Chapter 15: FIR Filter Design

Problem 15.1

$$H_{diff}(\Omega) = j\Omega e^{-jm\Omega} \quad 0 \le |\Omega| \le \pi$$

$$h_{diff}[k] = \Im\left\{H_{diff}(\Omega)\right\} = \tfrac{1}{2\pi}\int_{-\pi}^{\pi} j\Omega e^{-jm\Omega}e^{jk\Omega}d\Omega = \tfrac{j}{2\pi}\int_{-\pi}^{\pi}\Omega e^{j(k-m)\Omega}d\Omega$$

$$= \begin{cases} 0 & k = m \\ \tfrac{j}{2\pi}\left[\dfrac{e^{j(k-m)\Omega}}{-(k-m)^2}\left\{j(k-m)\Omega - 1\right\}\right]_{-\pi}^{\pi} & k \ne m \end{cases}$$

When $k \ne m$, $h_{diff}[k]$ is given by

$$h_{diff}[k] = \tfrac{j}{2\pi}\left[\dfrac{e^{j(k-m)\Omega}}{-(k-m)^2}\left\{j(k-m)\Omega - 1\right\}\right]_{-\pi}^{\pi} = -\tfrac{j}{2\pi(k-m)^2}\left[e^{j(k-m)\Omega}\left\{j(k-m)\Omega - 1\right\}\right]_{-\pi}^{\pi}$$

$$= -\tfrac{j}{2\pi(k-m)^2}\left[e^{j(k-m)\pi}\left\{j(k-m)\pi - 1\right\} - e^{-j(k-m)\pi}\left\{-j(k-m)\pi - 1\right\}\right]$$

$$= -\tfrac{j}{2\pi(k-m)^2}\left[e^{j(k-m)\pi}\left\{j(k-m)\pi - 1\right\} + e^{-j(k-m)\pi}\left\{j(k-m)\pi + 1\right\}\right]$$

$$= -\tfrac{j}{2\pi(k-m)^2}\left[j(k-m)\pi \times 2\cos((k-m)\pi) - 2j\underbrace{\sin((k-m)\pi)}_{=0}\right]$$

$$= \tfrac{1}{2\pi(k-m)^2}\left[(k-m)\pi \times 2\cos((k-m)\pi)\right]$$

$$= \tfrac{\cos((k-m)\pi)}{k-m} = \tfrac{(-1)^{k-m}}{k-m}.$$

Combining the above results, we obtain, $h_{diff}[k] = \begin{cases} 0 & k = m \\ \frac{(-1)^{k-m}}{k-m} & k \ne m \end{cases}$

For the special case of m=0 (zero delay), $h_{diff}[k] = \begin{cases} 0 & k = 0 \\ \frac{(-1)^{k}}{k} & k \ne 0. \end{cases}$ ∎

Problem 15.2

(i) The normalized cut-off frequency is given by $\Omega_n = \dfrac{2}{8/2} = 0.5$.

The ideal lowpass filter for the above normalized cut-off frequency is given by (see Table 14.1 in the text)

$$h_{ilp}[k] = 0.5\sin c\left(0.5k\right) = \tfrac{\sin(0.5\pi k)}{\pi k}.$$

(ii) The normalized cut-off frequency is given by $\Omega_n = \dfrac{2}{16/2} = 0.25$.

The ideal lowpass filter for the above normalized cut-off frequency is given by

$$h_{ilp}[k] = 0.25 \operatorname{sinc}(0.25k) = \frac{\sin(0.25\pi k)}{\pi k}.$$

(iii) The normalized cut-off frequency is given by $\Omega_n = \dfrac{2}{44.1/2} = 0.0907$.

The ideal lowpass filter for the above normalized cut-off frequency is given by

$$h_{ilp}[k] = 0.0907 \operatorname{sinc}(0.0907k) = \frac{\sin(0.0907\pi k)}{\pi k}.$$

Problem 15.3

The amplitude of the 5-tap ($N = 21$) rectangular, Hanning, Hamming, and Blackman windows are listed in the following table, and plotted in Fig. S15.3.

Table: Amplitude of the 5-tap (N = 5) Rectangular, Hanning, Hamming, and Blackman windows.

Window	Time Index (k)				
	0	1	2	3	4
Rectangular	1	1	1	1	1
Hanning	0	0.5	1	0.5	0
Hamming	0.08	0.54	1	0.54	0.08
Blackman	0	0.34	1	0.34	0

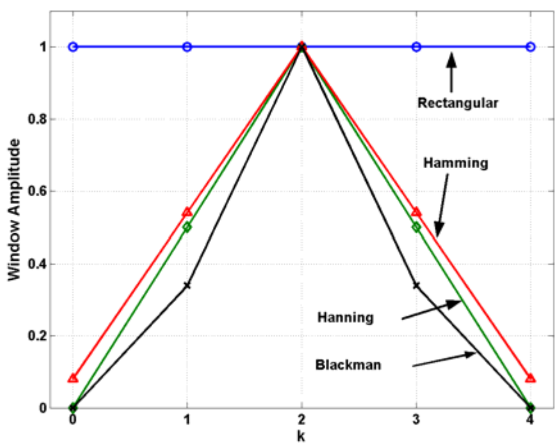


Fig. S15.3: 5-tap (N = 5) rectangular, Hanning, Hamming, and Blackman windows. The markers (x,o) corresponds to the actual amplitude of the discrete windows.

Program 15.3: MATLAB code to generate the amplitude of 5-tap (N = 5) Rectangular, Hanning, Hamming, and Blackman windows.

```
N=5;
k=0:N-1;
hannw  = 0.5-0.5*cos(2*pi*k/(N-1))
hammw = hamming(N)
blacw = blackman(N)

rectw = ones(1,5);
plot(k,rectw,k,hannw,k,hammw,k,blacw);
axis([-1  5  0  1.1])
xlabel('k')
ylabel('Window Amplitude');
print -dtiff plot.tiff
```

Problem 15.4

The minimum stopband attenuation is 35 dB. From Table 15.2 in the text, it is observed that Hanning, Hamming and Blackman windows will satisfy the stopband attenuation requirement.

The normalized transition bandwidth, $\Delta\Omega_n = \dfrac{\Delta\Omega_c}{\pi} = \dfrac{\Omega_s - \Omega_p}{\pi} = \dfrac{0.3\pi}{\pi} = 0.3$

Using Table 15.2, the length corresponding to various windows is given by

Hanning: $N \geq \dfrac{6.2}{\Delta\Omega_n} = \dfrac{6.2}{0.3} = 20.66$, or $N = 21$

Hamming: $N \geq \dfrac{6.6}{\Delta\Omega_n} = \dfrac{6.6}{0.3} = 22$. $N = 22$. If an odd-length filter is desired, N=23.

