specifications are used to design a normalized lowpass Butterworth filter using MATLAB.

Lowpass Butterworth filter:

The above specifications are used to design a normalized lowpass Butterworth filter. Here, we use the following MATLAB code to design the Butterworth filter. The same can be derived using the design steps outlined in the text.

```
>> wp=1; ws=1.3793; Rp=4; Rs=20 ;          % specify design parameters
>> [N,wc]=buttord(wp,ws,Rp,Rs,'s');        % determine order and cut-off freq
>> [num,den]=butter(N,wc,'s');             % determine num and denom coeff.
>> Ht = tf(num,den);                       % determine transfer function
```

The transfer function of the lowpass Butterworth filter is given by

$$H(S)=\frac{0.9545}{S^7+4.464S^6+9.964S^5+14.3S^4+14.21S^3+9.768S^2+4.318S+0.9545}.$$

To derive the transfer function of the required bandstop filter, we use transformation (7.74) with $\xi_{p1}=25$ radians/s and $\xi_{p2}=325$ radians/s. The transformation is given by

$$S=\frac{300s}{s^2+8125},$$

from which the transfer function of the bandstop filter is calculated as

$$H(s)=H(S)\Big|_{S=\frac{300s}{s^2+8125}}=\frac{0.9545}{\left[\frac{300s}{s^2+8125}\right]^7+4.464\left[\frac{300s}{s^2+8125}\right]^6+9.964\left[\frac{300s}{s^2+8125}\right]^5+14.3\left[\frac{300s}{s^2+8125}\right]^4+14.21\left[\frac{300s}{s^2+8125}\right]^3+9.768\left[\frac{300s}{s^2+8125}\right]^2+4.318\left[\frac{300s}{s^2+8125}\right]+0.9545},$$

which reduces to

$$H(s)=\frac{0.9545(s^2+8125)^7}{(300s)^7+4.464(300s)^6(s^2+8125)+9.964(300s)^5(s^2+8125)^2+14.3(300s)^4(s^2+8125)^3+14.21(300s)^3(s^2+8125)^4+9.768(300s)^2(s^2+8125)^5+4.318(300s)(s^2+8125)^6+0.9545(s^2+8125)^7}.$$

The magnitude spectrum of the bandstop filter is included in Fig. S7.20, which confirms that the given specifications for the bandstop filter are indeed satisfied. ▌

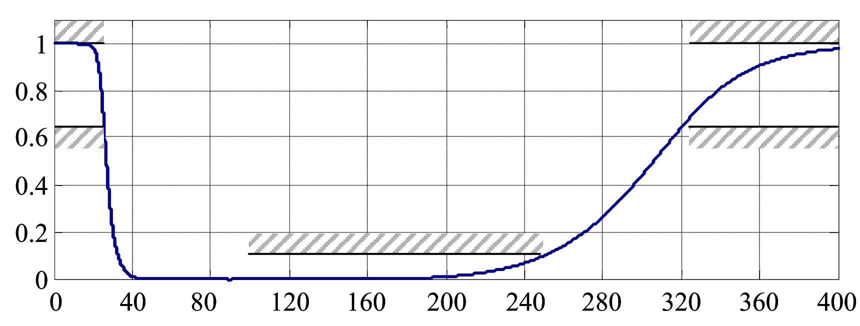

Figure S7.20: Magnitude spectrum of the Butterworth bandstop filter in Problem 7.20.

Problem 7.21

The transformation equation is given by

$$\omega = \frac{300\xi}{8125 - \xi^2},$$

with the following lowpass specifications:

Pass band ($0 \le |\omega| < 1$ radians/s): $-4\text{dB} \le 20\log_{10}|H(\omega)| \le 0$.

Stop band ($|\omega| \ge \min(1.3793, 16)$ radians/s: $20\log_{10}|H(\omega)| \le -20\text{dB}$.

Type I Chebyshev filter:

The above specifications are used to design a normalized Type I Chebyshev filter using MATLAB.

```
>> wp=1; ws=1.3793; Rp=4; Rs=20 ;          % specify design parameters
>> [N,wn] = cheb1ord(wp,ws,Rp,Rs,'s');     % determine order and natural freq
>> [num,den] = cheby1(N,Rp,wn,'s');        % determine num and denom coeff.
>> Ht = tf(num,den);                       % determine transfer function
```

The transfer function of the Type I Chebyshev filter is given by

$$H(S) = \frac{0.1017}{S^4 + 0.4882S^3 + 1.119S^2 + 0.3326S + 0.1611}.$$

To derive the transfer function of the required bandstop filter, we use the transformation

$$S = \frac{300s}{s^2 + 8125},$$

which results in the following transfer function for the bandstop filter

$$H(s) = H(S)\Big|_{S=\frac{300s}{s^2+8125}} = \frac{0.1017}{\left[\frac{300s}{s^2+8125}\right]^4 + 0.4882\left[\frac{300s}{s^2+8125}\right]^3 + 1.119\left[\frac{300s}{s^2+8125}\right]^2 + 0.3326\left[\frac{300s}{s^2+8125}\right] + 0.1611}$$

which reduces to

$$H(s) = \frac{0.1017(s^2+8125)^4}{(300s)^4 + 0.4882(300s)^3(s^2+8125) + 1.119(300s)^2(s^2+8125)^2 + 0.3326(300s)(s^2+8125)^3 + 0.1611(s^2+8125)^4}.$$

The magnitude spectrum of the bandstop filter is included in Fig. S7.21, which confirms that the given specifications for the bandstop filter are indeed satisfied. ∎

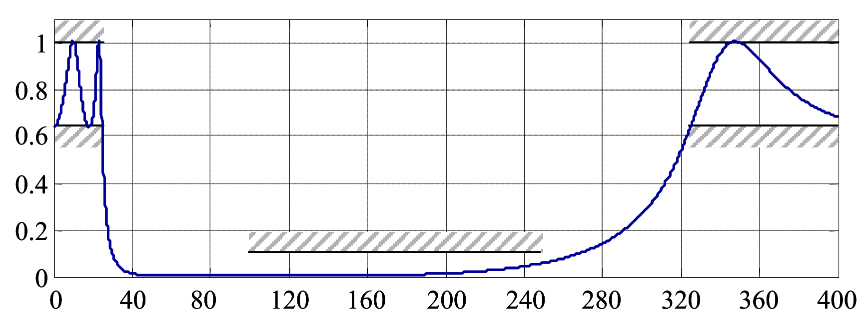

Figure S7.21: Magnitude spectrum of the Type I Chebyshev bandstop filter in Problem 7.21.

