Chapter 10: Time Domain Analysis of DT Systems

Problem 10.1

(a) Expressing the difference equation as

$$
\begin{aligned}
y[k] &= 2y[k-1] + x[k-1] \\
&= 2y[k-1] + 2u(k-1) \qquad \{\because x[k] = 2u[k]\} \\
&= \begin{cases} 2y[k-1] & k < 1 \\ 2y[k-1] + 2 & k \geq 1 \end{cases}
\end{aligned}
$$

Starting from $k = 0$, we iterate to calculate the output as

$$
\begin{aligned}
y[0] &= 2y[-1] = 4 \\
y[1] &= 2y[0] + 2 = 10 \\
y[2] &= 2y[1] + 2 = 22 \\
y[3] &= 2y[2] + 2 = 46 \\
y[4] &= 2y[3] + 2 = 94 \\
y[5] &= 2y[4] + 2 = 190
\end{aligned}
$$

(b) and (c) The zero-state and zero-input responses are calculated below

Zero state response	Zero input response
$y_{zs}[k] = \begin{cases} 2y_{zs}[k-1] & k < 1 \\ 2y_{zs}[k-1] + 2 & k \geq 1 \end{cases}, \; y[-1] = 0$ Iterating with respect to k, we get $y_{zs}[0] = 2y_{zs}[-1] = 0$ $y_{zs}[1] = 2y_{zs}[0] + 2 = 2$ $y_{zs}[2] = 2y_{zs}[1] + 2 = 6$ $y_{zs}[3] = 2y_{zs}[2] + 2 = 14$ $y_{zs}[4] = 2y_{zs}[3] + 2 = 30$ $y_{zs}[5] = 2y_{zs}[4] + 2 = 62$	$y_{zi}[k] = 2y_{zi}[k-1]$ with $y[-1] = 2$ Iterating with respect to k, we get $y_{zi}[0] = 2y[-1] = 4$ $y_{zi}[1] = 2y_{zi}[0] = 8$ $y_{zi}[2] = 2y_{zi}[1] = 16$ $y_{zi}[3] = 2y_{zi}[2] = 32$ $y_{zi}[4] = 2y_{zi}[3] = 64$ $y_{zi}[5] = 2y_{zi}[4] = 128$

(d) It can be easily verified that $y[k] = y_{zi}[k] + y_{zs}[k]$. ▌

Problem 10.2

(a) Expressing the difference equation as

$$y[k] = y[k-1] - 0.5y[k-2] + x[k-2]$$

or,

$$y[k] = \begin{cases} y[k-1] - 0.5y[k-2], & k < 2 \\ y[k-1] - 0.5y[k-2] + 0.5^{k-2} & k \geq 2. \end{cases}$$

Starting from $k = 0$, we iterate to calculate the output as

$$
\begin{aligned}
y[0] &= y[-1] - 0.5y[-2] = 0 - 0.5(1) = -0.5 \\
y[1] &= y[0] - 0.5y[-1] = -0.5 - 0.5(0) = -0.5 \\
y[2] &= y[1] - 0.5y[0] + 1 = -0.5 - 0.5(-0.5) + 1 = 0.75 \\
y[3] &= y[2] - 0.5y[1] + 0.5 = 0.75 - 0.5(-0.5) + 0.5 = 1.5 \\
y[4] &= y[3] - 0.5y[2] + 0.5^2 = 1.5 - 0.5(0.75) + 0.25 = 1.375 \\
y[5] &= y[4] - 0.5y[3] + 0.5^3 = 1.375 - 0.5(1.5) + 0.125 = 0.75.
\end{aligned}
$$

(b) and (c) The zero-state and zero-input responses are calculated below

Zero state response	**Zero input response**
$y[k] = \begin{cases} y[k-1] - 0.5y[k-2], & k < 2 \\ y[k-1] - 0.5y[k-2] + 0.5^{k-2} & k \geq 2. \end{cases}$	$y[k] = y[k-1] - 0.5y[k-2]$
with $y[-1] = y[-2] = 0$.	with $y[-1] = 0$ and $y[-2] = 1$.
Iterating with respect to k, we get	Iterating with respect to k, we get
$y_{zs}[0] = 0$	$y_{zi}[0] = y_{zi}[-1] - 0.5y_{zi}[-2] = -0.5$
$y_{zs}[1] = 0$	$y_{zs}[1] = y_{zi}[0] - 0.5y_{zi}[-1] = -0.5$
$y_{zs}[2] = y_{zs}[1] - 0.5y_{zs}[0] + 1 = 1$	$y_{zs}[2] = y_{zs}[1] - 0.5y_{zs}[0] = -0.25$
$y_{zs}[3] = y_{zs}[2] - 0.5y_{zs}[1] + 0.5 = 1.5$	$y_{zs}[3] = y_{zs}[2] - 0.5y_{zs}[1] = 0$
$y_{zs}[4] = y_{zs}[3] - 0.5y_{zs}[2] + 0.5^2 = 1.25$	$y_{zs}[4] = y_{zs}[3] - 0.5y_{zs}[2] = 0.125$
$y_{zs}[5] = y_{zs}[4] - 0.5y_{zs}[3] + 0.5^3 = 0.625.$	$y_{zs}[5] = y_{zs}[4] - 0.5y_{zs}[3] = 0.125.$

(d) By adding the zero-state and zero input responses obtained in parts (b) and (c) and comparing with the output in (a), it is verified that $y[k] = y_{zi}[k] + y_{zs}[k]$.

Problem 10.3

(a) Expressing the difference equation as

$$y[k] = 0.75y[k-1] - 0.125y[k-2] + x[k-2]$$

or,

$$y[k] = \begin{cases} 0.75y[k-1] - 0.125y[k-2], & k < 2 \\ 0.75y[k-1] - 0.125y[k-2] + (-1)^{k-2} & k \geq 2. \end{cases}$$

Starting from $k = 0$, we iterate to calculate the output as

$$
\begin{aligned}
y[0] &= 0.75y[-1] - 0.125y[-2] = 0.75 - 0.125(-1) = 0.875 \\
y[1] &= 0.75y[0] - 0.125y[-1] = 0.75(0.875) - 0.125(1) = 0.53125 \\
y[2] &= 0.75y[1] - 0.125y[0] + 1 = 0.75(0.53125) - 0.125(0.875) + 1 = 1.28906 \\
y[3] &= 0.75y[2] - 0.125y[1] - 1 = 0.75(1.28906) - 0.125(0.53125) - 1 = -0.09961 \\
y[4] &= 0.75y[3] - 0.125y[2] + 1 = 0.75(-0.09961) - 0.125(1.28906) + 1 = 0.76416 \\
y[5] &= 0.75y[4] - 0.125y[3] - 1 = 0.75(0.76416) - 0.125(-0.09961) - 1 = -0.41443.
\end{aligned}
$$

(b) and (c) The zero-state and zero-input responses are calculated below

Zero state response	**Zero input response**
$y_{zs}[k]=\begin{cases} 0.75y_{zs}[k-1]-0.125y_{zs}[k-2], & k<2 \\ 0.75y_{zs}[k-1]-0.125y_{zs}[k-2]+(-1)^{k-2} & k\geq 2 \end{cases}$ with $y_{zs}[-1] = y_{zs}[-2] = 0$. Iterating with respect to k, we get $y_{zs}[0]=0$ $y_{zs}[1]=0$ $y_{zs}[2]=0.75y_{zs}[1]-0.125y_{zs}[0]+1=1$ $y_{zs}[3]=0.75y_{zs}[2]-0.125y_{zs}[1]-1=-0.25$ $y_{zs}[4]=0.75y_{zs}[3]-0.125y_{zs}[2]+1=0.6875$ $y_{zs}[5]=0.75y_{zs}[4]-0.125y_{zs}[3]-1=-0.45312.$	$y_{zi}[k]=0.75y_{zi}[k-1]-0.125y_{zi}[k-2]$ with $y_{zi}[-1] = 1$ and $y_{zi}[-2] = 1$. Iterating with respect to k, we get $y_{zi}[0]=0.75y_{zi}[-1]-0.125y_{zi}[-2]=0.875$ $y_{zs}[1]=0.75y_{zi}[0]-0.125y_{zi}[-1]=0.53125$ $y_{zs}[2]=0.75y_{zs}[1]-0.125y_{zs}[0]=0.28906$ $y_{zs}[3]=0.75y_{zs}[2]-0.125y_{zs}[1]=0.15039$ $y_{zs}[4]=0.75y_{zs}[3]-0.125y_{zs}[2]=0.07869$ $y_{zs}[5]=0.75y_{zs}[4]-0.125y_{zs}[3]=0.03988.$

(d) By adding the zero-state and zero input responses obtained in parts (b) and (c) and comparing with the output in (a), it is verified that $y[k] = y_{zi}[k] + y_{zs}[k]$. ∎

Problem 10.4

$$a^k u[k]*b^k u[k]=\sum_{m=-\infty}^{\infty} a^m u[m]\times b^{k-m}u[k-m]=\sum_{m=0}^{\infty} a^m\times b^{k-m}u[k-m]=b^k\sum_{m=0}^{\infty}(a/b)^m u[k-m]$$

Noting that $u[k-m] = u[-(m-k)] = 0$ for $m > k$, we obtain

$$a^k u[k]*b^k u[k]=b^k\sum_{m=0}^{k}(a/b)^m=\begin{cases}(k+1)b^k & a=b\\ b^k\frac{1-(a/b)^{k+1}}{1-(a/b)} & a\neq b\end{cases}$$

$$=\begin{cases}(k+1)a^k & a=b\\ b^{k+1}\frac{1-(a/b)^{k+1}}{b-a} & a\neq b\end{cases}=\begin{cases}(k+1)a^k & a=b\\ \frac{b^{k+1}-a^{k+1}}{b-a} & a\neq b\end{cases}$$

$$=\begin{cases}(k+1)a^k & a=b\\ \frac{a^{k+1}-b^{k+1}}{a-b} & a\neq b\end{cases}$$

Problem 10.5

(a) The graphical approach is illustrated in Fig. 10.5(a).