Blackman: $N \geq \dfrac{11}{\Delta\Omega_n} = \dfrac{11}{0.3} = 36.66$, or, $N = 37$. ∎

Problem 15.5

As the minimum stopband attenuation is 35 dB, Eq. (15.20) in the text yields,

$$\beta = 0.5842(35-21)^{0.4} + 0.0789(35-21) = 1.6789 + 1.1046 \approx 2.783.$$

It was shown in the solution of Problem 15.4 that $\Delta\Omega_n = 0.3$. Therefore, the length of the Kaiser window is obtained from Eq. (15.21) as follows:

$$N \geq \frac{A - 7.95}{2.285\pi \times \Delta\Omega_n} = \frac{35 - 7.95}{2.285\pi \times 0.3} = 12.56$$

which is rounded off to the closest higher odd number as 13. ∎

Problem 15.6

The normalized cut-off frequency, $\Omega_n = \dfrac{\Omega_c}{\pi} = \dfrac{1}{\pi}$. Therefore, the impulse response of the DT filter is given by

$$h_{ilp}[k] = \tfrac{1}{\pi}\sin c\left(\tfrac{k}{\pi}\right)$$

The rectangular window with 51 taps is given by

$$w_R[k] = \begin{cases} 1 & 0 \le k \le 50 \\ 0 & otherwise \end{cases}$$

Right-shifting the ideal lowpass filter impulse response by 25 time units, and multiplying with the rectangular window, the designed FIR filter impulse response is obtained as:

$$h'_{rect}[k] = h_{ilp}[k]w_R[k] = \begin{cases} \tfrac{1}{\pi}\sin c(\tfrac{k-25}{\pi}) & 0 \le k \le 50 \\ 0 & otherwise. \end{cases}$$

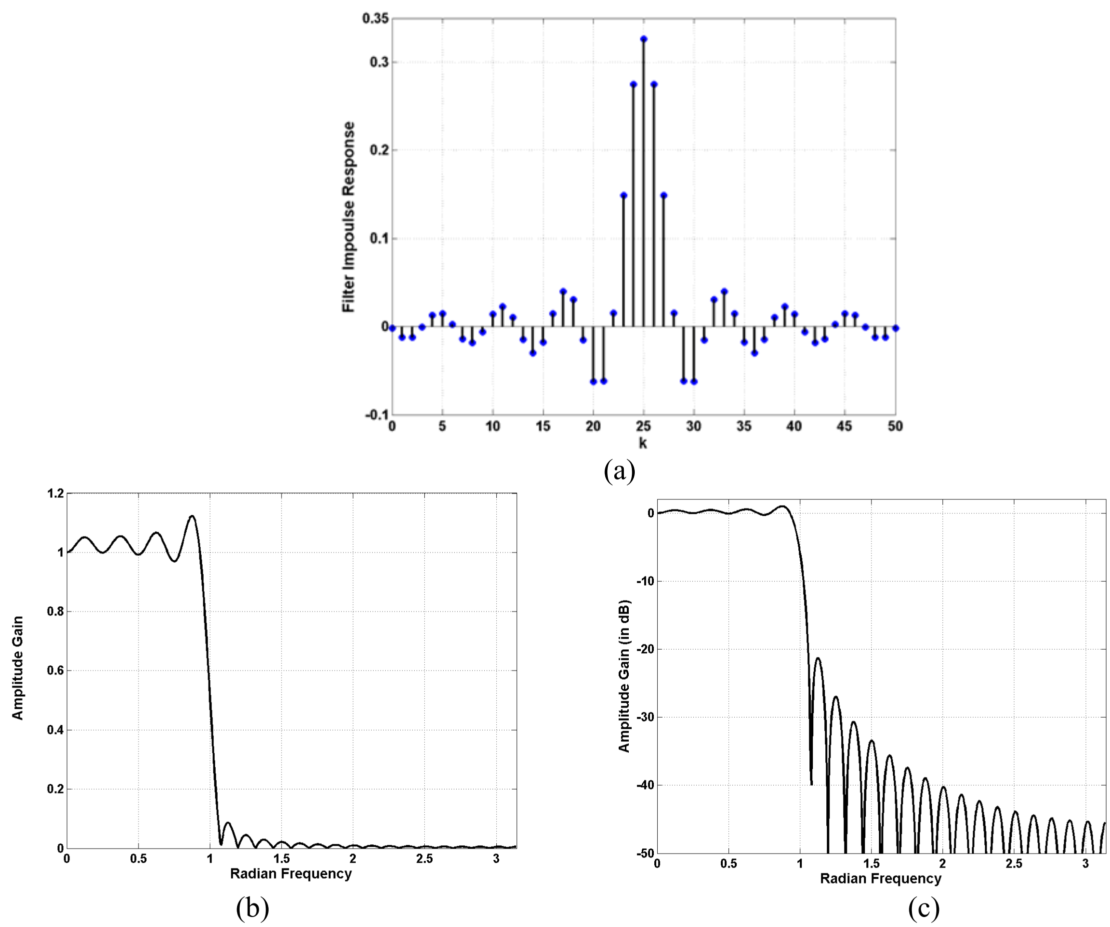


Figure S15.6: Filter design using windows in Problem 15.6. (a) 51 tap filter obtained using the rectangular window, (b) amplitude gain (in absolute scale) of the filter, and (c) amplitude gain (in dB scale) of the filter.

In order to make the DC gain unity, the filter impulse response is divided by $\sum h'_{rect}[k] = 0.9754$:

$$h_{rect}[k] = \frac{h'_{rect}[k]}{0.9754}.$$

The impulse response $h_{rect}[k]$ and the frequency characteristics of the filter are shown in Fig. S15.6. ▌

Program 15.6: MATLAB Program for calculating and plotting the Filter responses

clear; clf
N=51 ; ;   % Number of filter taps
M=(N-1)/2 ;
k = 0:N-1;
 	
filter_ideal = (1/pi)*sinc((k-M)/pi) ;
window_rect = ones(1,N) ;
filter_rect = filter_ideal.*window_rect ;
S=sum(filter_rect)
filter_rect = filter_rect/S;
% the filter impulse response is scaled so that the
% the DC gain is one.
 	
% Plotting the filter impulse response
stem(k, filter_rect, 'filled'),grid
ylabel('Filter Impulse Response');
xlabel('k')
print -dtiff plot.tiff
 	
% Calculating the freq. response
[H, w] = freqz(filter_rect,1) ;
 	
%Plot in absolute scale
plot(w, abs(H)), grid
axis([0 pi 0 1.1]);
xlabel('Frequency (rad/s)')
ylabel('Amplitude Gain');
print -dtiff plot.tiff
 	
%Plot in dB scale
Hr = 20*log10(abs(H)+eps) ;
plot(w, Hr), grid
axis([0   pi   -50   2]);
xlabel('Frequency (rad/s)')
ylabel('Amplitude Gain (in dB)');
print -dtiff plot.tiff
	clear; clf
	N=51 ; ; % Number of filter taps
	M=(N-1)/2 ;
	k = 0:N-1;
	
	filter_ideal = (1/pi)*sinc((k-M)/pi) ;
	window_hamming = 0.54-0.46*cos(2*pi*k/(N-1)) ;
	filter_hamming = filter_ideal.*window_hamming ;
	S=sum(filter_hamming)
	filter_hamming = filter_hamming/S;
	% the filter impulse response is scaled so that the
	% DC gain is one.
	