<u>Problem 7.22</u>

The transformation equation is given by

$$\omega = \frac{300\xi}{8125-\xi^2},$$

with the following lowpass specifications

Pass band ($0 \le |\omega| < 1$ radians/s): $-4\text{dB} \le 20\log_{10}|H(\omega)| \le 0$.

Stop band ($|\omega| \ge \min(1.3793,16)$ radians/s: $20\log_{10}|H(\omega)| \le -20\text{dB}$.

Type II Chebyshev filter:

The above specifications are used to design a normalized Type II Chebyshev filter using MATLAB.

```
>> wp=1; ws=1.3793; Rp=4; Rs=20 ;          % specify design parameters
>> [N,wn] = cheb2ord(wp,ws,Rp,Rs,'s');     % determine order and natural freq
>> [num,den] = cheby2(N,Rs,wn,'s');        % determine num and denom coeff.
>> Ht = tf(num,den);                       % determine transfer function
```

The transfer function of the Type II Chebyshev filter is given by

$$H(S) = \frac{0.03081}{S^4 + 0.08204S^3 + 1.569S^2 + 0.08313S + 0.3081}.$$

To derive the transfer function of the required bandstop filter, we use the transformation

$$S = \frac{300s}{s^2+8125},$$

which results in the following transfer function for the bandstop filter

$$H(s) = H(S)\Big|_{S=\frac{300s}{s^2+8125}} = \frac{0.03081}{\left[\frac{300s}{s^2+8125}\right]^4 + 0.082\left[\frac{300s}{s^2+8125}\right]^3 + 1.569\left[\frac{300s}{s^2+8125}\right]^2 + 0.0831\left[\frac{300s}{s^2+8125}\right] + 0.3081}$$

which reduces to

$$H(s) = \frac{0.03081(s^2+8125)^4}{(300s)^4 + 0.082(300s)^3(s^2+8125) + 1.569(300s)^2(s^2+8125)^2 + 0.0831(300s)(s^2+8125)^4 + 0.3081(s^2+8125)^4}$$

The magnitude spectrum of the bandstop filter is included in Fig. S7.22, which confirms that the given specifications for the bandstop filter are indeed satisfied. ▌

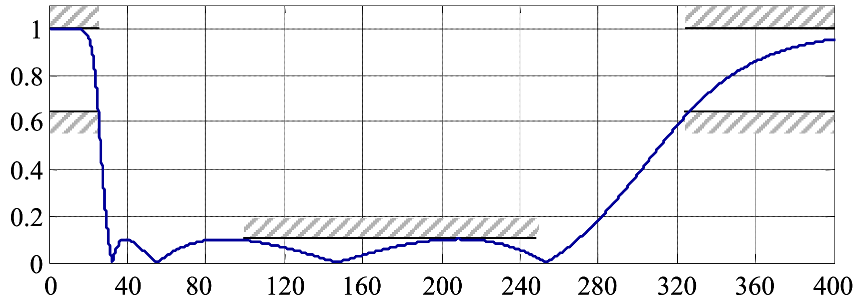

Figure S7.22: Magnitude spectrum of the Type II Chebyshev bandstop filter in Problem 7.22.

Problem 7.23

The MATLAB code for designing the lowpass filter specified in Problem 7.7 is shown in Program S7.23.

Program S7.23: MATLAB code for designing the lowpass filter in Problem 7.23

```
wp=10; ws=20; rp=0.9151; rs=20;        % specify design parameters
% Rp = -20*log10(0.9)= 0.9151dB
% Rs = -20*log10(0.1)=20dB
% Butterworth filter
[N,wc]=buttord(wp,ws,rp,rs,'s');    % determine order and cut-off freq
[num1,den1]=butter(N,wc,'s');       % determine num and denom coeff.
Ht1 = tf(num1,den1);                      % determine transfer function
[H1,w1] = freqs(num1,den1);         % determine magnitude spectrum
subplot(411); plot(w1,abs(H1));           % plot magnitude spectrum
grid on, title('Butterworth filter'); ax = axis;
% Type I Chebyshev filter
[N,wn] = cheb1ord(wp,ws,rp,rs,'s'); % determine order and natural freq
[num2,den2] = cheby1(N,rp,wn,'s');  % determine num and denom coeff.
Ht2 = tf(num2,den2);                      % determine transfer function
[H2,w2] = freqs(num1,den1);         % determine magnitude spectrum
subplot(412); plot(w2,abs(H2));           % plot magnitude spectrum
grid on, title('Type I Chebyshev filter'); axis(ax);
% Type II Chebyshev filter
[N,wn] = cheb2ord(wp,ws,rp,rs,'s'); % determine order and natural freq
[num3,den3] = cheby2(N,rs,ws,'s');  % determine num and denom coeff.
Ht3 = tf(num3,den3);                      % determine transfer function
[H3,w3] = freqs(num3,den3);         % determine magnitude spectrum
subplot(413); plot(w3,abs(H3));           % plot magnitude spectrum
grid on, title('Type II Chebyshev filter'); axis(ax);
% Elliptic filter
[N,wn] = ellipord(wp,ws,rp,rs,'s'); % determine order and natural freq
[num4,den4] = ellip(N,rp,rs,wn,'s');      %  determine  num  and  denom
coeff.
Ht4 = tf(num4,den4);                      % determine transfer function
[H4,w4] = freqs(num4,den4);         % determine magnitude spectrum
subplot(414); plot(w4,abs(H4));           % plot magnitude spectrum
grid on, title('Elliptic filter'); axis(ax);
```