Case ($k<-6$), Fig. S10.5(a) subplot (v): $x_1[k]*x_2[k]=0$.

Case ($-6\leq k\leq 2$), Fig. S10.5(b) subplot (vi):

$$x_1[k]*x_2[k]=\sum_{m=-2}^{k+4}1=(k+7).$$

Case ($2\leq k\leq 6$), Fig. S10.5(b) subplot (vii):

$$x_1[k]*x_2[k] = \sum_{m=k-4}^{2} 1 = (6-k)\,.$$

Case ($k > 6$), Fig. S10.5(b) subplot (viii): $x_1[k]*x_2[k] = 0$

The convolution sum is plotted as a function of k in subplot (ix) of Fig. S10.5(a).

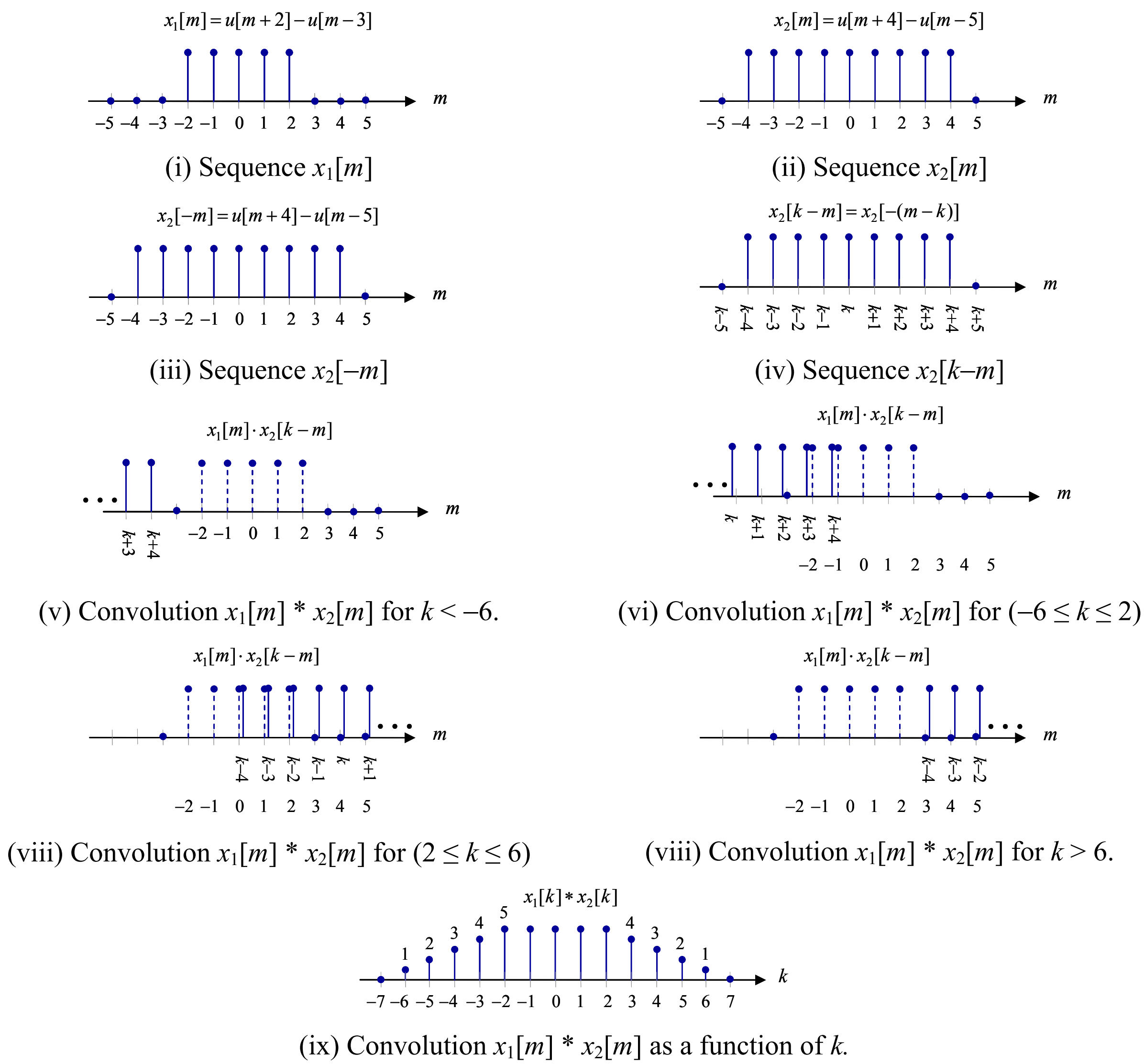


Fig. S10.5(a) Convolution sum for part (a) in Problem 10.5.

(b) From Problem 10.4, we know that

$$a^k u[k] * b^k u[k] = \tfrac{1}{(a-b)}(a^{k+1} - b^{k+1})u[k]\,.$$

Setting $a = 0.8$ and $b = 0.5$, we get

$$0.8^k u[k] * 0.5^k u[k] = \tfrac{10}{3}(0.8^{k+1} - 0.5^{k+1})u[k]\,.$$

Using the time shifting property,

$$0.8^{k-5}u[k-5]*0.5^k u[k] = \tfrac{10}{3}(0.8^{k-5+1} - 0.5^{k-5+1})u[k-5],$$

or, $$0.8^{k-5}u[k-5]*0.5^k u[k] = \tfrac{10}{3}(0.8^{k-4} - 0.5^{k-4})u[k-5],$$

which implies $$0.8^{k}u[k-5]*0.5^k u[k] = 0.8^5 \times \tfrac{10}{3}(0.8^{k-4} - 0.5^{k-4})u[k-5],$$

or, $$0.8^{k}u[k-5]*0.5^k u[k] = 1.09227(0.8^{k-4} - 0.5^{k-4})u[k-5].$$

(c) The graphical approach is illustrated in Fig. S10.5(c).

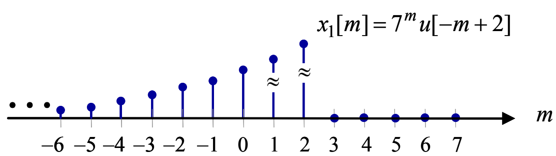
Case ($k \le -6$), Fig. S10.5(c) subplot (v):

$$x_1[k]*x_2[k] = \sum_{m=-\infty}^{k-4} 0.4^{k-m} \times 7^m = 0.4^k \sum_{m=-(k-4)}^{\infty} (2/35)^m = \frac{16}{33 \times 35^3} \times 7^k.$$

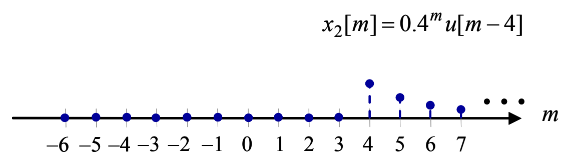
Case ($2 \le k \le 6$), Fig. S10.5(b) subplot (vii):

$$x_1[k]*x_2[k] = \sum_{m=-\infty}^{2} 0.4^{k-m} \times 7^m = 0.4^k \sum_{m=-2}^{\infty} (2/35)^m = \frac{35^3}{132} \times 0.4^k.$$

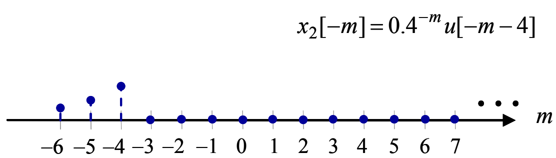
The convolution sum is plotted as a function of k in subplot (viii) of Fig. S10.5(c).



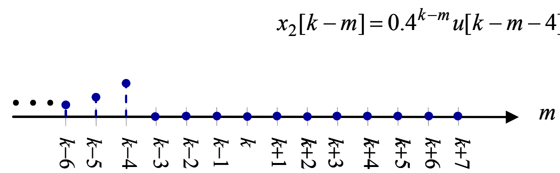
(i) Sequence $x_1[m]$



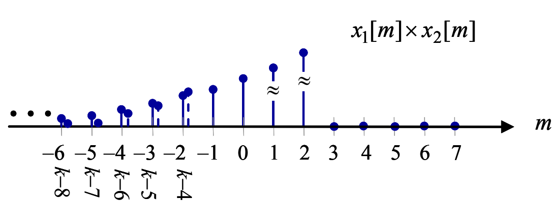
(ii) Sequence $x_2[m]$



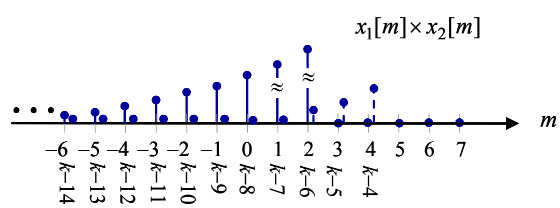
(iii) Sequence $x_2[-m]$



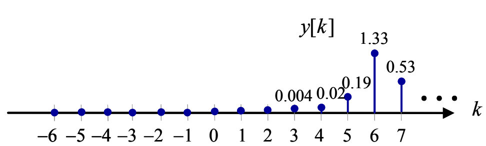
(iv) Sequence $x_2[k–m]$



(v) Convolution $x_1[m]$ * $x_2[m]$ for $k \le 6$



(vi) Convolution $x_1[m]$ * $x_2[m]$ for $k \ge 6$



(vii) Convolution $x_1[m] * x_2[m]$ as a function of k.

Fig. S10.5(c) Convolution sum for part (c) in Problem 10.5.

(d) The graphical approach is illustrated in Fig. S10.5(d).