	% Plotting the filter impulse response
	stem(k, filter_hamming, 'filled'),grid
	ylabel('Filter Impulse Response');
	xlabel('k')
	print -dtiff plot.tiff
	
	% Calculating the freq. response
	[H, w] = freqz(filter_hamming,1) ;
	
	%Plot in absolute scale
	plot(w, abs(H)), grid
	axis([0 pi 0 1.1]);
	xlabel('Frequency (rad/s)')
	ylabel('Amplitude Gain');
	print -dtiff plot.tiff
	
	%Plot in dB scale
	H = 20*log10(abs(H)+eps) ;
	plot(w, H), grid
	axis([0 pi -80 5]);
	xlabel('Frequency (rad/s)')
	ylabel('Amplitude Gain (in dB)');
	print -dtiff plot.tiff
	
	%Plot in dB scale
	plot(w, H,w,Hr), grid
	axis([0 pi -80 5]);
	xlabel('Frequency (rad/s)')
	ylabel('Amplitude Gain (in dB)');
	print -dtiff plot.tiff

Problem 15.7

The normalized cut-off frequency, $\Omega_n = \dfrac{\Omega_c}{\pi} = \dfrac{1}{\pi}$. Therefore, the impulse response of the DT filter is given by

$$h_{ilp}[k] = \tfrac{1}{\pi}\sin c\left(\tfrac{k}{\pi}\right) = \tfrac{\sin(k)}{k\pi}$$

The Hamming window with 51 taps are given by

$$w_H[k] = \begin{cases} 0.54 - 0.46\cos\left(\frac{\pi k}{25}\right) & 0 \le k \le 50 \\ 0 & otherwise \end{cases}$$

Right-shifting the ideal lowpass filter impulse response by 25 time units, and multiplying with the Hamming window, the designed FIR filter impulse response is obtained as:

$$h'_{Hamm}[k] = h_{ilp}[k] w_H[k] = \begin{cases} \frac{1}{\pi}\left[0.54 - 0.46\cos\left(\frac{\pi k}{25}\right)\right]\mathrm{sinc}(\frac{k-25}{\pi}) & 0 \le k \le 50 \\ 0 & otherwise. \end{cases}$$

In order to make the DC gain unity, the filter impulse response is divided by $\sum h'_{Hamm}[k] = 0.9982$:

$$h_{rect}[k] = \frac{h'_{rect}[k]}{0.9982}.$$

The impulse response $h_{Hamm}[k]$ is shown in Fig. S15.7(a). The frequency characteristic of the filter is shown in Fig. S15.7(b) and (c). Fig. S15.7(d) compares the frequency characteristics of the designed filter with that of the filter obtained in Problem 15.6.

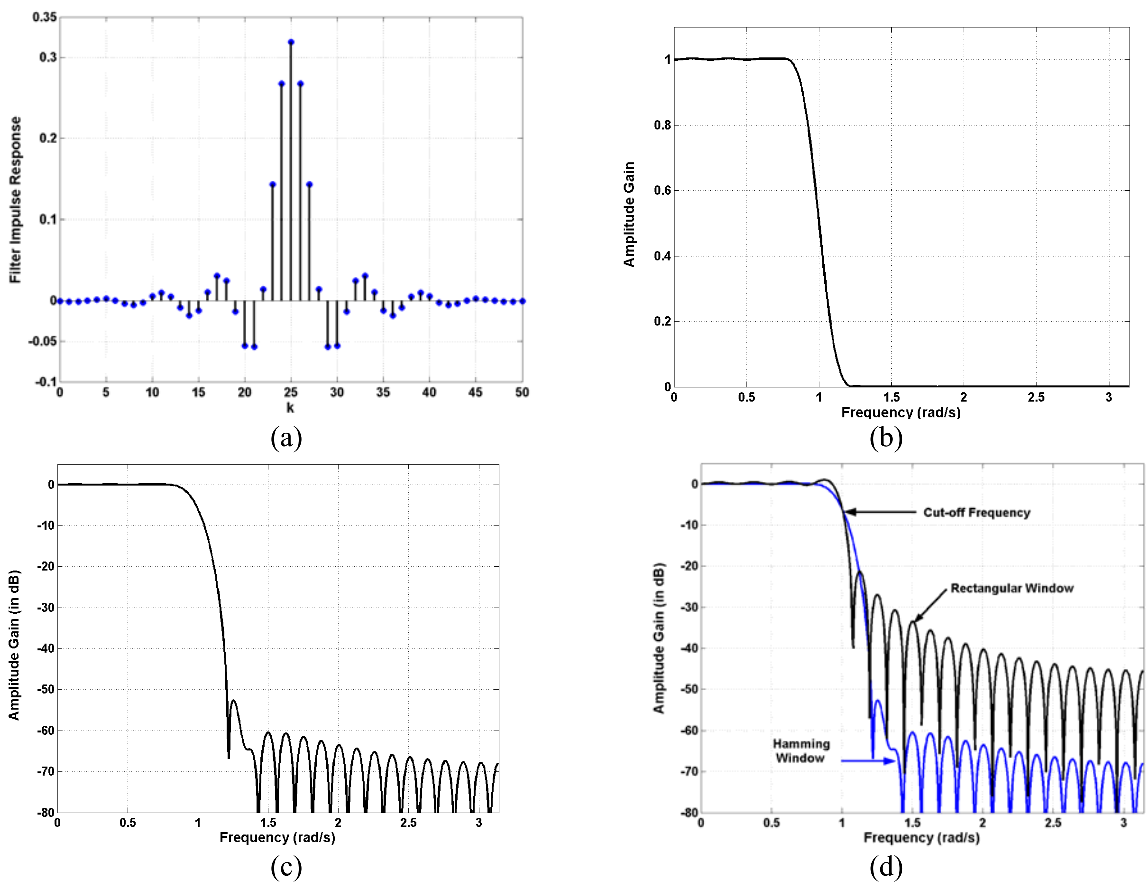


Figure S15.7: Filter design using Hamming window in Problem 15.7. (a) Impulse response of the 51-tap FIR filter, (b) the amplitude gain characteristics of the filter in absolute scale, (c) the amplitude gain characteristics of the filter in dB scale and (d) comparison of the amplitude gain characteristic with the filter obtained in Problem 15.6 (using Rectangular window).

Problem 15.8

(a) As the minimum stopband attenuation is 45 dB, several windows such as Hamming, Hanning, and Blackman will satisfy the specification.

(b) The cut-off frequency of the filter is calculated to be

f_c = passband edge frequency + 0.5*transition bandwidth = 10.025 KHz + 0.5 KHz = 10.525 KHz

The normalized cut-off frequency is given by

$$\Omega_n = \frac{10.525}{44.1/2} = 0.4773 \cdot$$

The ideal lowpass filter for the above normalized cut-off frequency is given by

$$h_l[k] = \frac{\sin(0.4773\pi k)}{\pi k} = 0.4773 \sin c\left(0.4773k\right).$$

The normalized transition bandwidth, $\Delta\Omega_n = \frac{1\ KHz}{22.05\ KHz} = 0.0454$.