The transfer functions for the four implementations are given as follows:

Butterworth filter: $$H(s)=\frac{3.216\times10^{5}}{(s^{5}+40.88s^{4}+835.5s^{3}+1.055\times10^{4}s^{2}+8.239\times10^{4}s+3.216\times10^{5})}.$$

Type I Chebyshev filter: $$H(s)=\frac{516.2}{s^{3}+10.21s^{2}+127.2\times10^{3}s+516.2}.$$

Type II Chebyshev filter: $$H(s)=\frac{6.03s^{2}+3216}{s^{3}+28.11s^{2}+376.8s+3216}.$$

Elliptic filter: $$H(s)=\frac{3.26s^{2}+692}{s^{3}+9.982s^{2}+126.8s+692}.$$

Note that the expressions for the transfer function are the same as obtained in Problems 7.7 (for Butterworth filter), 7.9 (for Type I Chebyshev filter), and 7.10 (for Type II Chebyshev filter). In case of the elliptic filter, the order N was evaluated in Problem 7.11(a) as 3, which is observed to be the same in the above expression.

The magnitude spectra for the four implementations are plotted in Fig. S7.23, which satisfy the given specifications.

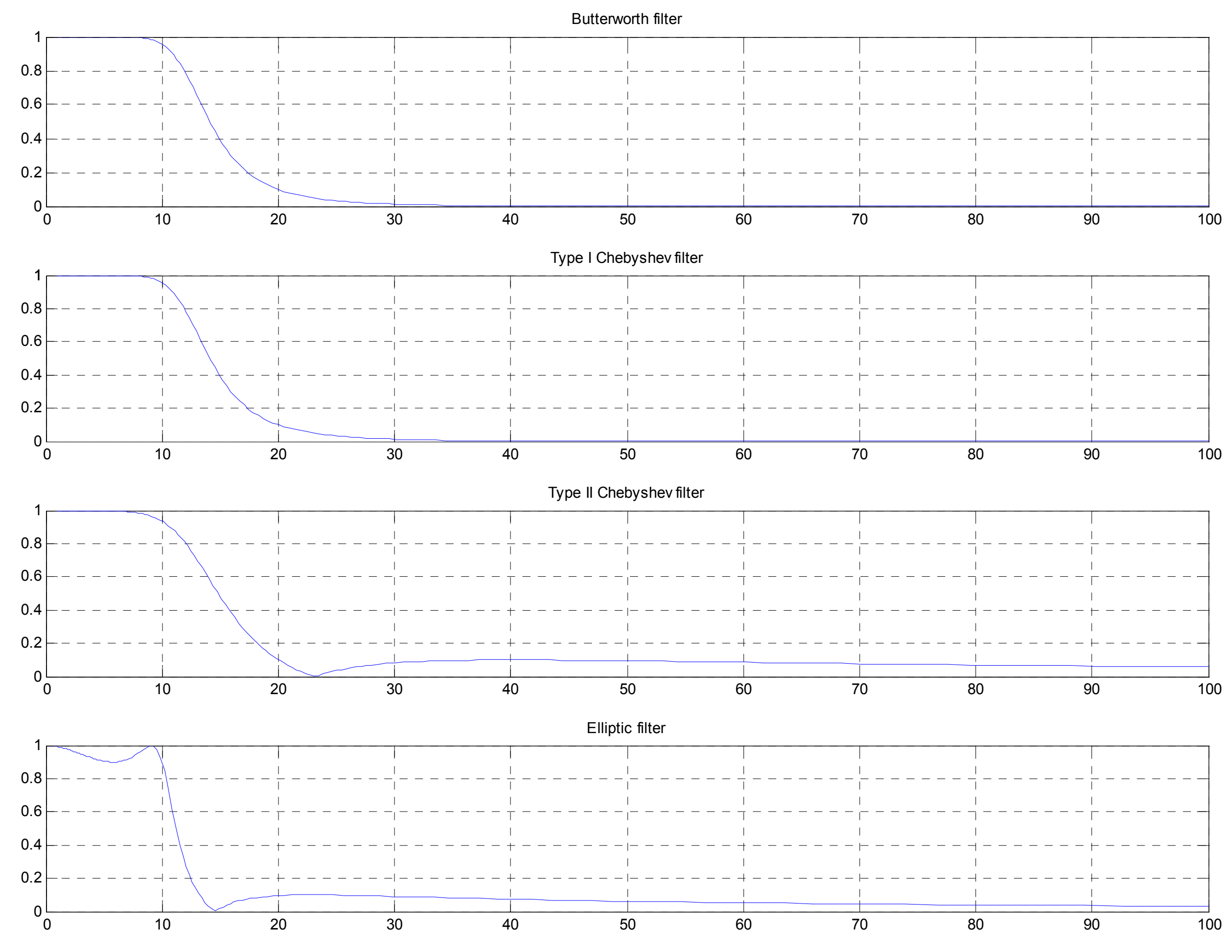

Figure S7.23: Magnitude Spectra of the four implementations of the low pass filter for Problem 7.23.

Problem 7.24

The Matlab code is similar to that for Problem 7.23 except for the design parameters, and is shown in Program S7.24.