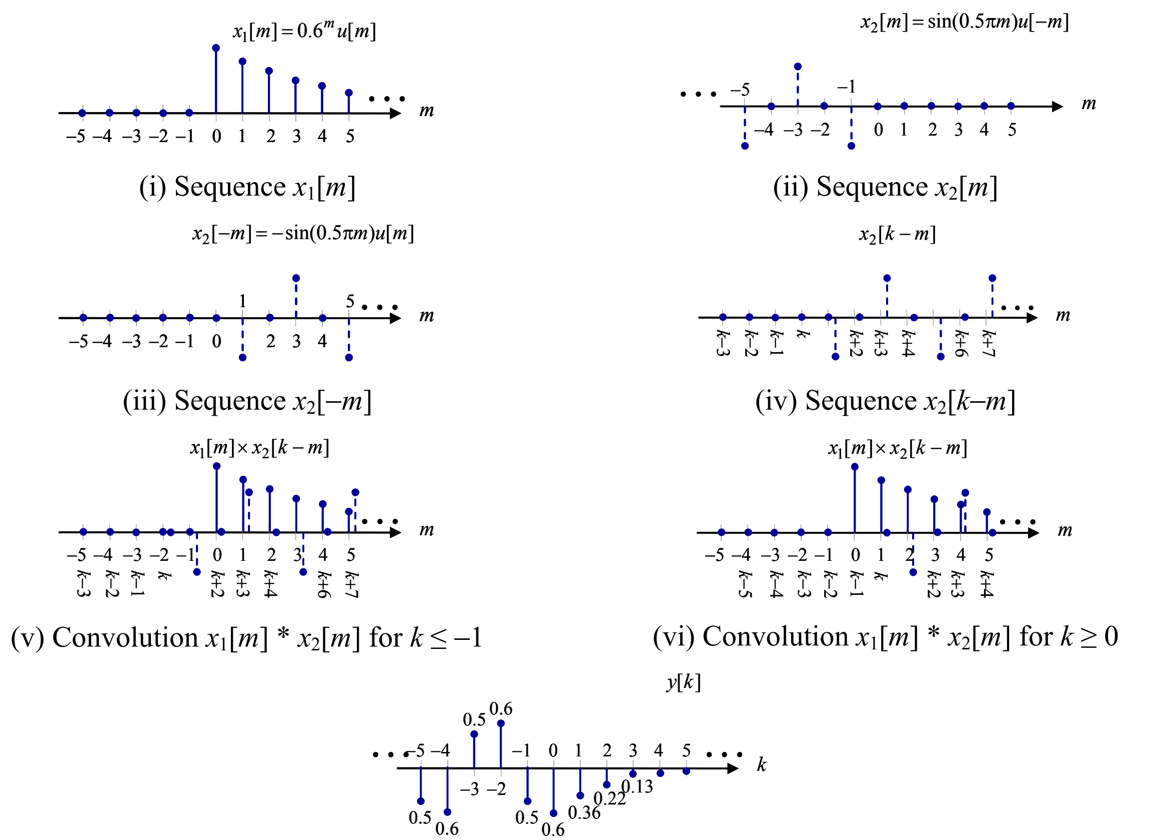
Case ($k \le -1$), Fig. S10.5(d) subplot (v):

$$x_1[k] * x_2[k] = \sum_{m=0}^{\infty} 0.6^m \sin(0.5\pi(k-m)) = \frac{1}{j2}\sum_{m=0}^{\infty} 0.6^m \times e^{j0.5\pi(k-m)} - \frac{1}{j2}\sum_{m=0}^{\infty} 0.6^m \times e^{-j0.5\pi(k-m)},$$

which reduces to

$$x_1[k] * x_2[k] = \frac{1}{j2} e^{j0.5\pi k} \sum_{m=0}^{\infty} (0.6e^{-j0.5\pi})^m - \frac{1}{j2} e^{-j0.5\pi k} \sum_{m=0}^{\infty} (0.6e^{j0.5\pi})^m,$$

or,
$$x_1[k] * x_2[k] = \frac{1}{j2} e^{j0.5\pi k} \frac{1}{1-0.6e^{-j0.5\pi}} - \frac{1}{j2} e^{-j0.5\pi k} \frac{1}{1-0.6e^{j0.5\pi}}.$$



(vii) Convolution $x_1[m] * x_2[m]$ as a function of k.

Fig. S10.5(d) Convolution sum for part (d) in Problem 10.5.

Simplifying, we get

$$x_1[k]*x_2[k]=\frac{1}{j2}e^{-j0.5\pi k}\left[e^{j\pi k}\,\frac{1}{1+j0.6}-\frac{1}{1-j0.6}\right].$$

If k is even, $\exp(j\pi k)=1$, therefore,

$$x_1[k]*x_2[k]=\frac{1}{j2}e^{-j0.5\pi k}\left[\frac{1}{1+j0.6}-\frac{1}{1-j0.6}\right]=-0.6e^{-j0.5\pi k}.$$

If k is odd, $\exp(j\pi k)=-1$, therefore,

$$x_1[k]*x_2[k]=\frac{1}{j2}e^{-j0.5\pi k}\left[-\frac{1}{1+j0.6}-\frac{1}{1-j0.6}\right]=je^{-j0.5\pi k}.$$

Case ($k\geq 0$), Fig. S10.5(d) subplot (vi):

$$x_1[k]*x_2[k]=\frac{1}{j2}\sum_{m=k}^{\infty}0.6^m\times e^{j0.5\pi(k-m)}-\frac{1}{j2}\sum_{m=k}^{\infty}0.6^m\times e^{-j0.5\pi(k-m)},$$

which reduces to

$$x_1[k]*x_2[k]=\frac{1}{j2}e^{j0.5\pi k}\sum_{m=k}^{\infty}(0.6e^{-j0.5\pi})^m-\frac{1}{j2}e^{-j0.5\pi k}\sum_{m=k}^{\infty}(0.6e^{j0.5\pi})^m,$$

or, $$x_1[k]*x_2[k]=\frac{1}{j2}e^{j0.5\pi k}\,\frac{0.6^k\,e^{-j0.5\pi k}}{1-0.6e^{-j0.5\pi}}-\frac{1}{j2}e^{-j0.5\pi k}\,\frac{0.6^k\,e^{j0.5\pi k}}{1-0.6e^{j0.5\pi}},$$

or, $$x_1[k]*x_2[k]=\frac{0.6^k}{j2}\left[\frac{1}{1-0.6e^{-j0.5\pi}}-\frac{1}{1-0.6e^{j0.5\pi}}\right]=\frac{0.6^k}{j2}\times(-1.2j\sin(0.5\pi)),$$

or, $$x_1[k]*x_2[k]=\frac{0.6^k}{j2}\times(-1.2j\sin(0.5\pi))=-0.6^{k+1}.$$

The convolution sum is plotted as a function of k in subplot (vii) of Fig. S10.5(d). Note that the result of convolution sum is always real as two real valued sequences are being convolved.

(e) The graphical approach is illustrated in Fig. 10.5(e).

Case ($k<0$), Fig. S10.5(e) subplot (v):

$$x_1[k]*x_2[k]=\sum_{m=-\infty}^{k-1}0.5^{-m}\times 0.8^{(k-m)}+\sum_{m=k}^{0}0.5^{-m}\times 0.8^{-(k-m)}+\sum_{m=1}^{\infty}0.5^{m}\times 0.8^{-(k-m)},$$

which reduces to

$$x_1[k]*x_2[k]=0.8^k\sum_{m=-\infty}^{k-1}(0.4)^{-m}+0.8^{-k}\sum_{m=k}^{0}0.625^{-m}+0.8^{-k}\sum_{m=1}^{\infty}0.4^m,$$

or, $$x_1[k]*x_2[k]=0.8^k\sum_{m=-(k-1)}^{\infty}0.4^m+0.8^{-k}\sum_{m=0}^{-k}0.625^m+0.8^{-k}\sum_{m=1}^{\infty}0.4^m,$$

or, $$x_1[k]*x_2[k]=0.8^k\,\frac{0.4^{-(k-1)}}{1-0.4}+0.8^{-k}\,\frac{(1-0.625^{-k+1})}{1-0.625}+0.8^{-k}\,\frac{0.4}{1-0.4}$$

or, $$x_1[k]*x_2[k]=\frac{2}{3}\times\left[2^k+4\times 0.8^{-k}(1-0.625^{-k+1})+0.8^{-k}\right].$$

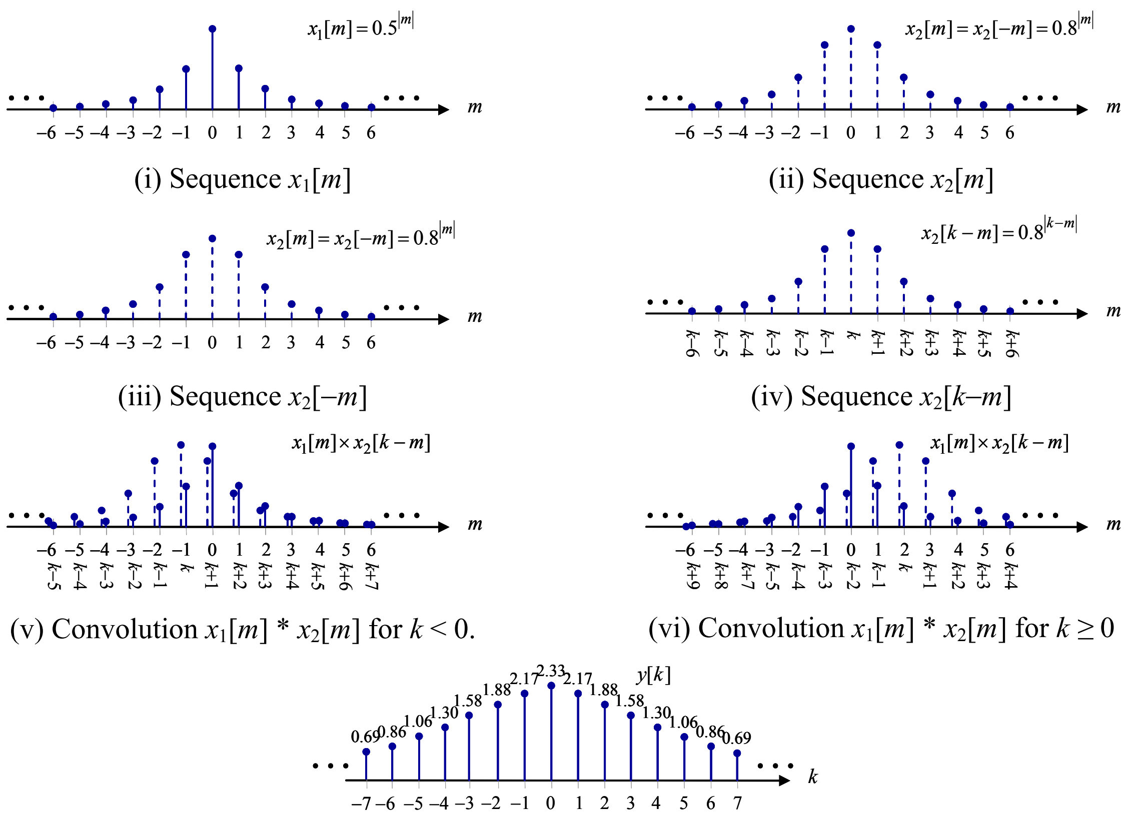
Case ($k\geq 0$), Fig. S10.5(e) subplot (v):