From Table 14.3, we know that for Hamming window, $\Delta\Omega_n = \frac{6.6}{N}$.

Therefore, $N \geq \frac{6.6}{\Delta\Omega_n} = \frac{6.6}{0.0454} = 145.4$. We can choose, *N*=146 (even length) or 147 (odd length). Note that the 146 tap filter will have a fractional delay (72.5 units) and the 147 tap filter will have an integer delay of 73 time units.

Case 1: *N*=146

The Hamming window is given by

$$w_H[k] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi k}{145}\right) & 0 \leq k \leq 145 \\ 0 & otherwise \end{cases}$$

Right-shifting the ideal lowpass filter by 73 time units, and multiplying with the Hamming window, the designed FIR filter impulse response is obtained as:

$$h[k] = h_l[k] w_H[k] = \begin{cases} 0.4773\left[0.54 - 0.46\cos\left(\frac{2\pi k}{145}\right)\right]\sin c\left(0.4773(k-72.5)\right) & 0 \leq k \leq 145 \\ 0 & otherwise. \end{cases}$$

In this case, $\sum h[k] = 1.0004$, and hence the scaling of $h[k]$ can be ignored.

Case 2: *N*=147

The Hamming window is given by

$$w_H[k] = \begin{cases} 0.54 - 0.46\cos\left(\frac{\pi k}{73}\right) & 0 \leq k \leq 146 \\ 0 & otherwise \end{cases}$$

Right-shifting the ideal lowpass filter by 73 time units, and multiplying with the Hamming window, the designed FIR filter impulse response is obtained as:

$$h[k] = h_l[k] w_H[k] = \begin{cases} 0.4773\left[0.54 - 0.46\cos\left(\frac{\pi k}{73}\right)\right]\sin c\left(0.4773(k-73)\right) & 0 \leq k \leq 146 \\ 0 & otherwise. \end{cases}$$

In this case, $\sum h[k] = 1.0005$, and hence the scaling of $h[k]$ can be ignored.

(c) The frequency response of the 146-tap and 147-tap filters is shown in Fig. S15.8(i), and Fig. S15.8(ii), respectively. Note that, because of the shift, both filters are causal.

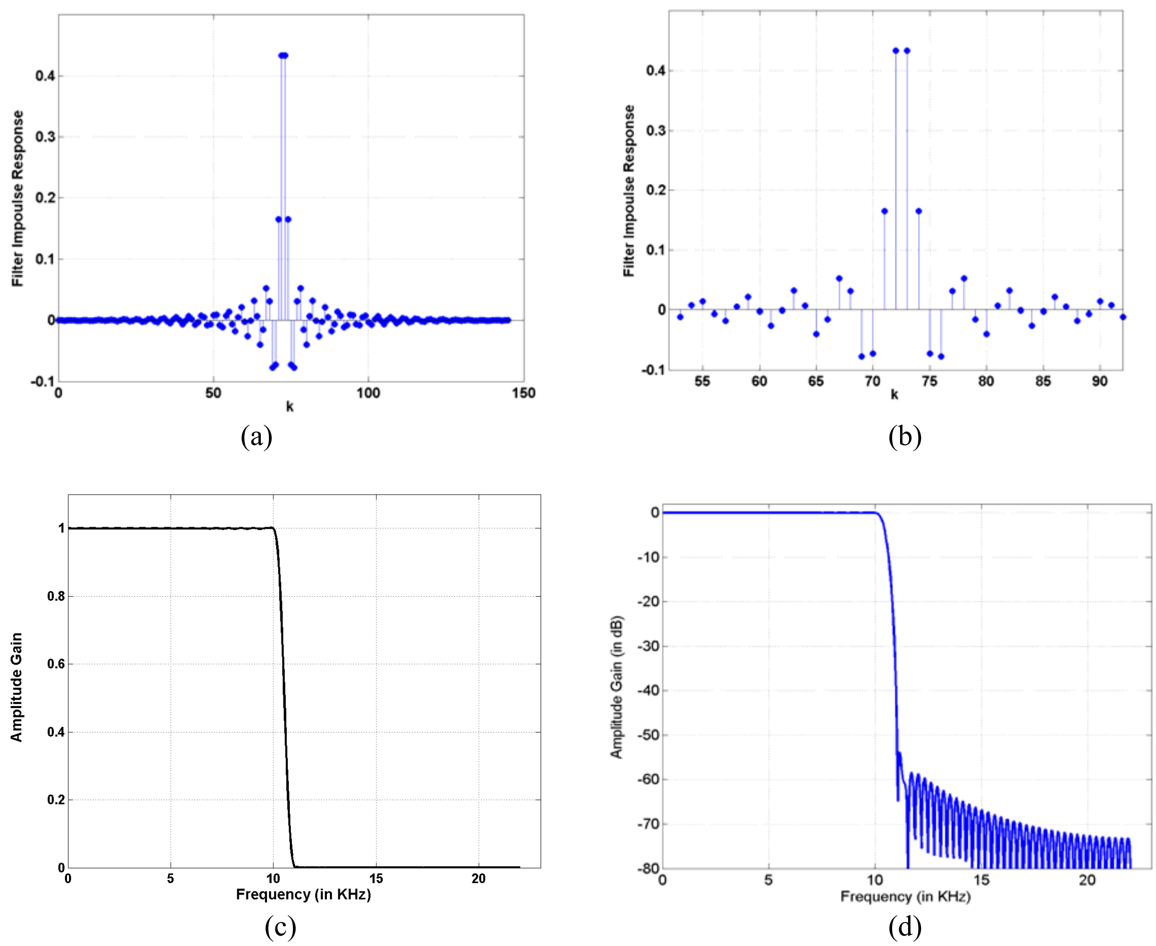


Figure S15.8(i). 146-tap FIR filter designed using Hamming Window. a) The impulse response, b) the blow up of the impulse response showing the middle 40 impulses, c) the amplitude-frequency response in absolute scale, and (d) the amplitude-frequency response in dB scale. Note that as the filter has even number of taps, the middle two impulses in Fig (b) have identical amplitude.