Program S7.24: MATLAB code for designing lowpass filter in Problem 7.24

```
wp=50; ws=65; rp=1; rs=25;                % specify design parameters
%
% Butterworth filter
[N,wc]=buttord(wp,ws,rp,rs,'s');    % determine order and cut-off freq
[num1,den1]=butter(N,wc,'s');       % determine num and denom coeff.
Ht1 = tf(num1,den1);                      %     determine     transfer
function
[H1,w1] = freqs(num1,den1);         % determine magnitude spectrum
subplot(411); plot(w1,abs(H1));           % plot magnitude spectrum
grid on, title('Butterworth filter'); ax = axis;
% Type I Chebyshev filter
[N,wn] = cheb1ord(wp,ws,rp,rs,'s'); % determine order and natural freq
[num2,den2] = cheby1(N,rp,wn,'s');  % determine num and denom coeff.
Ht2 = tf(num2,den2);                      %     determine     transfer
function
[H2,w2] = freqs(num1,den1);         % determine magnitude spectrum
subplot(412); plot(w2,abs(H2));           % plot magnitude spectrum
grid on, title('Type I Chebyshev filter'); axis(ax);
% Type II Chebyshev filter
[N,wn] = cheb2ord(wp,ws,rp,rs,'s'); % determine order and natural freq
[num3,den3] = cheby2(N,rs,ws,'s');  % determine num and denom coeff.
Ht3 = tf(num3,den3);                      %     determine     transfer
function
[H3,w3] = freqs(num3,den3);         % determine magnitude spectrum
subplot(413); plot(w3,abs(H3));           % plot magnitude spectrum
grid on, title('Type II Chebyshev filter'); axis(ax);
% Elliptic filter
[N,wn] = ellipord(wp,ws,rp,rs,'s'); % determine order and natural freq
[num4,den4] = ellip(N,rp,rs,wn,'s'); % determine num and denom coeff.
Ht4 = tf(num4,den4);                % determine transfer function
[H4,w4] = freqs(num4,den4);         % determine magnitude spectrum
subplot(414); plot(w4,abs(H4));           % plot magnitude spectrum
grid on, title('Elliptic filter'); axis(ax);
```

The transfer functions for the four implementations are given as follows:

Butterworth filter:

$$H(s) = \frac{1.354\times10^{24}}{(s^{14}+472.7s^{13}+1117.3s^{12}+1.746\times10^{7}s^{11}+2.01\times10^{9}s^{10}+1.801\times10^{11}s^{9}+\ldots}$$

$$\overline{\ldots+1.295\times10^{13}s^{8}+7.577\times10^{14}s^{7}+3.627\times10^{16}s^{6}+1.414\times10^{18}s^{5}+4.418\times10^{19}s^{4}+\ldots}$$

$$\overline{\ldots+1.075\times10^{21}s^{3}+1.927\times10^{22}s^{2}+2.284\times10^{23}s+1.354\times10^{24})}.$$

Type I Chebyshev filter:

$$H(S) = \frac{0.9596 \times 10^9}{S^6 + 46.41S^5 + 4827S^4 + 1.503 \times 10^5 S^3 + 5.871 \times 10^6 S^2 + 9.596 \times 10^7 S + 1.077 \times 10^9}.$$

Type II Chebyshev filter:

$$H(s) = \frac{0.05623s^6 + 4277s^4 + 4.818 \times 10^7 s^2 + 1.357 \times 10^{11}}{s^6 + 250.9s^5 + 3.172 \times 10^4 s^4 + 2.53 \times 10^6 s^3 + 1.436 \times 10^8 s^2 + 5.089 \times 10^9 s + 1.357 \times 10^{11}}.$$

Elliptic filter ($N = 4$):

$$H(s) = \frac{0.05622s^2 + 970.9s^2 + 2.785 \times 10^6}{s^4 + 46.02s^3 + 4027s^2 + 1.072 \times 10^5 s + 3.125 \times 10^6}.$$

Note that the expressions for the transfer function are the same as obtained in Problems 7.8 (for Butterwoth filter), 7.9(b) (for Type I Chebyshev filter), and 7.10(b) (for Type II Chebyshev filter). In case of the elliptic filter, the order N was evaluated in Problem 7.11(b) as 4, which is observed to be the same in the above expression.