$$x_1[k]*x_2[k]=\sum_{m=-\infty}^{-1}0.5^{-m}\times 0.8^{(k-m)}+\sum_{m=0}^{k}0.5^{m}\times 0.8^{(k-m)}+\sum_{m=k+1}^{\infty}0.5^{m}\times 0.8^{-(k-m)},$$

which reduces to

$$x_1[k]*x_2[k]=0.8^k\sum_{m=-\infty}^{-1}0.4^{-m}+0.8^k\sum_{m=0}^{k}0.625^{m}+0.8^{-k}\sum_{m=k+1}^{\infty}0.4^{m},$$

or, $$x_1[k]*x_2[k]=0.8^k\sum_{m=1}^{\infty}0.4^{m}+0.8^k\sum_{m=0}^{k}0.625^{m}+0.8^{-k}\sum_{m=k+1}^{\infty}0.4^{m},$$



(i) Sequence $x_1[m]$

(ii) Sequence $x_2[m]$

(iii) Sequence $x_2[-m]$

(iv) Sequence $x_2[k-m]$

(v) Convolution $x_1[m] * x_2[m]$ for $k < 0$.

(vi) Convolution $x_1[m] * x_2[m]$ for $k \geq 0$

(vii) Convolution $x_1[m] * x_2[m]$ as a function of k.

Fig. S10.5(e): Convolution sum for part (e) in Problem 10.5.

or, $$x_1[k] * x_2[k] = 0.8^k \frac{0.4}{1-0.4} + 0.8^k \frac{(1-0.625^{k+1})}{1-0.625} + 0.8^{-k} \frac{0.4^{k+1}}{1-0.4}$$

or, $$x_1[k] * x_2[k] = \frac{2}{3} \times \left[0.8^k + 4 \times 0.8^k (1-0.625^{k+1}) + 2^{-k}\right].$$

The convolution sum is plotted as a function of k in subplot (vii) of Fig. S10.5(e). ▌

<u>Problem 10.6</u>

(a) <u>Direct Convolution Approach:</u>

Expressing $$x[k] = \delta[k-1] + 2\delta[k-2] + 3\delta[k-3]$$

and $$h[k] = 2\delta[k+1] + 2\delta[k] + 2\delta[k-1] + 2\delta[k-2],$$

the convolution is given by

$$x[k] * h[k] = (\delta[k-1] + 2\delta[k-2] + 3\delta[k-3]) * (2\delta[k+1] + 2\delta[k] + 2\delta[k-1] + 2\delta[k-2]),$$

which reduces to

$$\begin{aligned} x[k] * h[k] = {} & (2\delta[k] + 2\delta[k-1] + 2\delta[k-2] + 2\delta[k-3]) + (4\delta[k-1] + 4\delta[k-2] \\ & + 4\delta[k-3] + 4\delta[k-4]) + (6\delta[k-2] + 6\delta[k-3] + 6\delta[k-4] + 6\delta[k-5]), \end{aligned}$$

or, $$x[k] * h[k] = 2\delta[k] + 6\delta[k-1] + 12\delta[k-2] + 12\delta[k-3] + 10\delta[k-4] + 6\delta[k-5].$$

<u>Sliding tape method:</u> The convolution using the sliding tape method is shown in Table S10.6(a).

Table S10.6(a): Convolution of $x[k]$ and $h[k]$ using the sliding tape method Problem 10.6(a).

	…	–5	–4	–3	–2	–1	0	1	2	3	4	5	6	7	…	k	$y[k]$
$x[m]$								1	2	3							
$h[m]$						2	2	2	2								
$h[-m]$					2	2	2	2									
$h[-1-m]$				2	2	2	2									–1	0
$h[0-m]$					2	2	2	2								0	2
$h[1-m]$						2	2	2	2							1	6
$h[2-m]$							2	2	2	2						2	12
$h[3-m]$								2	2	2	2					3	12
$h[4-m]$									2	2	2	2				4	10
$h[5-m]$										2	2	2	2			5	6
$h[6-m]$											2	2	2	2		6	0

The results from the two approaches are the same.

(b) <u>Direct Convolution Approach:</u>

$$x[k] = 2\delta[k+2] + \delta[k+1] + \delta[k-1] + 2\delta[k-2]$$

$$h[k] = \delta[k] + 0.5\delta[k-1] + 0.25\delta[k-2] + 0.125\delta[k-3],$$

The convolution of $x[k]$ and $h[k]$ is given by

$$\begin{aligned}
x[k]*h[k] &= (2\delta[k+2]+\delta[k+1]+\delta[k-1]+2\delta[k-2])*(\delta[k]+0.5\delta[k-1]+0.25\delta[k-2]+0.125\delta[k-3]) \\
&= (2\delta[k+2]+\delta[k+1]+0.5\delta[k]+0.25\delta[k-1]) \\
&\quad +(\delta[k+1]+0.5\delta[k]+0.25\delta[k-1]+0.125\delta[k-2]) \\
&\quad +(\delta[k-1]+0.5\delta[k-2]+0.25\delta[k-3]+0.125\delta[k-4]) \\
&\quad +(2\delta[k-2]+\delta[k-3]+0.5\delta[k-4]+0.25\delta[k-5]) \\
&= 2\delta[k+2]+2\delta[k+1]+\delta[k]+1.5\delta[k-1]+2.625\delta[k-2]+1.25\delta[k-3]+0.625\delta[k-4] \\
&\quad +0.25\delta[k-5]
\end{aligned},$$

or, $x[k]*h[k]=2\delta[k+2]+2\delta[k+1]+\delta[k]+1.5\delta[k-1]+2.625\delta[k-2]+1.25\delta[k-3]+0.625\delta[k-4]+0.25\delta[k-5]$.

<u>Sliding tape method:</u> The convolution using the sliding tape method is shown in Table S10.6(b). The results from the two approaches are the same. ▌

Table S10.6(a): Convolution of $x[k]$ and $h[k]$ using the sliding tape method Problem 10.6(b).

	…	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	…	k	$y[k]$
$x[m]$						2	1	0	1	2							
$h[m]$								1	0.5	0.25	0.125						
$h[-m]$					0.125	0.25	0.5	1									
$h[-3-m]$		0.125	0.25	0.5	1											−3	0
$h[-2-m]$			0.125	0.25	0.5	1										−2	2
$h[-1-m]$				0.125	0.25	0.5	1									−1	2
$h[0-m]$					0.125	0.25	0.5	1								0	1
$h[1-m]$						0.125	0.25	0.5	1							1	1.5
$h[2-m]$							0.125	0.25	0.5	1						2	2.625
$h[3-m]$								0.125	0.25	0.5	1					3	1.25
$h[4-m]$									0.125	0.25	0.5	1				4	0.625
$h[5-m]$										0.125	0.25	0.5	1			5	0.25
$h[6-m]$											0.125	0.25	0.5	1		6	0

Problem 10.7

The convolution using the sliding tape method is shown in Table S10.7.

Table S10.7: Convolution of $x[k]$ and $h[k]$ using the sliding tape method Problem 10.7.

	…	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	…	k	$y[k]$
$h[m]$							1	2	3	4	5						
$x[m]$							2	2	2								
$x[-m]$					2	2	2										
$x[-1-m]$				2	2	2										−1	0
$x[0-m]$					2	2	2									0	2
$x[1-m]$						2	2	2								1	6
$x[2-m]$							2	2	2							2	12
$x[3-m]$								2	2	2						3	18
$x[4-m]$									2	2	2					4	24
$x[5-m]$										2	2	2				5	18
$x[6-m]$											2	2	2			6	10
$x[7-m]$												2	2	2		7	0

The result matches with that obtained in Example 10.8. ▌

<u>Problem 10.8</u>

The convolution using the sliding tape method is shown in Table S10.8.

Table S10.8: Convolution of $x[k]$ and $h[k]$ using the sliding tape method Problem 10.8.

	…	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	…	k	$y[k]$
$x[m]$							-1	1	2								
$h[m]$							3	1	-2	3	-2						
$h[-m]$					-2	3	-2	1	3								
$h[-3-m]$		-2	3	-2	1	3										−3	0
$h[-2-m]$			-2	3	-2	1	3									−2	-3
$h[-1-m]$				-2	3	-2	1	3								−1	2
$h[0-m]$					-2	3	-2	1	3							0	9
$h[1-m]$						-2	3	-2	1	3						1	-3
$h[2-m]$							-2	3	-2	1	3					2	1
$h[3-m]$								-2	3	-2	1	3				3	4
$h[4-m]$									-2	3	-2	1	3			4	-4
$h[5-m]$										-2	3	-2	1	3		5	0

The result matches with that obtained in Example 10.9. ▌

<u>Direct Convolution Approach:</u>

$x[k] = -\delta[k+1] + \delta[k] + 2\delta[k-1]$

$h[k] = 3\delta[k+1] + \delta[k] - 2\delta[k-1] + 3\delta[k-2] - 2\delta[k-3]$,

The convolution is given by

$$\begin{aligned} x[k]*h[k] &= (-\delta[k+1]+\delta[k]+2\delta[k-1])*(3\delta[k+1]+\delta[k]-2\delta[k-1]+3\delta[k-2]-2\delta[k-3]) \\ &= (-3\delta[k+2]-\delta[k+1]+2\delta[k]-3\delta[k-1]+2\delta[k-2]) \\ &\quad +(3\delta[k+1]+\delta[k]-2\delta[k-1]+3\delta[k-2]-2\delta[k-3]) \\ &\quad +(6\delta[k]+2\delta[k-1]-4\delta[k-2]+6\delta[k-3]-4\delta[k-4]) \\ &= -3\delta[k+2]+2\delta[k+1]+9\delta[k]-3\delta[k-1]+\delta[k-2]+4\delta[k-3]-4\delta[k-4]. \end{aligned}$$

<u>Problem 10.9</u>

The periodic convolution with $K_0 = 10$ is shown in Table S10.9(a).