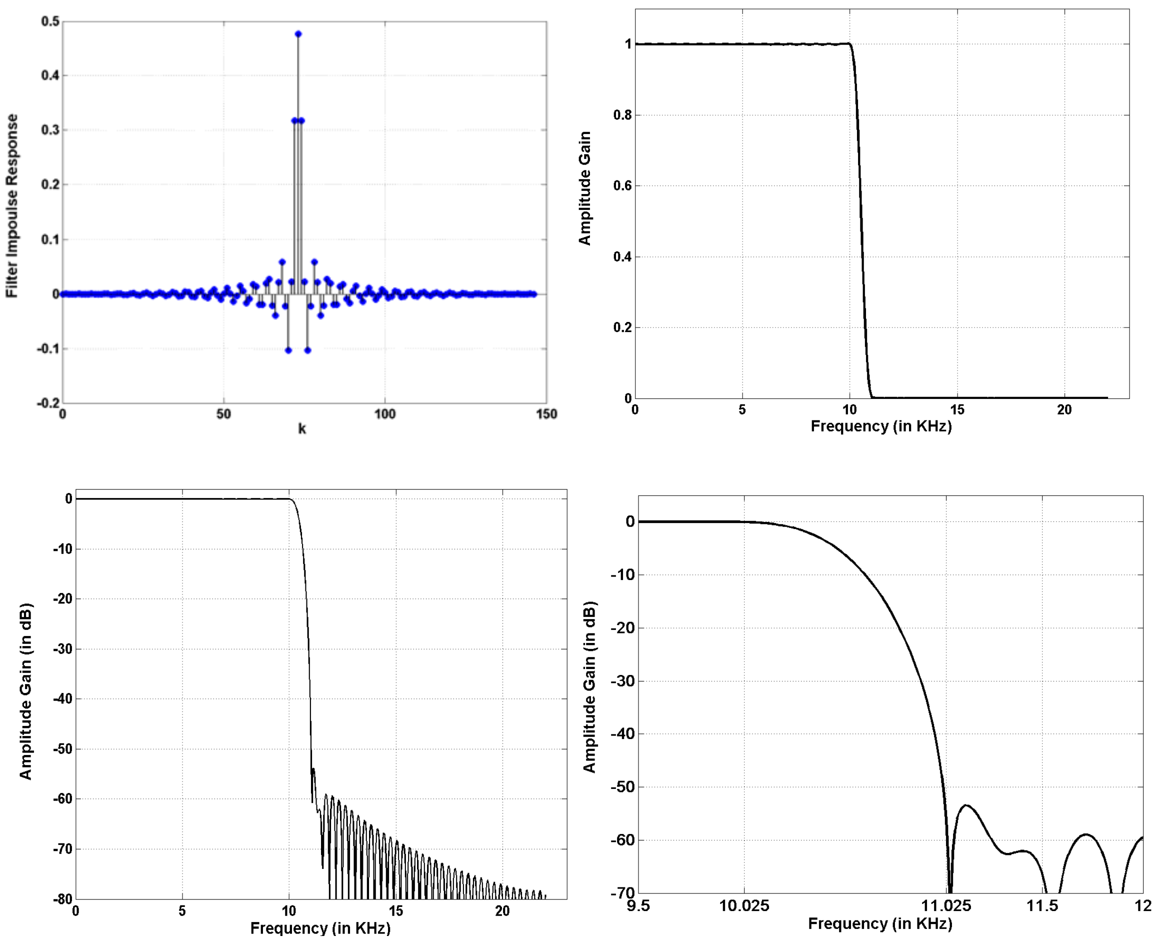


Figure S15.8(ii). 147-tap FIR filter designed using Hamming Window. a) The impulse response, b) the amplitude-frequency response in absolute scale, (c) the amplitude-frequency response in dB scale, and (d) the blow-up of amplitude-frequency response near the cut-off frequency.

Program 15.8: MATLAB Program for calculating and plotting the Filter responses

```
clear; clf
N=146 ; ;              % Number of filter taps
M=(N-1)/2 ;
k = 0:N-1;

filter_ideal = 0.4773*sinc(0.4773*(k-M)) ;
window_hamming = 0.54-0.46*cos(2*pi*k/(N-1)) ;
%win_hamm = hamming(N) ;
filter_hamming = filter_ideal.*window_hamming ;
S=sum(filter_hamming)
filter_hamming = filter_hamming/S;
% the filter impulse response is scaled so that the
% sum is one.

% Plotting the filter impulse response
stem(k, filter_hamming, 'filled'),grid
ylabel('Filter Impulse Response');
xlabel('k')
```

```
clear; clf
N=147 ; ;              % Number of filter taps
M=(N-1)/2 ;
k = 0:N-1;

filter_ideal = 0.4773*sinc(0.4773*(k-M)) ;
window_hamming = 0.54-0.46*cos(2*pi*k/(N-1)) ;
%win_hamm = hamming(N) ;
filter_hamming = filter_ideal.*window_hamming ;
S=sum(filter_hamming)
filter_hamming = filter_hamming/S;
% the filter impulse response is scaled so that the
% sum is one.

% Plotting the filter impulse response
stem(k, filter_hamming, 'filled'),grid
ylabel('Filter Impulse Response');
xlabel('k')
```

print -dtiff plot.tiff
 	
% Calculating the freq. response
[H, w] = freqz(filter_hamming,1) ;
 	
%Plot in absolute scale
plot(w/pi*22.05, abs(H)), grid
axis([0 23 0 1.1]);
xlabel('Frequency (in KHz)')
ylabel('Amplitude Gain');
print -dtiff plot.tiff
 	
%Plot in dB scale
H = 20*log10(abs(H)+eps) ;
plot(w/pi*22.05, H), grid
axis([0 23 -80 2]);
xlabel('Frequency (in KHz)')
ylabel('Amplitude Gain (in dB)');
print -dtiff plot.tiff
	print -dtiff plot.tiff
	
	% Calculating the freq. response
	[H, w] = freqz(filter_hamming,1) ;
	
	%Plot in absolute scale
	plot(w/pi*22.05, abs(H)), grid
	axis([0 23 0 1.1]);
	xlabel('Frequency (in KHz)')
	ylabel('Amplitude Gain');
	print -dtiff plot.tiff
	
	%Plot in dB scale
	H = 20*log10(abs(H)+eps) ;
	plot(w/pi*22.05, H), grid
	axis([0 23 -80 2]);
	xlabel('Frequency (in KHz)')
	ylabel('Amplitude Gain (in dB)');
	print -dtiff plot.tiff

Problem 15.9

As the normalized cut-off frequency $\Omega_n = 0.4773$, the ideal (IIR) impulse response is given by

$$h[k] = 0.4773 \sin c\left(0.4773k\right).$$

The passband ripples requirement is not specified. The stopband attenuation should be at least 45 dB. Therefore, *A*=45. The shape parameter is then calculated to be

$$\beta = 0.5842(A-21)^{0.4} + 0.078(A-21) = 0.5842(45-21)^{0.4} + 0.078(45-21) \approx 3.9548$$

The normalized transition bandwidth, $\Delta\Omega_n = \frac{1\ KHz}{22.05\ KHz} = 0.0454$. Therefore, the window length *N* is given by

$$N \geq \frac{45-7.95}{7.18 \times 0.0454} = 113.78 \text{ or } 114.$$

Substituting β = 3.9548 and *N* = 114 in Eq. (15.18), the Kaiser window is given by