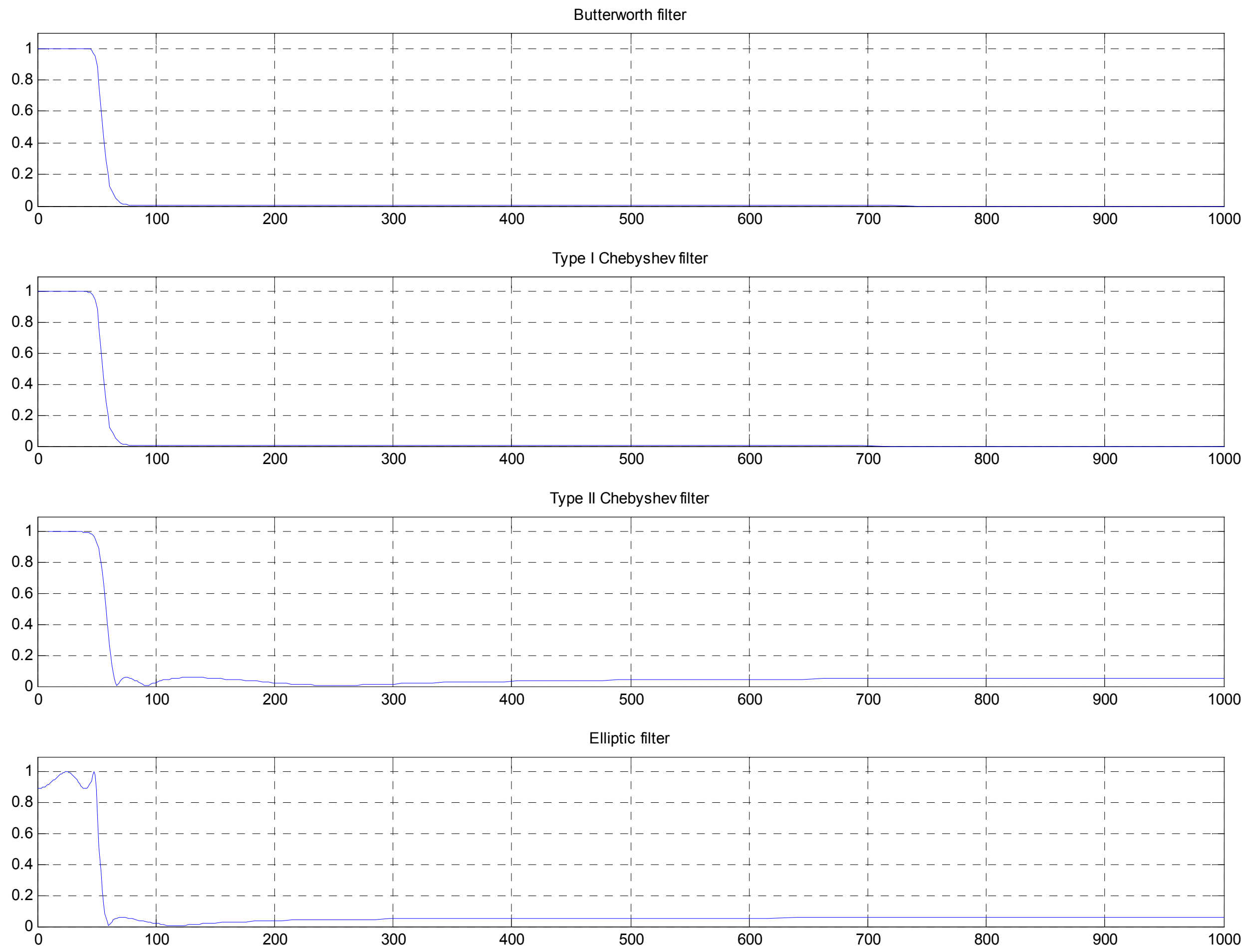

Figure S7.24: Magnitude Spectra of the four implementations of the lowpass filter for Problem 7.24.

The magnitude spectra for the four implementations are plotted in Fig. S7.24, which satisfy the given specifications. ▮

Problem 7.25

Expressing the pass-band and stop-band gains in dB, we get

Rs = 20*log(0.15) = –16.4782 and Rp = 20*log(0.85) = –1.4116.

The MATLAB code for designing the highpass filter specified in Problem 7.14 is shown in Program S7.25.

Program S7.25: MATLAB code for designing highpass filter in Problem 7.25

```
% MATLAB code for designing highpass filter in Problem 7.25
wp=30; ws=15; Rp=1.4116; Rs=16.4782 ;      % design specifications high
                                           % pass Butterworth filter
[N, wc] = buttord(wp,ws,Rp,Rs,'s'); % determine order and cut off
[num1,den1] = butter(N,wc,'high','s') ;% determine transfer function
Ht1 = tf(num1,den1);
[H1,w1] = freqs(num1,den1);         % determine magnitude spectrum
subplot(411); plot(w1,abs(H1));           % plot magnitude spectrum
grid on, title('Butterworth filter'); ax = axis;
%%%%%                               % Type I Chebyshev filter
[N, wn] = cheb1ord(wp,ws,Rp,Rs,'s') ;
[num2,den2] = cheby1(N,Rp,wn,'high','s') ;
Ht2 = tf(num2,den2);
[H2,w2] = freqs(num2,den2);         % determine magnitude spectrum
subplot(412); plot(w2,abs(H2));           % plot magnitude spectrum
grid on, title('Type I Chebyshev filter'); axis(ax);
%%%%%                               % Type II Chebyshev filter
[N,wn] = cheb2ord(wp,ws,Rp,Rs,'s') ;
[num3,den3] = cheby2(N,Rs,wn,'high','s') ;
Ht3 = tf(num3,den3);
[H3,w3] = freqs(num3,den3);         % determine magnitude spectrum
subplot(413); plot(w3,abs(H3));           % plot magnitude spectrum
grid on, title('Type II Chebyshev filter'); axis(ax);
%%%%%                               % Elliptic filter
[N,wn] = ellipord(wp,ws,Rp,Rs,'s') ;
[num4,den4] = ellip(N,Rp,Rs,wn,'high','s') ;
Ht4 = tf(num4,den4);
[H4,w4] = freqs(num4,den4);         % determine magnitude spectrum
subplot(414); plot(w4,abs(H4));           % plot magnitude spectrum
grid on, title('Elliptic filter'); axis(ax);
```

The following transfer functions are returned.

Butterworth filter: $$H(s) = \frac{s^4}{s^4 + 62.8s^3 + 1972s^2 + 3.628\times10^4 s + 3.337\times10^5}.$$

Type I Chebyshev filter: $$H(s) = \frac{s^3}{s^3 + 83.41s^2 + 1923s + 6.693\times10^4}.$$

Type II Chebyshev filter: $$H(s) = \frac{s^3 + 275.6s}{s^3 + 37.27s^2 + 970.2s + 1.161\times10^4}.$$

Elliptic filter: $$H(s) = \frac{0.85s^2 + 131.5}{s^2 + 24.83s + 876.6}.$$

Note that the transfer function of the Type II Chebyshev filter is different from the one obtained in Problem 7.16. This is because of the MATLAB implementation of the Type II Chebyshev highpass filter, which is slightly different from the one explained in the text. If all steps of the Type II Chebyshev filter are implemented as explained in the text, we get the same transfer function. Both transfer functions satisfy the design specifications. The magnitude spectra are plotted in Fig. S7.25. ▌