Table S10.9(a): Periodic convolution of $x_p[k]$ and $h_p[k]$ in Problem 10.9 with K_0 set to 10.

	0	1	2	3	4	5	6	7	8	9	k	$y_p[k]$
$h_p[k]$	1	2	3	4	5	0	0	0	0	0		
$x_p[k]$	2	2	2	0	0	0	0	0	0	0		
$x_p[-k]$	2	0	0	0	0	0	0	0	2	2	0	2
$x_p[1-k]$	2	2	0	0	0	0	0	0	0	2	1	6
$x_p[2-k]$	2	2	2	0	0	0	0	0	0	0	2	12

$x_p[3-k]$	0	2	2	2	0	0	0	0	0	0	3	18
$x_p[4-k]$	0	0	2	2	2	0	0	0	0	0	4	24
$x_p[5-k]$	0	0	0	2	2	2	0	0	0	0	5	18
$x_p[6-k]$	0	0	0	0	2	2	2	0	0	0	6	10
$x_p[7-k]$	0	0	0	0	0	2	2	2	0	0	7	0
$x_p[8-k]$	0	0	0	0	0	0	2	2	2	0	8	0

The periodic convolution with $K_0 = 13$ is shown in S10.9(b).

Table S10.9(b): Periodic convolution of $x_p[k]$ and $h_p[k]$ in Problem 10.9 with K_0 set to 13.

	0	1	2	3	4	5	6	7	8	9	10	11	12	k	$y_p[k]$
$h_p[k]$	1	2	3	4	5	0	0	0	0	0	0	0	0		
$x_p[k]$	2	2	2	0	0	0	0	0	0	0	0	0	0		
$x_p[-k]$	2	0	0	0	0	0	0	0	0	0	0	2	2	0	2
$x_p[1-k]$	2	2	0	0	0	0	0	0	0	0	0	0	2	1	6
$x_p[2-k]$	2	2	2	0	0	0	0	0	0	0	0	0	0	2	12
$x_p[3-k]$	0	2	2	2	0	0	0	0	0	0	0	0	0	3	18
$x_p[4-k]$	0	0	2	2	2	0	0	0	0	0	0	0	0	4	24
$x_p[5-k]$	0	0	0	2	2	2	0	0	0	0	0	0	0	5	18
$x_p[6-k]$	0	0	0	0	2	2	2	0	0	0	0	0	0	6	10
$x_p[7-k]$	0	0	0	0	0	2	2	2	0	0	0	0	0	7	0
$x_p[8-k]$	0	0	0	0	0	0	2	2	2	0	0	0	0	8	0
$x_p[9-k]$	0	0	0	0	0	0	0	2	2	2	0	0	0	9	0
$x_p[10-k]$	0	0	0	0	0	0	0	0	2	2	2	0	0	10	0
$x_p[11-k]$	0	0	0	0	0	0	0	0	0	2	2	2	0	11	0
$x_p[12-k]$	0	0	0	0	0	0	0	0	0	0	2	2	2	12	0

Since in both cases the fundamental period $K_0 \geq K_1 + K_2 - 1\ (= 7)$, the results of periodic convolution over one period are the same as that of linear convolution.

Problem 10.10

The periodic convolution with $K_0 = 10$ is shown in Table S10.10.

Table S10.10: Periodic convolution of $x_p[k]$ and $h_p[k]$ in Problem 10.10 with K_0 set equal to 10.

	0	1	2	3	4	5	6	7	8	9	k	$y_p[k]$
$h_p[k]$	1	2	3	4	5	0	0	0	0	0		
$x_p[k]$	2	2	2	0	0	0	0	0	0	0		
$x_p[-k]$	2	0	0	0	0	0	0	0	2	2	0	2
$x_p[1-k]$	2	2	0	0	0	0	0	0	0	2	1	6
$x_p[2-k]$	2	2	2	0	0	0	0	0	0	0	2	12
$x_p[3-k]$	0	2	2	2	0	0	0	0	0	0	3	18
$x_p[4-k]$	0	0	2	2	2	0	0	0	0	0	4	24
$x_p[5-k]$	0	0	0	2	2	2	0	0	0	0	5	18
$x_p[6-k]$	0	0	0	0	2	2	2	0	0	0	6	10
$x_p[7-k]$	0	0	0	0	0	2	2	2	0	0	7	0
$x_p[8-k]$	0	0	0	0	0	0	2	2	2	0	8	0

Since the fundamental period $K_0(=10) \geq K_1 + K_2 - 1\ (=7)$ of the two sequences, the result of periodic convolution over one period is the same as that of linear convolution.

Problem 10.11

The periodic convolution with $K_0 = 15$ is shown in Table S10.11.

Table S10.11: Periodic convolution of $x_p[k]$ and $h_p[k]$ in Problem 10.9 with K_0 set to 15.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	k	$y_p[k]$
$h_p[k]$	1	2	3	4	5	0	0	0	0	0	0	0	0	0	0		
$x_p[k]$	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0		
$x_p[-k]$	2	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	2
$x_p[1-k]$	2	2	0	0	0	0	0	0	0	0	0	0	0	0	2	1	6
$x_p[2-k]$	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	2	12
$x_p[3-k]$	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	3	18
$x_p[4-k]$	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	4	24
$x_p[5-k]$	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	5	18
$x_p[6-k]$	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0	6	10
$x_p[7-k]$	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0	7	0
$x_p[8-k]$	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0	8	0
$x_p[9-k]$	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	9	0
$x_p[10-k]$	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	10	0
$x_p[11-k]$	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	11	0
$x_p[12-k]$	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	12	0
$x_p[13-k]$	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	13	0
$x_p[14-k]$	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	14	0

Since the fundamental period $K_0(=15) \geq K_1 + K_2 - 1\ (=7)$ of the two sequences, the result of periodic convolution over one period is the same as that of linear convolution.

Problem 10.12

The periodic convolution with $K_0 = 8$ is shown in Table S10.12.

Table S10.12: Periodic convolution of $x_p[k]$ and $h_p[k]$ in Problem 10.12 with K_0 set to 8.

	0	1	2	3	4	5	6	7	k	$y_p[k]$
$h_p[k]$	5	5	0	0	0	0	0	0		
$x_p[k]$	0	1	2	3	0	0	0	0		
$x_p[-k]$	0	0	0	0	0	3	2	1	0	0
$x_p[1-k]$	1	0	0	0	0	0	3	2	1	5
$x_p[2-k]$	2	1	0	0	0	0	0	3	2	15
$x_p[3-k]$	3	2	1	0	0	0	0	0	3	25
$x_p[4-k]$	0	3	2	1	0	0	0	0	4	15
$x_p[5-k]$	0	0	3	2	1	0	0	0	5	0
$x_p[6-k]$	0	0	0	3	2	1	0	0	6	0
$x_p[7-k]$	0	0	0	3	2	1	0	3	7	0

Note that the non-zero values of the periodic convolution are the same as the ones obtained in Example 10.12. Since the period has been increased, additional zeros are added. ▌

Problem 10.13

The unit step response $s[k]$ of the DT system is the output of the system when a unit step $u[k]$ is applied at the input of the system. In other words,

$$s[k] = h[k] * u[k] = \sum_{p=-\infty}^{\infty} h[p]u[k-p]$$

$$= \sum_{p=-\infty}^{k} h[p] \qquad \left[\because\ u[k-p] = \begin{cases} 1 & p \le k \\ 0 & otherwise \end{cases}\right].$$

In other words, the unit step response is the running sum of the impulse response.

(a) $$s[k] = \sum_{p=-\infty}^{k} h[p] = \sum_{p=-\infty}^{k} \left(u[p+7] - u[p-8]\right) = \begin{cases} 0 & k < -7 \\ (k+8) & -7 \le k \le 7 \\ 15 & k > 7. \end{cases}$$

(b) $$s[k] = \sum_{p=-\infty}^{k} 0.4^p u[p] = \begin{cases} 0 & k < 0 \\ \sum_{p=0}^{k} 0.4^p & k \ge 0 \end{cases} = \begin{cases} 0 & k < 0 \\ \frac{5}{3}(1 - 0.4^{k+1}) & k \ge 0. \end{cases}$$

(c) $$s[k] = \sum_{p=-\infty}^{k} 2^p u[-p] = \begin{cases} \sum_{p=-\infty}^{k} 2^p & k < 0 \\ \sum_{p=-\infty}^{0} 2^p & k \ge 0 \end{cases} = \begin{cases} 2^{k+1} & k < 0 \\ 2 & k \ge 0. \end{cases}$$

(d) $$s[k] = \sum_{p=-\infty}^{k} 0.6^{|p|} = \begin{cases} \sum_{p=-\infty}^{k} 0.6^{-p} & k < 0 \\ \sum_{p=-\infty}^{0} 0.6^{-p} + \sum_{p=1}^{k} 0.6^{p} & k \ge 0 \end{cases} = \begin{cases} 2.5 \times 0.6^{-k} & k < 0 \\ 2.5 + 1.5 \times (1 - 0.6^k) & k \ge 0. \end{cases}$$

(e) $$h[k] = \sum_{m=-\infty}^{\infty} (-1)^m \delta(k-2m) = \begin{cases} 1 & k = 4n; \quad 0, \pm 4, \pm 8, ... \\ -1 & k = 4n+2; \quad \pm 2, \pm 6, ... \\ 0 & otherwise \end{cases}$$