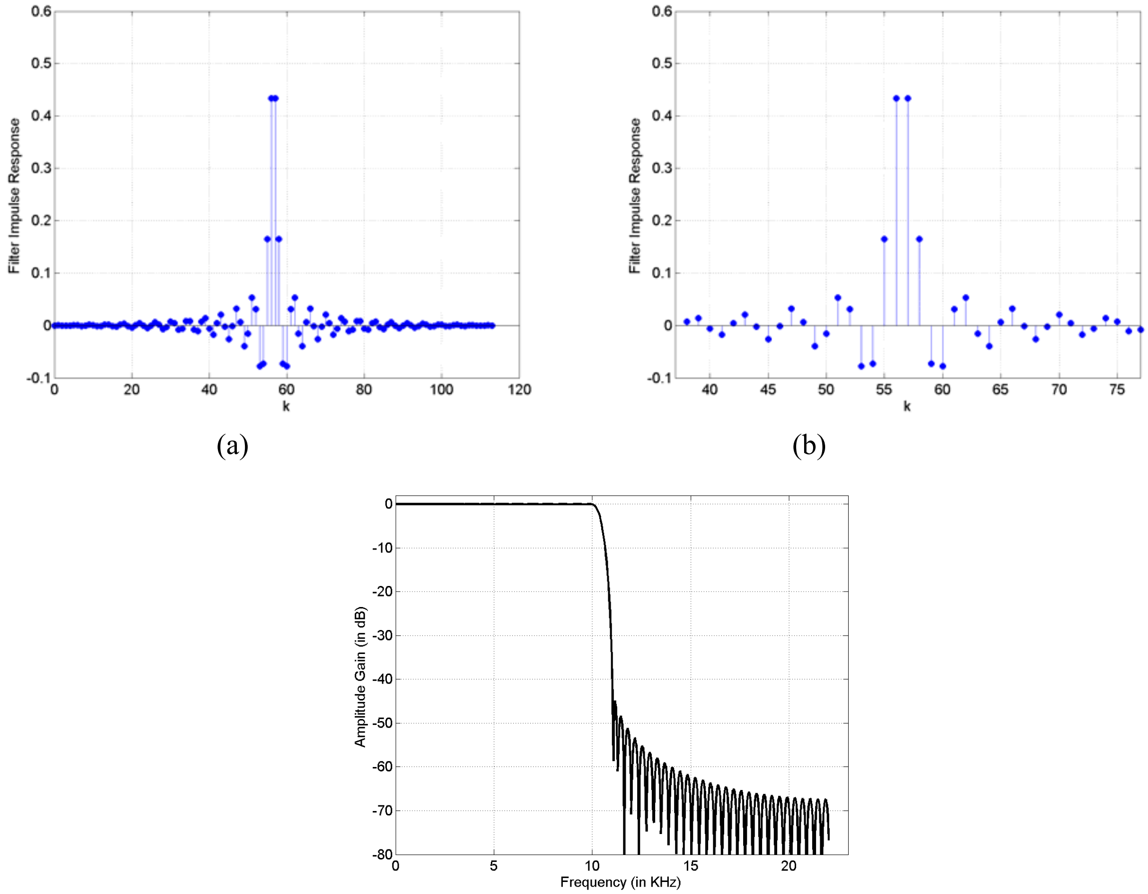
$$w_{kaiser}\left[k\right] = \begin{cases} \dfrac{I_0\left[3.9548\left(\sqrt{1-\left[\left(k-56.5\right)/56.5\right]^2}\right)\right]}{I_0\left[3.9548\right]} & 0 \leq k \leq 113 \\ 0 & \text{otherwise.} \end{cases}$$

By applying a right-shift to the ideal lowpass filter by 56.5 time units, and multiplying with the Kaiser window, the designed FIR filter impulse response is given by

$$h[k] = h_{ilp}[k] w_{kaiser}[k] = \begin{cases} 0.4773 \sin c\left(0.4773(k-56.5)\right) w_{kaiser}[k] & 0 \leq k \leq 113 \\ 0 & \textit{otherwise.} \end{cases}$$

In this case, $\sum h[k] = 1.0006$, and hence the scaling of *h*[*k*] can be ignored.

The magnitude response of the designed filter is plotted in Fig. S15.9, and it is observed that the given specifications of the filter are satisfied.



(a) (b) (c)

Figure S15.9. 114-tap FIR filter designed using Kaiser Window. (a) The impulse response, (b) the blow up of the impulse response showing the middle 40 impulses, and (c) the frequency response.

Program 15.9: MATLAB Program for Problem 15.9

```
clear; clf

A=45 ;
beta = 0.5842*((A-21)^0.4) + 0.078*(A-
21)
NTB = 1/44.1
N = (A-7.95)/(14.36*NTB)
N=ceil(N)

M=(N-1)/2 ;
k = 0:N-1;

filter_ideal=0.4773*sinc(0.4773*(k-M));
filter_kaiser =
filter_ideal.*(kaiser(N,beta))' ;
S=sum(filter_kaiser)
filter_kaiser = filter_kaiser/S;
```

```
% Plotting the filter impulse response
stem(k, filter_kaiser, 'filled'),grid
ylabel('Filter Impulse Response');
xlabel('k')
print -dtiff plot.tiff
% Calculating the frequency response
[H, w] = freqz(filter_kaiser,1) ;
H = 20*log10(abs(H)+eps) ;

plot(w,H) ;
plot(w/pi*22.05, H), grid
axis([0  23 -80 2]);
xlabel('Frequency (in KHz)')
ylabel('Amplitude Gain (in dB)');
print -dtiff plot.tiff
```

Problem 15.10

The cut-off frequency Ω_c of the filter is given by $\Omega_c = 0.64\pi - 0.3\pi/2 = 0.49\pi$. The normalized cut-off frequency Ω_n of the filter is $\Omega_c / \pi = 0.49$. The impulse response of the ideal high pass filter with a cut-off frequency of 0.49 is given by

$$h_{ihp}[k] = \delta[k-m] - 0.49\mathrm{sinc}[0.49(k-m)].$$

The maximum passband ripple is 0.002, and the maximum stopband ripple is 0.005. As the ripple characteristics are similar in passband and stopband, the effective maximum ripple = min(0.002, 0.005) = 0.002. The minimum attenuation A is therefore given by $A = 20\log_{10} 0.002 \approx 54$ dB.

The shape parameter is evaluated from Eq. (15.20) as follows:

$$\beta = 0.1102(A - 8.7) = 4.99.$$

The transition band $\Delta\Omega_c$ for the FIR filter is $(\Omega_p - \Omega_s) = 0.3\pi$. The normalized transition band $\Delta\Omega_n$ is therefore given by $\Delta\Omega_c / \pi = 0.3$. Using $\Delta\Omega_n = 0.3$, the length N of the Kaiser window is given by

$$N \geq \frac{54 - 7.95}{2.285\pi \times 0.3} = 21.38.$$

Rounding off to the higher closest odd number, we obtain $N = 23$.

The expression for the Kaiser window is given by

$$w_{kaiser}[k] = \begin{cases} \dfrac{I_0\left[4.99\left(\sqrt{1-\left[(k-11)/11\right]^2}\right)\right]}{I_0[4.99]} & 0 \leq k \leq 22 \\ 0 & \text{otherwise.} \end{cases}$$

The impulse response of the highpass FIR filter is given by

$$h_{hp}[k] = h_{ihp}[k]\, w_{kaiser}[k],$$

where $h_{ihp}[k]$ is specified above with $m = 11$. The filter gain at $\Omega = \pi$ is given by

$$H_{hp}(\pi) = \sum_{k=0,2,\ldots}^{N-1} h_{hp}[k] - \sum_{k=1,3,\ldots}^{N-1} h_{hp}[k] = 1.0002.$$

As $H(\pi) \approx 1$, the coefficients of $h[k]$ need not be normalized.