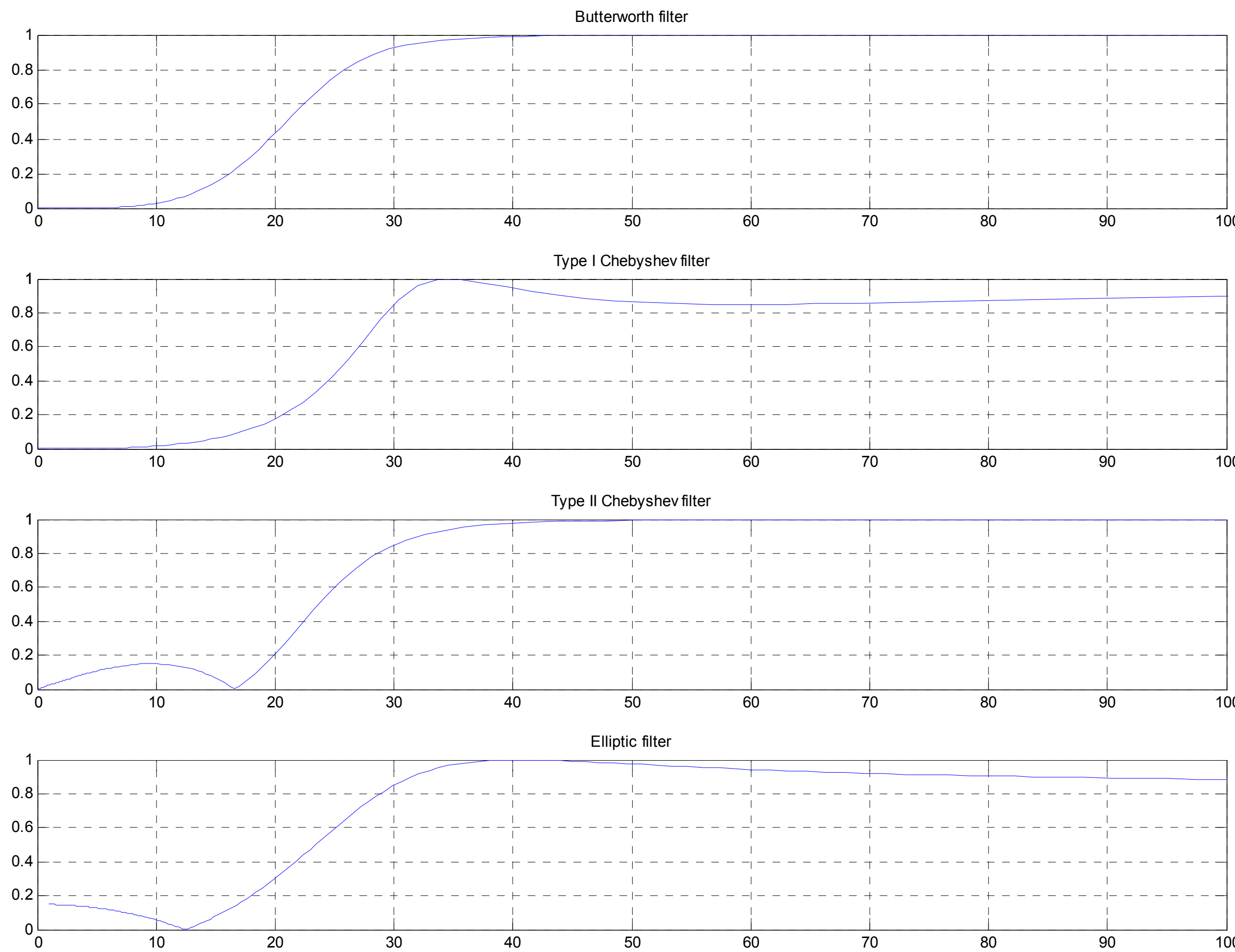

Figure S7.25: Magnitude Spectra of the four implementations of the low pass filter for Problem 7.25.

Problem 7.26

The MATLAB code for designing the bandpass filter specified in Problem 7.17 is shown in Program S7.26.

Program S7.26: MATLAB code for designing bandpass filter in Problem 7.26

```
% MATLAB code for designing bandpass filter in Problem 7.26
%
wp=[100 150]; ws=[75  175]; Rp=1; Rs=15 ; % Design Specifications
%%%%%
% Butterworth filter
[N, wc] = buttord(wp,ws,Rp,Rs,'s');
[num1,den1] = butter(N,wc,'s');
Ht1 = tf(num1,den1);
[H1,w1] = freqs(num1,den1);          % determine magnitude spectrum
```

```
subplot(411); plot(w1,abs(H1));            % plot magnitude spectrum
grid on, title('Butterworth filter'); ax = axis;
%%%%%
% Type I Chebyshev filter
[N, wn] = cheb1ord(wp,ws,Rp,Rs,'s');
[num2,den2] = cheby1(N,Rp,wn,'s');
Ht2 = tf(num2,den2);
[H2,w2] = freqs(num2,den2);         % determine magnitude spectrum
subplot(412); plot(w2,abs(H2));            % plot magnitude spectrum
grid on, title('Type I Chebyshev filter'); axis(ax);
%%%%%
% Type II Chebyshev filter
[N,wn] = cheb2ord(wp,ws,Rp,Rs,'s');
[num3,den3] = cheby2(N,Rs,wn,'s');
Ht3 = tf(num3,den3);
[H3,w3] = freqs(num3,den3);         % determine magnitude spectrum
subplot(413); plot(w3,abs(H3));            % plot magnitude spectrum
grid on, title('Type II Chebyshev filter'); axis(ax);
%%%%%
% Elliptic filter
[N,wn] = ellipord(wp,ws,Rp,Rs,'s');
[num4,den4] = ellip(N,Rp,Rs,wn,'s');
Ht4 = tf(num4,den4);
[H4,w4] = freqs(num4,den4);         % determine magnitude spectrum
subplot(414); plot(w4,abs(H4));            % plot magnitude spectrum
grid on, title('Elliptic filter'); axis(ax);
```

The aforementioned MATLAB code produces the following transfer functions for the four filters.

Butterworth filter:

$$H(s) = \frac{1.025\times 10^9 s^5}{\begin{array}{l} s^{10} + 205.2s^9 + 9.606\times 10^4 s^8 + 1.365\times 10^7 s^7 + 3.25\times 10^9 s^6 + 3.181\times 10^{11} s^5 \\ + 4.875\times 10^{13} s^4 + 3.071\times 10^{15} s^3 + 3.242\times 10^{17} s^2 + 1.039\times 10^{19} s + 7.594\times 10^{20} \end{array}}.$$

Type I Chebyshev filter:

$$H(s) = \frac{6.141\times 10^4 s^3}{s^6 + 49.42s^5 + 4.81\times 10^4 s^4 + 1.544\times 10^6 s^3 + 7.214\times 10^8 s^2 + 1.112\times 10^{10} s + 3.375\times 10^{12}}.$$

Type II Chebyshev filter:

$$H(s) = \frac{42.66s^5 + 1.632\times 10^6 s^3 + 9.599\times 10^9}{s^6 + 133.5s^5 + 5.3\times 10^4 s^4 + 4.357\times 10^6 s^3 + 7.95\times 10^8 s^2 + 3.004\times 10^{10} s + 3.375\times 10^{12}}.$$