The plot of $h[k]$ is shown in Fig. S10.13. It is observed that $h[k]$ is a zero-mean periodic signal with period 4. The running sum $s[k] = \sum_{p=-\infty}^{k} h[p]$ will depend on how $h[k]$ starts at $k = -\infty$. If $h[-\infty] = 1$, the sequence $s[k]$ will have the form $\left[\underset{\text{at } k=-\infty}{\underset{\uparrow}{1}}, 1, 0, 0, 1, 1, 0, 0, 1, 1, ... \right]$. On the other hand, if

$h[-\infty]=-1$, the sequence $s[k]$ will have the form $\left[\underbrace{-1}_{\text{at }k=-\infty},-1,0,0,-1,-1,0,0,-1,-1,\ldots\ldots\right]$. As we do not know if $\infty=4n\ or\ (4n+1)\ or\ (4n+2)\ or\ (4n+3)$, we cannot calculate $s[k]$ exactly. The only observation we can make with certainty is that $s[k]$ will have three possible values $-1,0,1$. ▌

Problem 10.14

(a) $(x[k]+2\delta[k-1]) * \delta[k-2] = x[k] * \delta[k-2] + 2\delta[k-1] * \delta[k-2]$ [using distributive property]

$$= x[k-2] + 2\delta[k-3].$$

(b) Using the distributive property

$$(x[k]+2\delta[k-1]) * (\delta[k+1]+\delta[k-2])$$
$$= x[k] * \delta[k+1] + 2\delta[k-1] * \delta[k+1] + x[k] * \delta[k-2] + 2\delta[k-1] * \delta[k-2]$$
$$= x[k+1] + 2\delta[k] + x[k-2] + 2\delta[k-3]$$
$$= x[k+1] + x[k-2] + 2\delta[k] + 2\delta[k-3].$$

(c) $(x[k]-u[k-1]) * \delta[k-2] = x[k] * \delta[k-2] - u[k-1] * \delta[k-2]$ [using distributive property]

$$= x[k-2] - u[k-3].$$

(d) $(x[k]-x[k-1]) * u[k] = (x[k]-x[k-1]) * \sum_{m=0}^{\infty}\delta[k-m]$

$$=\sum_{m=0}^{\infty}x[k-m]-\sum_{m=0}^{\infty}x[k-1-m]$$ [using distributive property]

$$=x[k]$$ ▌

Problem 10.15

The result is straightforward to prove because zero padding ensures that there is no overlap between two replicas of the linear convolution of $x[k]$ and $h[k]$.

Problem 10.16

Using the Eq. (10.25) (convolution with impulse function property), the left hand side of Eq. (10.24) can be expressed as

$$x_1[k-k_1] * x_2[k-k_2] = (x_1[k] * \delta[k-k_1]) * (x_2[k] * \delta[k-k_2])$$

Using the commutative and associative property, the above expression can be expressed as

$$x_1[k-k_1] * x_2[k-k_2] = \underbrace{(x_1[k] * x_2[k])}_{g[k]} * \underbrace{(\delta[k-k_1]) * \delta[k-k_2])}_{\delta[k-k_1-k_2]}$$
$$= g[k] * \delta[k-k_1-k_2]$$
$$= g[k-k_1-k_2].$$ ▌

Problem 10.17

Note that the finite duration DT sequences $x_1[k]$ and $x_2[k]$ can be expressed as

$$x_1[k] = x_1[k_{\ell 1}]\delta[k-k_{\ell 1}] + x_1[k_{\ell 1}+1]\delta[k-k_{\ell 1}-1] + \cdots + x_1[k_{u1}]\delta[k-k_{u1}]$$

and
$$x_2[k] = x_2[k_{\ell 2}]\delta[k-k_{\ell 2}] + x_2[k_{\ell 2}+1]\delta[k-k_{\ell 2}-1] + \cdots + x_2[k_{u2}]\delta[k-k_{u2}].$$

When $x_1[k]$ is convolved by $x_2[k]$, the earliest sample will be the result of the convolution of the term $x_1[k_{\ell 1}]\delta[k-k_{\ell 1}]$ with $x_2[k_{\ell 2}]\delta[k-k_{\ell 2}]$, while the latest sample will be the result of the convolution of the term $x_1[k_{u1}]\delta[k-k_{u1}]$ with $x_2[k_{u2}]\delta[k-k_{u2}]$. In other words,

$$\text{Earliest term} = x_1[k_{\ell 1}]\delta[k-k_{\ell 1}] * x_2[k_{\ell 2}]\delta[k-k_{\ell 2}] = x_1[k_{\ell 1}]x_2[k_{\ell 2}]\delta[k-k_{\ell 1}-k_{\ell 2}],$$

and
$$\text{Latest term} = x_1[k_{u1}]\delta[k-k_{u1}] * x_2[k_{u2}]\delta[k-k_{u2}] = x_1[k_{u1}]x_2[k_{u2}]\delta[k-k_{u1}-k_{u2}],$$

which proves that the linear convolution $y[k]$ is time-limited within the range ($k_{\ell 1} + k_{\ell 2} \le k \le k_{u1} + k_{u2}$). ▌

Problem 10.18

(a) $$h[k] = u[k+7] - u[k-8] = \begin{cases} 1 & -7 \le k \le 7 \\ 0 & \text{otherwise.} \end{cases}$$

We observe that:

(i) $h[k] \ne 0$ for all $k \ne 0$. Therefore, the system is NOT memoryless.

(ii) $h[k] \ne 0$ for $k < 0$, e.g., $h[-1] = 1$. Therefore, the system is NOT causal.

(iii) $\Sigma|h[k]| = 15 < \infty$. Therefore, the system is bounded-input-bounded-output (BIBO) stable.

(b) $$h[k] = \sin(\pi k/8)u[k] = \begin{cases} \sin(\pi k/8) & k \ge 0 \\ 0 & k < 0. \end{cases}$$

We observe that:

(i) $h[k] \ne 0$ for all $k \ne 0$. Therefore, the system is NOT memoryless.

(ii) $h[k] = 0$ for $k < 0$. Therefore, the system is causal.

(iii) $\Sigma|h[k]| = \infty$. Therefore, the system is NOT stable.

(c) $$h[k] = 6^k u[-k] = \begin{cases} 6^k & k \le 0 \\ 0 & k > 0. \end{cases}$$

We observe that:

(i) $h[k] \ne 0$ for all $k \ne 0$. Therefore, the system is NOT memoryless.

(ii) $h[k] \ne 0$ for $k < 0$, e.g., $h[-2] = 1/36$. Therefore, the system is NOT causal.

(iii) $\Sigma|h[k]| = 1/(1 - 1/6) = 1.2 < \infty$. Therefore, the system is stable.

(d) $h[k] = 0.9^{|k|} = \begin{cases} 0.9^{k} & k \ge 0 \\ 0.9^{-k} & k < 0. \end{cases}$

we observe that:

(i) $h[k] \neq 0$ for all $k \neq 0$. Therefore, the system is NOT memoryless.

(ii) $h[k] \neq 0$ for $k < 0$, e.g., $h[-2] = 0.81$. Therefore, the system is NOT causal.

(iii) $\Sigma|h[k]| = 21 < \infty$. Therefore, the system is BIBO stable.

(e) $h[k] = \sum_{m=-\infty}^{\infty} (-1)^m \delta(k-2m) = \begin{cases} 1 & k = 4m \\ -1 & k = 4m+2 \\ 0 & k = odd \end{cases}$

we observe that:

(i) $h[k] \neq 0$ for all $k \neq 0$. Therefore, the system is NOT memoryless.

(ii) $h[k] \neq 0$ for $k < 0$, e.g., $h[-2] = -1$. Therefore, the system is NOT causal.

(iii) Since $h[k]$ includes an infinite number of impulse functions, $\Sigma|h[k]| = \infty$.Therefore, the system is NOT stable.

Problem 10.19

(a) $h_1[k] * h_2[k] = u[-k-1] * (\delta[k-1] - \delta[k]) = u[-k-1] * \delta[k-1] - u[-k-1] * \delta[k]$

$$= u[-k] - u[-k-1] = \delta[k]$$

Therefore, the pair corresponds to inverse systems.

(b) $h_1[k] * h_2[k] = 0.5^k u[k] * (\delta[k] - 0.5\delta[k-1]) = 0.5^k u[k] * \delta[k] - 0.5 \times 0.5^k u[k] * \delta[k-1]$

$$= 0.5^k u[k] - 0.5^k u[k-1] = \delta[k]$$

Therefore, the pair corresponds to inverse systems.

(c) $h_1[k] * h_2[k] = 0.8^k ku[k] * (0.8\delta[k-1] - 2\delta[k] + 1.25\delta[k+1])$

$$\begin{aligned} &= 0.8^k ku[k] * 0.8\delta[k-1] - 2 \times 0.8^k ku[k] * \delta[k] + 1.25 \times 0.8^k ku[k] * \delta[k+1] \\ &= 0.8^k (k-1)u[k-1] - 2 \times 0.8^k ku[k] + 1.25 \times 0.8^{k+1} (k+1)u[k+1] \\ &= 0.8^k u[k-1] \times ((k-1) - 2k + (k+1)) + 1.25 \times 0.8^1 \delta[k] \\ &= \delta[k]. \end{aligned}$$

Therefore, the pair corresponds to inverse systems.

(d) $h_1[k] * h_2[k] = ku[k] * (\delta[k+1] - 2\delta[k] + \delta[k-1])$

$$
\begin{aligned}
&= ku[k]*\delta[k+1] - 2ku[k]*\delta[k] + ku[k]*\delta[k-1] \\
&= (k+1)u[k+1] - 2ku[k] + (k-1)u[k-1] \\
&= \delta[k] + u[k-1]\times((k+1) - 2k + (k-1)) \\
&= \delta[k].
\end{aligned}
$$

Therefore, the pair corresponds to inverse systems.