The magnitude response of the highpass FIR filter is plotted in Fig. S15.10, and it is observed that the given specifications of the filter are satisfied. ▌

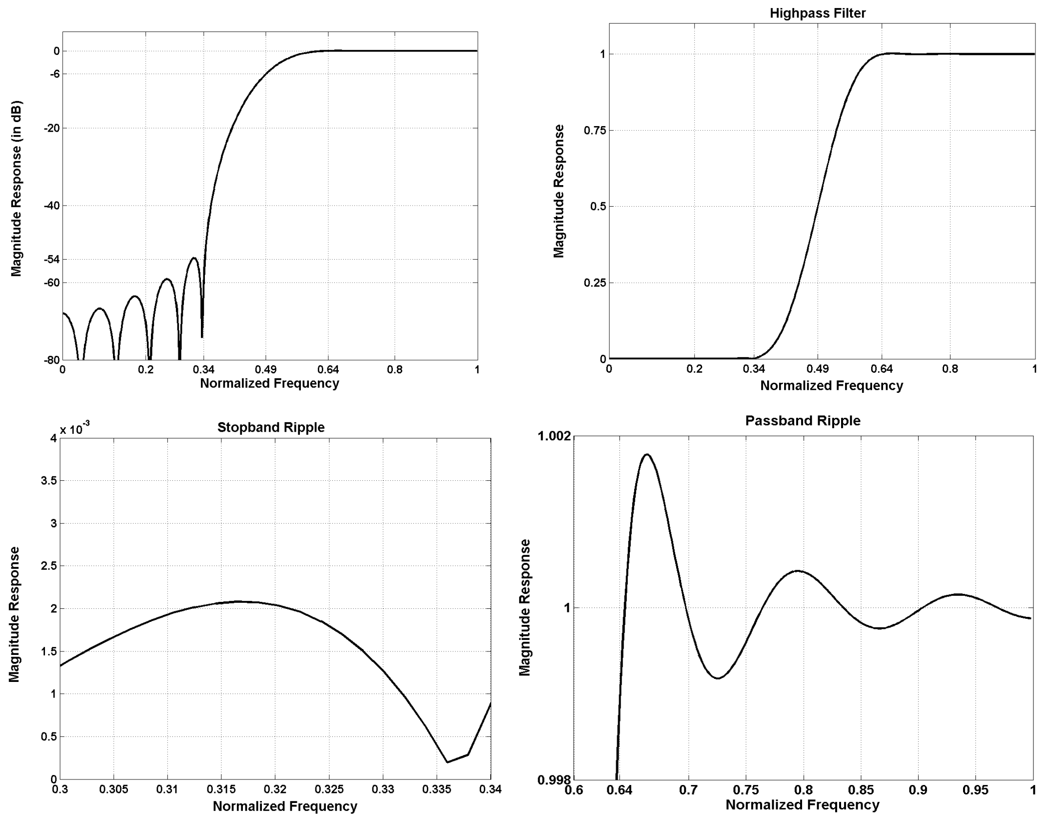


Fig. S15.10. Magnitude response of the highpass FIR filter designed in Problem 15.10.

Program 15.10: MATLAB Program for Problem 15.10

```
wn = 0.49 ;           % Normalized cutoff frequency
beta = 4.99;          % Shape parameter
N  = 23;              % Impulse response length
M  = (N-1)/2;         % Delay
k = [0:(N-1)];
d = [zeros(1,M) 1 zeros(1,M)];      % delayed impulse
hihp = d - wn*sinc(wn*(k-M))
h = hihp.* kaiser(N,beta)'  ;
S=sum(h.*((-1).^(k-1)));     % =0.9999
[H, w] = freqz(h,1,512);
freq = (w/pi) ; % Horizontal axis for plotting freq. response
H2dB = 20*log10(abs(H)) ;

plot(freq, H2dB), grid
axis([0  1  -80  5]);
%title('Kaiser Window')
xlabel('Normalized Frequency')
ylabel('Magnitude Response (in dB)');
plot(freq, abs(H)), grid
axis([0  1  0  1.1]);
title('Highpass Filter')
xlabel('Normalized Frequency')
ylabel('Magnitude Response');
print -dtiff plot.tiff
```

Problem 15.11

The cut-off frequencies of the bandpass filter are given by

$$\Omega_{c1} = 0.5(0.2\pi + 0.4\pi) = 0.3\pi \text{ and}$$

$$\Omega_{c2} = 0.5(0.6\pi + 0.8\pi) = 0.7\pi .$$

The normalized cut-off frequencies are given by $\Omega_{n1} = \Omega_{c1}/\pi = 0.3$ and $\Omega_{n2} = \Omega_{c2}/\pi = 0.7$. The impulse response of an ideal bandpass filter is given by

$$h_{ibp}[k] = 0.7\text{sinc}[0.7(k-m)] - 0.3\text{sinc}[0.3(k-m)].$$

The maximum ripple = min(0.02,0.009) = 0.009. Therefore, the minimum attenuation $A = 20\log_{10} 0.009 \approx 41$ dB.

The shape parameter β of the Kaiser window is computed as

$$\beta = 0.5842(A-21)^{0.4} + 0.0789(A-21) = 3.51.$$

The transition bands $\Delta\Omega_{c1}$ and $\Delta\Omega_{c2}$ for the bandpass FIR filter are given by

$$\Delta\Omega_{c1} = 0.4\pi - 0.2\pi = 0.2\pi \text{ and}$$

$$\Delta\Omega_{c2} = 0.8\pi - 0.6\pi = 0.2\pi ,$$

which lead to the normalized transition BW of $\Delta\Omega_n = 0.2$.

The length N of the Kaiser window is given by

$$N \geq \frac{41 - 7.95}{2.285\pi \times 0.2} = 23.02 .$$

Rounded to the closest higher odd number, $N = 25$ and the value of m is 12. The expression for the Kaiser window is as follows:

$$w_{kaiser}[k] = \begin{cases} \dfrac{I_0\left[3.51\left(\sqrt{1-\left[(k-12)/12\right]^2}\right)\right]}{I_0[3.51]} & 0 \leq k \leq 24 \\ 0, & \text{otherwise.} \end{cases}$$

The impulse response of the bandpass FIR filter is given by

$$h_{bp}[k] = h_{ibp}[k]\, w_{kaiser}[k],$$

where $h_{ihp}[k]$ is specified above with $m = 12$. The filter gain at $\Omega = 0.5\pi$ (mid passband) is given by

$$H_{bp}(0.5\pi) = \sum_{k=0}^{N-1} h_{hp}[k] e^{-j0.5\pi k} = 0.995 .$$

Therefore, the coefficients of $h[k]$ are normalized, as $h_{bp}[k] = h_{bp}[k]/0.995$.