Elliptic filter:

$$H(s) = \frac{23.56s^5 + 8.011\times 10^5 s^3 + 5.3\times 10^9 s}{s^6 + 49.03s^5 + 4.806\times 10^4 s^4 + 1.565\times 10^6 s^3 + 7.209\times 10^8 s^2 + 1.103\times 10^{10} s + 3.375\times 10^{12}}.$$

The magnitude spectra are plotted in Fig. S7.26. ▌

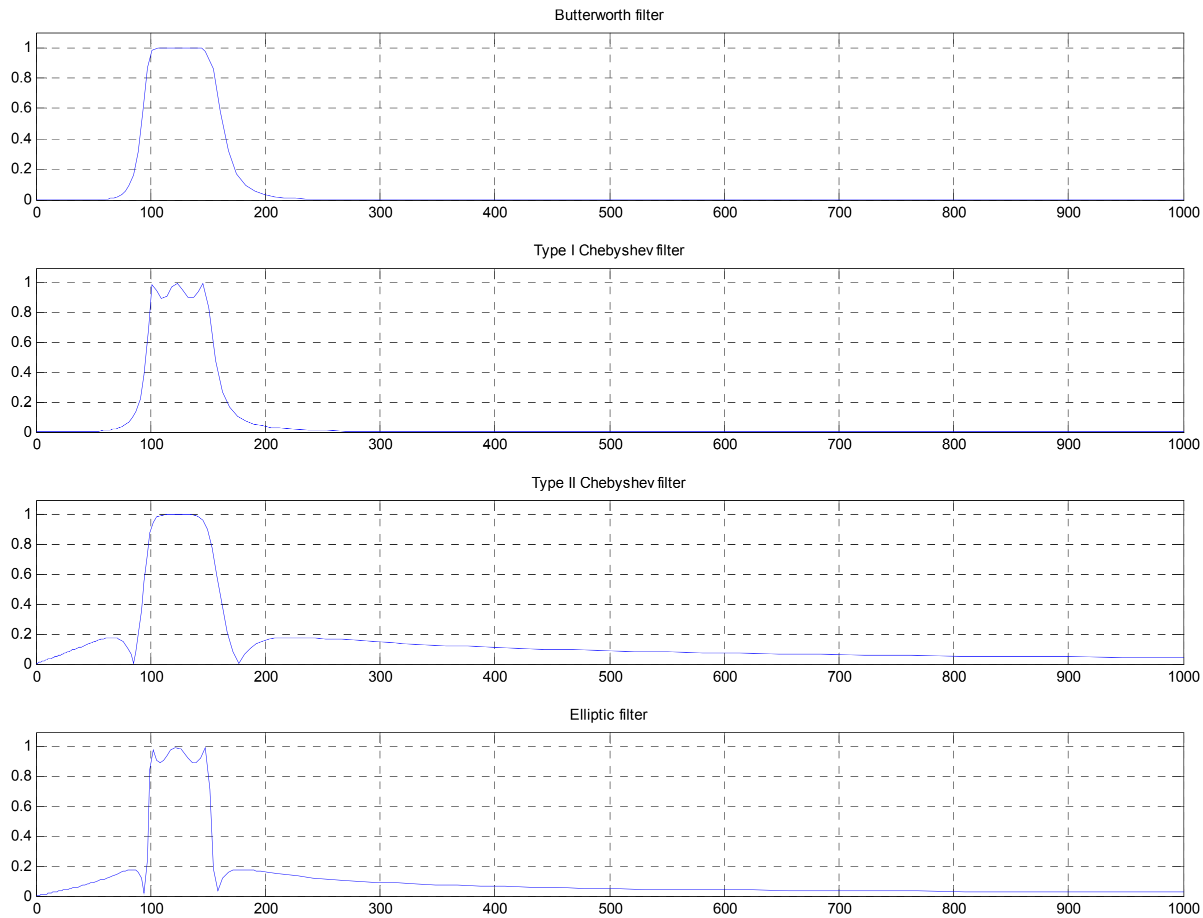

Figure S7.26: Magnitude Spectra of the four implementations of the band pass filter for Problem 7.26.

Problem 7.27

The MATLAB code for the design of the bandstop filter is shown in Program S7.27.

Program S7.27: MATLAB code for designing bandstop filter in Problem 7.27

```
% MATLAB code for designing bandstop filter
wp=[25 325]; ws=[100  250]; Rp=4; Rs=20;        % Specifications
% Butterworth Filter
[N, wn] = buttord(wp,ws,Rp,Rs,'s') ;
[num1,den1] = butter(N,wn,'stop','s');
Ht1 = tf(num1,den1);
[H1,w1] = freqs(num1,den1);          % determine magnitude spectrum
subplot(411); plot(w1,abs(H1));      % plot magnitude spectrum
grid on, title('Butterworth filter'); ax = axis;
% Type I Chebyshev filter
[N, wn] = cheb1ord(wp,ws,Rp,Rs,'s') ;
[num2,den2] = cheby1(N,Rp,wn,'stop','s');
Ht2 = tf(num2,den2);
[H2,w2] = freqs(num2,den2);          % determine magnitude spectrum
subplot(412); plot(w2,abs(H2));      % plot magnitude spectrum
grid on, title('Type I Chebyshev filter'); axis(ax);
% Type II Chebyshev filter
[N,wn] = cheb2ord(wp,ws,Rp,Rs,'s') ;
```

```
[num3,den3] = cheby2(N,Rs,wn,'stop','s');
Ht3 = tf(num3,den3);
[H3,w3] = freqs(num3,den3);          % determine magnitude spectrum
subplot(413); plot(w3,abs(H3));            % plot magnitude spectrum
grid on, title('Type II Chebyshev filter'); axis(ax);
% Elliptic filter
[N,wn] = ellipord(wp,ws,Rp,Rs,'s') ;
[num4,den4] = ellip(N,Rp,Rs,wn,'stop','s');
Ht4 = tf(num4,den4);
[H4,w4] = freqs(num4,den4);          % determine magnitude spectrum
subplot(414); plot(w4,abs(H4));      % plot magnitude spectrum
grid on, title('Elliptic filter'); axis(ax);
```