(e) $h_1[k]*h_2[k] = (k+1)0.8^k u[k]*(\delta[k] - 1.6\delta[k-1] + 0.64\delta[k-2])$

$$
\begin{aligned}
&= (k+1)0.8^k u[k]*\delta[k] - 1.6(k+1)0.8^k u[k]*\delta[k-1] + 0.64(k+1)0.8^k u[k]*\delta[k-2]) \\
&= (k+1)0.8^k u[k] - 1.6k\times 0.8^{k-1}u[k-1] + 0.64(k-1)0.8^{k-2}u[k-2] \\
&= [\underbrace{(k+1) - 2k + (k-1)}_{=0}]0.8^k u[k-2] + \delta[k] + \underbrace{2\times 0.8\delta[k-1] - 1.6\delta[k-1]}_{=0} \\
&= \delta[k]
\end{aligned}
$$

Therefore, the pair corresponds to inverse systems.

Problem 10.20

The MATLAB code for computing the first 50 samples of the output response y[k] in Problems 10.1 – 10.3 is given below:

```
>> % Problem 10.1: y[k] - 2y[k-1] = x[k-1]
>> k1 = [0:49];                          % time index k = [-1, 0, 1, … 49]
>> x1 = 2*(k1 >= 0);                     % Input Signal x[k] = 2*u[k]
>> A1 = [1 -2];                          % Coefficients for y[k]
>> B1 = [0 1];                           % Coefficients for x[k]
>> zi1 = filtic(B1,A1,[2]);              % Initial Conditions
>> y1 = filter(B1,A1,x1,zi1);            % Calculate output
>> subplot(3,1,1); stem(k1,y1,'fill'); title('Problem 10.1'); xlabel('k');
>> axis tight; grid on
>>
>> % Problem 10.2: y[k] - y[k-1] + 0.5y[k-2] = x[k-2]
>> k2 = [0:49];                          % time index k = [-1, 0, 1, … 49]
>> x2 = (0.5.^k2).*(k2 >= 0);            % Input Signal x[k] = 0.5^k u[k]
>> A2 = [1 -1 0.5];                      % Coefficients for y[k]
>> B2 = [0 0 1];                         % Coefficients for x[k]
>> zi2 = filtic(B2,A2,[0 1]);            % Initial Conditions
>> y2 = filter(B2,A2,x2,zi2);            % Calculate output
>> subplot(3,1,2); stem(k2,y2,'fill'); title('Problem 10.2'); xlabel('k');
>> axis tight; grid on
>>
>> % Problem 10.3: y[k] - 0.75y[k-1] + 0.125y[k-2] = x[k-2]
>> k3 = [0:49];                          % time index k = [-1, 0, 1, … 49]
>> x3 = ((-1).^k3).*(k3 >= 0);           % Input Signal x[k] = (-1)^k u[k]
>> A3 = [1 -0.75 0.125];                 % Coefficients for y[k]
>> B3 = [0 0 1];                         % Coefficients for x[k]
>> zi3 = filtic(B3,A3,[1 -1]);           % Initial Conditions
>> y3 = filter(B3,A3,x3,zi3);            % Calculate output
>> subplot(3,1,3); stem(k3,y3,'fill'); title('Problem 10.3'); xlabel('k');
>> axis tight; grid on
```

The stem plots for the three outputs are shown in Fig. S10.20.

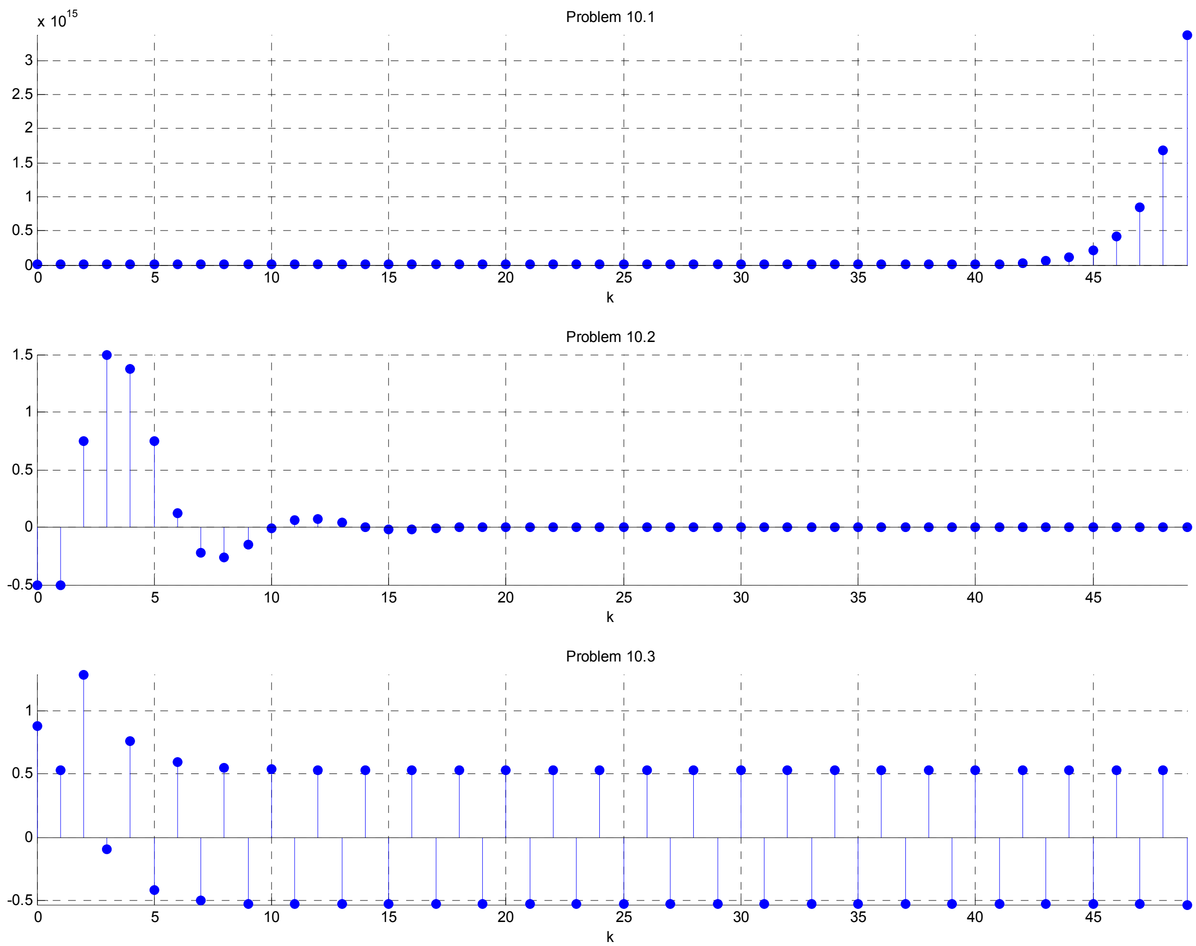


Fig. S10.20: Stem plots for the outputs of the DT systems in Problem 10.20.

Problem 10.21

The MATLAB code for computing the convolution sums in Problem 10.5 is given below:

```
>> % Problem 10.5(a)
>> kx1 = [-2:2];                           % time indices where x1 is nonzero
>> x1 = 1*ones(length(kx1),1);             % Sample values for DT sequence x1
>> kx2 = [-4:4];                           % time indices where x2 is nonzero
>> x2 = 1*ones(length(kx2),1);             % Sample values for DT sequence x2
>> y1 = conv(x1,x2);                       % Convolve x1 with x2
>> ky1 = kx1(1)+kx2(1):kx1(length(kx1))+kx2(length(kx2));% ky1=indices for y1
>> subplot(5,1,1); stem(ky1,y1,'fill'); title('Problem 10.5(a)');
>> xlabel('k'); axis tight; grid on
>> % Problem 10.5(b)
>> kx1 = [0:ceil(log(0.001)/log(0.5))]; % time indices where x1 is nonzero
>> x1 = 0.5.^kx1.*(kx1>=0);               % Sample values for DT sequence x1
>> kx2 = [0:ceil(log(0.001)/log(0.8))]; % time indices where x2 is nonzero
>> x2 = 0.8.^kx2.*(kx2>=0);               % Sample values for DT sequence x2
>> y2 = conv(x1,x2);                       % Convolve x1 with x2
>> ky2 = kx1(1)+kx2(1):kx1(length(kx1))+kx2(length(kx2));% ky2=indices for y2
>> subplot(5,1,2); stem(ky2,y2,'fill'); title('Problem 10.5(b)');
>> xlabel('k'); axis tight; grid on
```

```
>> % Problem 10.5(c)
>> kx1 = [floor(log(0.049)/log(7)):2];  % time indices where x1 is nonzero
>> x1 = 7.^kx1.*(kx1<=2);               % Sample values for DT sequence x1
>> kx2 = [4:ceil(log(0.001*0.4^4)/log(0.4))];
                                        % time indices where x2 is nonzero
>> x2 = 0.4.^kx2.*(kx2>=4);             % Sample values for DT sequence x2
>> y3 = conv(x1,x2);                    % Convolve x1 with x2
>> ky3 = kx1(1)+kx2(1):kx1(length(kx1))+kx2(length(kx2));% ky3=indices for y3
>> subplot(5,1,3); stem(ky3,y3,'fill'); title('Problem 10.5(c)');
>> xlabel('k'); axis tight; grid on
>> % Problem 10.5(d)
>> kx1 = [0:ceil(log(0.001)/log(0.6))]; % time indices where x1 is nonzero
>> x1 = 0.6.^kx1.*(kx1>=0);             % Sample values for DT sequence x1
>> kx2 = [-30:0];                       % time indices where x2 is nonzero
>> x2 = sin(pi*kx2/2).*(kx2<=0);        % Sample values for DT sequence x2
>> y4 = conv(x1,x2);                    % Convolve x1 with x2
>> ky4 = kx1(1)+kx2(1):kx1(length(kx1))+kx2(length(kx2));% ky4=indices for y4
>> subplot(5,1,4); stem(ky4,y4,'fill'); title('Problem 10.5(d)');
>> xlabel('k'); axis tight; grid on
>> % Problem 10.5(e)
>> kx1 = [-floor(log(0.001)/log(0.5)):ceil(log(0.001)/log(0.5))];
                                        % time indices where x1 is nonzero
>> x1 = 0.5.^abs(kx1);                  % Sample values for DT sequence x1
>> kx2 = [-floor(log(0.001)/log(0.8)):ceil(log(0.001)/log(0.8))];
                                        % time indices where x2 is nonzero
>> x2 = 0.8.^abs(kx2);                  % Sample values for DT sequence x2
>> y5 = conv(x1,x2);                    % Convolve x1 with x2
>> ky5 = kx1(1)+kx2(1):kx1(length(kx1))+kx2(length(kx2));% ky5=indices for y5
>> subplot(5,1,5); stem(ky5,y5,'fill'); title('Problem 10.5(e)');
>> xlabel('k'); axis tight; grid on
```

The stem plots for the five convolution sums are shown in Fig. S10.21.