The magnitude response of the bandpass FIR filter is plotted in Fig. S15.11, and it is observed that the given specifications of the filter are satisfied. ▌

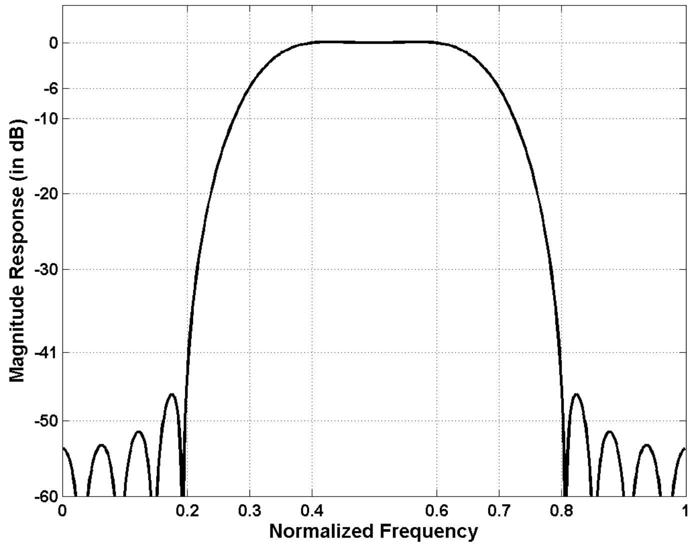


Fig. S15.11. Magnitude response of the bandpass FIR filter designed in Problem 15.11.

Program 15.11: MATLAB Program for Problem 15.11

```
wn1 = 0.3 ;           % Normalized cutoff frequency-1
wn2 = 0.7 ;           % Normalized cutoff frequency-2
beta = 3.51;          % Shape parameter
N  = 25;              % Impulse response length
M  = (N-1)/2;         % Delay
k = [0:(N-1)];
hibp = wn2*sinc(wn2*(k-M)) - wn1*sinc(wn1*(k-M));
h = hibp.* kaiser(N,beta)' ;

S=sum(h.*exp(-j*0.5*pi*(k-1)))     % =0.9950
h = h/abs(S) ;

[H, w] = freqz(h,1,512);
freq = (w/pi) ; % Horizontal axis for plotting freq. response
H2dB = 20*log10(abs(H)) ;

plot(freq, H2dB), grid
axis([0  1  -80  5]);
%title('Kaiser Window')
xlabel('Normalized Frequency')
ylabel('Magnitude Response (in dB)');
print -dtiff plot.tiff
```

Problem 15.12

The cut-off frequencies of the bandstop filter are given by

$$\Omega_{c1} = 0.5(0.3\pi + 0.4\pi) = 0.35\pi \text{ and}$$

$$\Omega_{c2} = 0.5(0.6\pi + 0.7\pi) = 0.65\pi .$$

The normalized cut-off frequencies are given by $\Omega_{n1} = 0.35$ and $\Omega_{n2} = 0.65$. The impulse response of an ideal bandstop filter is given by

$$h_{ibs}[k] = \delta[k-m] - 0.65\text{sinc}[0.65(k-m)] + 0.35\text{sinc}[0.35(k-m)].$$

The maximum ripple = 0.05. Therefore, the minimum attenuation $A = 20\log_{10} 0.05 \approx 26.02$ dB.

The shape parameter β of the Kaiser window is computed as

$$\beta = 0.5842(A-21)^{0.4} + 0.0789(A-21) = 1.51.$$

The transition bands $\Delta\Omega_{c1}$ and $\Delta\Omega_{c2}$ for the bandpass FIR filter are given by

$$\Delta\Omega_{c1} = (0.4\pi - 0.3\pi) = 0.1\pi \text{ and}$$

$$\Delta\Omega_{c2} = (0.7\pi - 0.6\pi) = 0.1\pi,$$

which leads to the normalized transition BW of $\Delta\Omega_n = 0.1$.

The length N of the Kaiser window is given by

$$N \geq \frac{26.02 - 7.95}{2.285\pi \times 0.1} = 25.17.$$

Rounded to the closest higher odd number, $N = 27$ and the value of m is 13.

The expression for the Kaiser window is as follows:

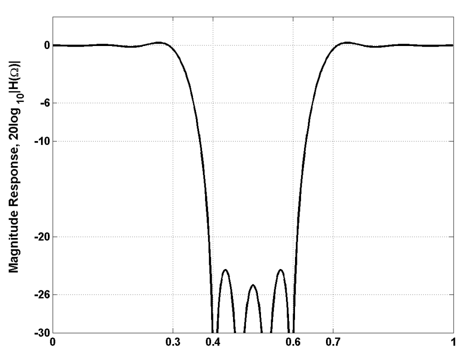
$$w_{kaiser}[k] = \begin{cases} \dfrac{I_0\left[1.51\left(\sqrt{1-\left[(k-13)/13\right]^2}\right)\right]}{I_0[1.51]}, & 0 \leq k \leq 26 \\ 0, & \text{otherwise.} \end{cases}$$

The impulse response of the bandstop FIR filter is given by

$$h_{bs}[k] = h_{ibs}[k]\, w_{kaiser}[k],$$

where $h_{ibs}[k]$ is specified above with $m = 13$.

The magnitude response of the bandstop FIR filter is plotted in Fig. S15.12. It is observed that the bandstop filter satisfies the design specifications.



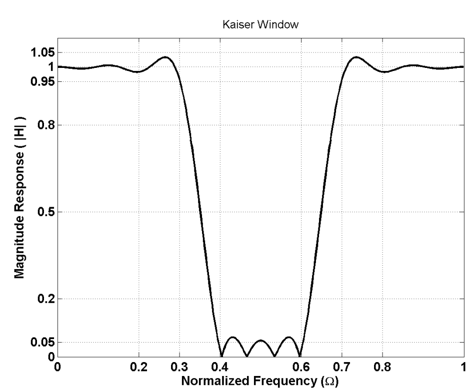


Fig. S15.12. Magnitude response of the bandstop FIR filter designed in Problem 15.12.

Program 15.12: MATLAB Program for Problem 15.12

```
wn1 = 0.35 ;           % Normalized cutoff frequency-1
wn2 = 0.65 ;           % Normalized cutoff frequency-2
beta = 1.51;          % Shape parameter
N  = 27;              % Impulse response length
M  = (N-1)/2;         % Delay
k = [0:(N-1)];
d = [zeros(1,M) 1 zeros(1,M)];      % delayed impulse
hibs = d - wn2*sinc(wn2*(k-M)) + wn1*sinc(wn1*(k-M));
h = hibs.* kaiser(N,beta)' ;

S=sum(h)     % =0.
h = h/abs(S) ;

[H, w] = freqz(h,1,512);
freq = (w/pi) ; % Horizontal axis for plotting freq. response
HdB = 20*log10(abs(H)) ;

plot(freq, HdB), grid
axis([0  1  -80  5]);
%title('Kaiser Window')
xlabel('Normalized Frequency (\Omega)')
ylabel('Magnitude Response, 20log_{ 10}|H(\Omega)|');
print -dtiff plot.tiff
%
plot(freq, abs(H)), grid
%axis([0  1  -80  5]);
title('Kaiser Window')
xlabel('Normalized Frequency (\Omega)')
ylabel('Magnitude Response (|H|)');
print -dtiff plot.tiff
```

The solution of Remaining problems (15.13-15.25) will be added soon.