The resulting transfer functions are:

Butterworth filter:

$$H(s) = \frac{s^{10} + 1.25\times10^{5}s^{8} + 6.25\times10^{9}s^{6} + 1.562\times10^{14}s^{4} + 1.953\times10^{18}s^{2} + 9.766\times10^{21}}{s^{10} + 768.6s^{9} + 4.203\times10^{5}s^{8} + 1.47\times10^{8}s^{7} + 3.87\times10^{10}s^{6} + 7.145\times10^{12}s^{5} + 9.674\times10^{14}s^{4} + 9.187\times10^{16}s^{3} + 6.568\times10^{18}s^{2} + 3.002\times10^{20}s + 9.766\times10^{21}}.$$

Type I Chebyshev filter:

$$H(s) = \frac{s^{6} + 7.5\times10^{4}s^{4} + 1.875\times10^{9}s^{2} + 1.562\times10^{13}}{s^{6} + 1068s^{5} + 2.265\times10^{5}s^{4} + 1.285\times10^{8}s^{3} + 5.662\times10^{9}s^{2} + 6.674\times10^{11}s + 1.562\times10^{13}}.$$

Type II Chebyshev filter:

$$H(s) = \frac{s^{6} + 9.662\times10^{4}s^{4} + 2.416\times10^{9}s^{2} + 1.562\times10^{13}}{s^{6} + 397.9s^{5} + 1.758\times10^{5}s^{4} + 3.207\times10^{7}s^{3} + 4.395\times10^{9}s^{2} + 2.487\times10^{11}s + 1.562\times10^{13}}.$$

Elliptic filter:

$$H(s) = \frac{0.631s^{4} + 4.009\times10^{4}s^{2} + 3.943\times10^{8}}{s^{4} + 167.9s^{3} + 1.354\times10^{5}s^{2} + 4.197\times10^{6}s + 6.25\times10^{8}}.$$

Note that the transfer functions for the bandstop filters are different than the ones obtained in Problems 7.20 to 7.22. Both versions satisfy the specifications though the transfer functions obtained using MATLAB are of lower order. MATLAB uses a different transformation between the bandstop and lowpass domains resulting in a different answer.

The magnitude spectra are plotted in Fig. S7.27. ▌

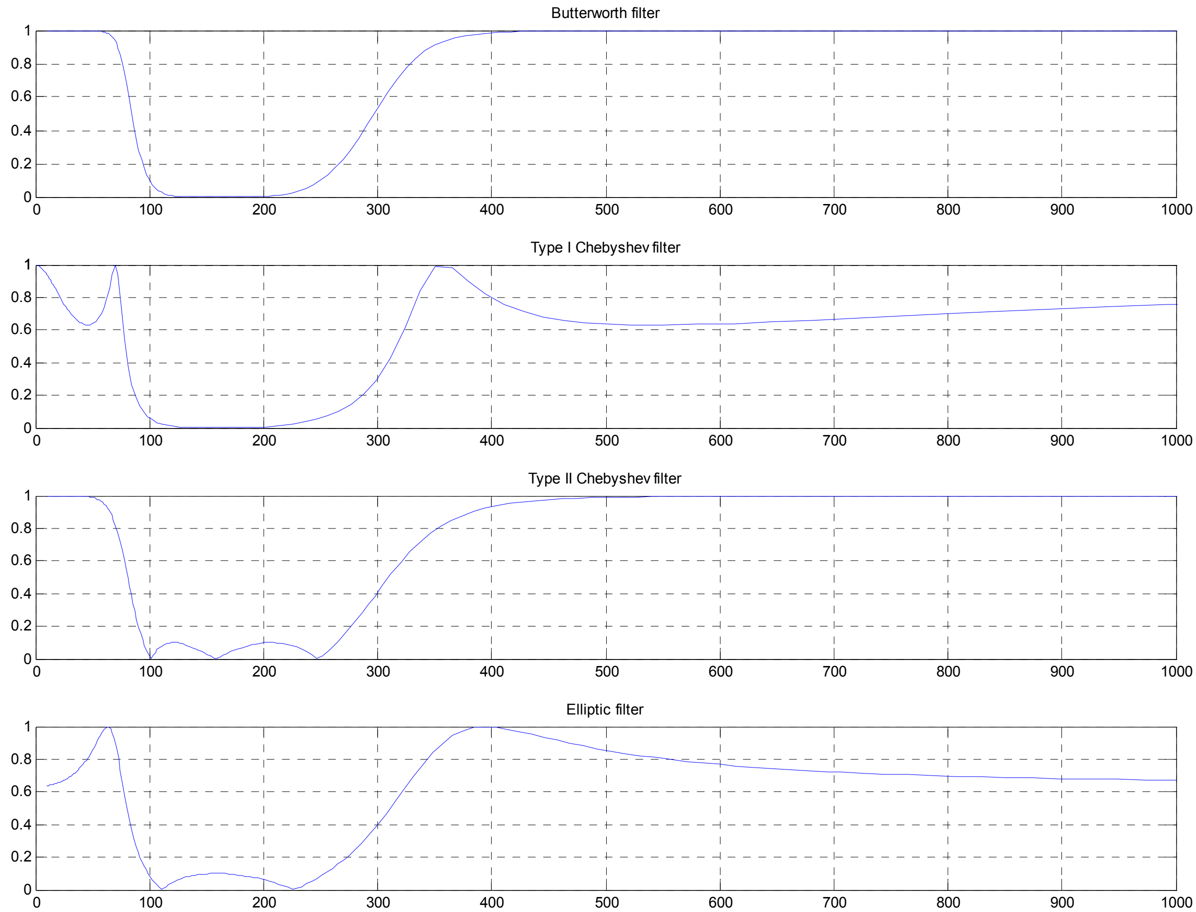

Figure S7.27: Magnitude Spectra of the four implementations of the bandstop filter for Problem 7.27.