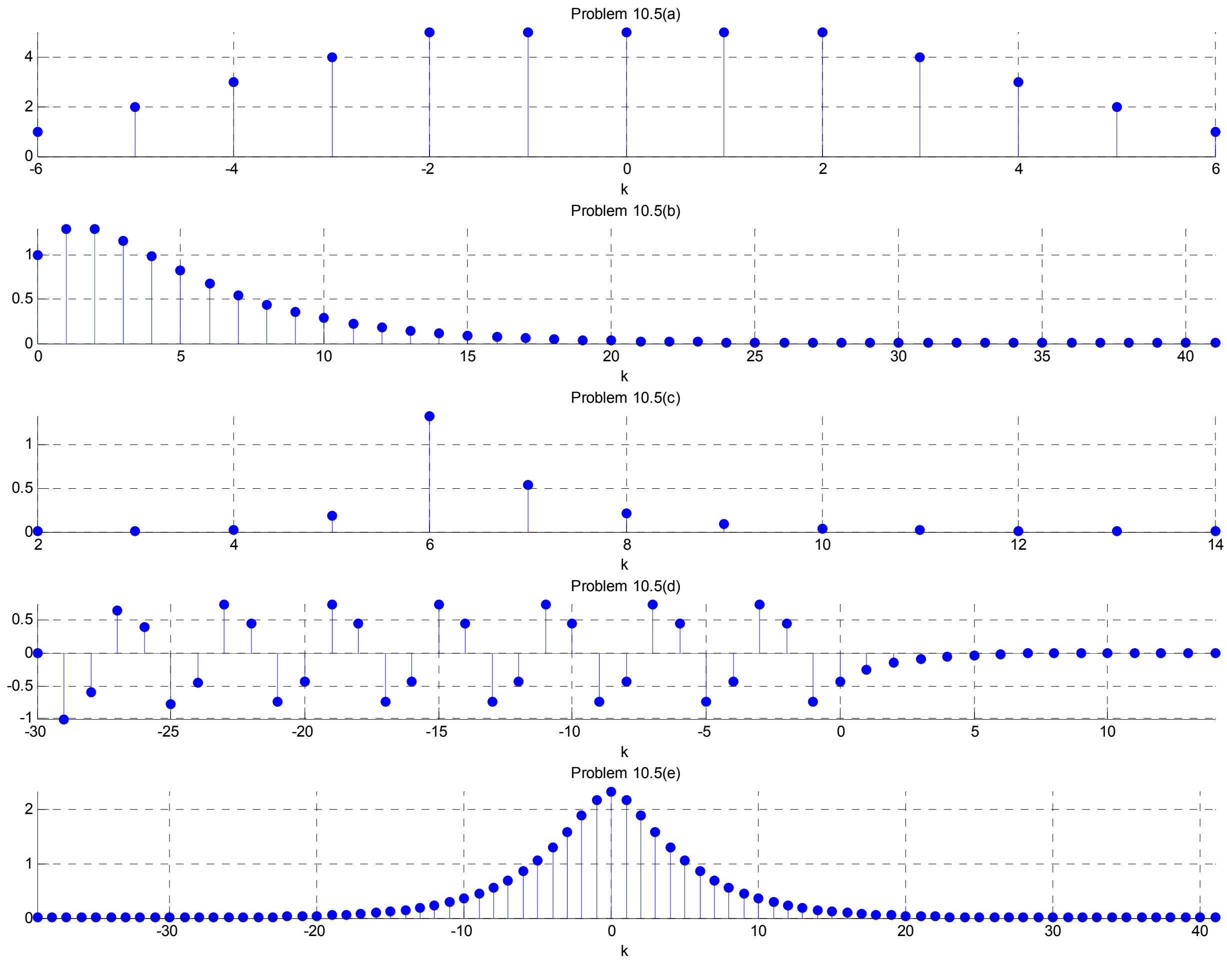


Fig. S10.21: Stem plots for the results of convolution sums in Problem 10.21.

Problem 10.22

The MATLAB code for determining the impulse responses of the systems specified in Problem 10.1 – 10.3 is as follows:

```
>> % Problem 10.1: y[k] - 2y[k-1] = x[k-1]
>> A1 = [1 -2];       % Coefficients for y[k]
>> B1 = [0 1]; % Coefficients for x[k]
>> [h1,k1] = impz(B1,A1,50);                    % Calculate impulse response
>> subplot(3,1,1); stem(k1,h1,'fill');
>> title('Problem 10.1'); xlabel('k');
>> axis tight; grid on
>> % Problem 10.2: y[k] - y[k-1] + 0.5y[k-2] = x[k-2]
>> A2 = [1 -1 0.5];  % Coefficients for y[k]
>> B2 = [0 0 1];      % Coefficients for x[k]
>> [h2,k2] = impz(B2,A2,50);                    % Calculate impulse response
>> subplot(3,1,2); stem(k2,h2,'fill');
>> title('Problem 10.2'); xlabel('k');
>> axis tight; grid on
>> % Problem 10.3: y[k] - 0.75y[k-1] + 0.125y[k-2] = x[k-2]
>> A3 = [1 -0.75 0.125];   % Coefficients for y[k]
>> B3 = [0 0 1];      % Coefficients for x[k]
```

```
>> [h3,k3] = impz(B3,A3,50);                 % Calculate impulse response
>> subplot(3,1,3); stem(k3,h3,'fill');
>> title('Problem 10.3'); xlabel('k');
>> axis tight; grid on
```

The impulse responses of the DT systems are shown in Fig. S10.22.

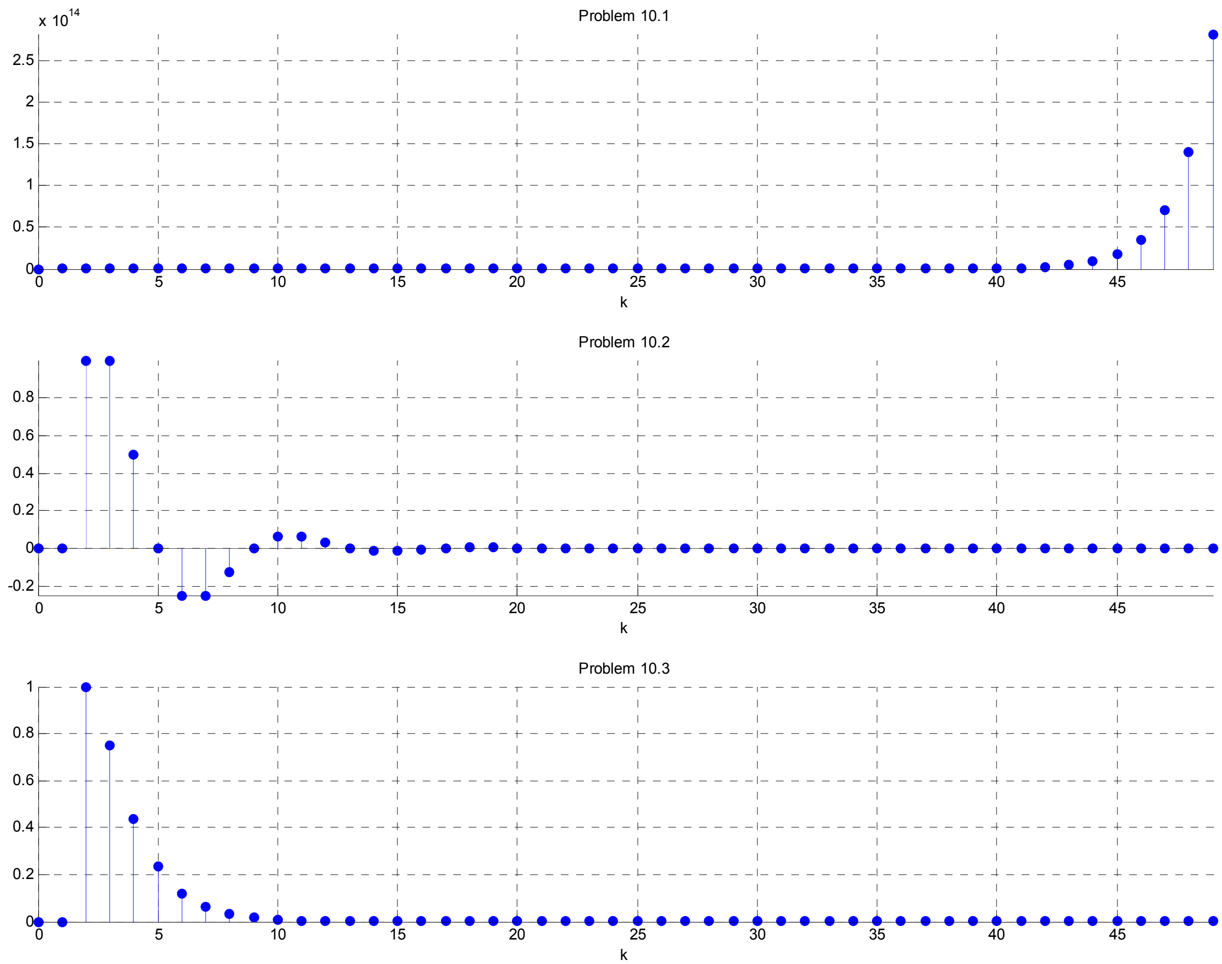


Fig. S10.22: Impulse responses for systems defined using difference equations in Problem 10.22.