

## Problems for Chapter 24 of ‘Ultra Low Power Bioelectronics’

### Problem 24.1

- a) Find differential equations that correspond to the acid dissociation reaction of Equation (24.4) and show that Figure 24.2 (b) is a circuit representation of them.
- b) Draw a dissociation feedback loop analogous to the association feedback loop of Figure 24.3.

### Problem 24.2

Show that the variance in the number of heads obtained after  $N_t$  tosses of a coin with a probability of heads =  $p$  is given by Equation (24.7).

### Problem 24.3

The circuit shown in Figure 24.4 is a current-mode implementation of the forward half of the chemical reaction  $A + B \rightleftharpoons C$ . Draw a circuit that is a current-mode implementation of the backward half of the reaction ( $C \rightarrow A + B$ ).

### Problem 24.4

Show how to modify the circuit of Figure 24.8 such that the transcription binding subcircuit has a programmable Hill coefficient.

### Problem 24.5

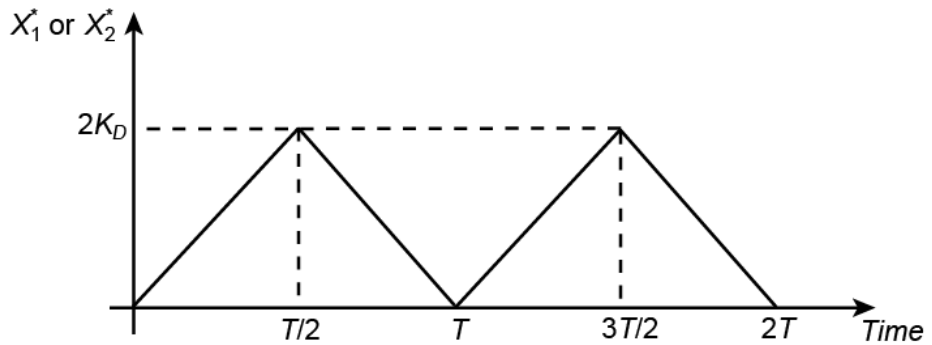
If  $[S]$  may not be assumed constant, modify Figure 24.3 into a nonlinear interacting set of feedback loops that model changes in dynamics in  $[E]$  and  $[S]$ .

### Problem 24.6

If the "logic circuit" of Figure 24.11 is described by the following truth table for mRNA production rates

$u(X_1^*/K_D - 1)$	$u(X_2^*/K_D - 1)$	$\beta_{mRNA}$
0	0	$\beta_0$
0	1	$10\beta_0$
1	0	$10\beta_0$
1	1	$100\beta_0$

plot the output mRNA concentration if  $X_1^*$  and  $X_2^*$  are both described by the symmetric triangle wave shown below. You may assume that  $R_\alpha^{mRNA} C \approx 0$  because  $C$  is small but that  $R_\alpha^{mRNA}$  is finite.



**Figure P24.6:**  $X_1^*$  and  $X_2^*$  waveforms.

Problem 24.7

Repeat Problem 24.6 for  $R_\alpha^{mRNA} C = \frac{T}{10}$ . Justify any approximations.

Problem 24.8

Modify Figure 24.10 to implement an analog AND function.

Problem 24.9

Derive Equation (24.33) from Figure 24.17 (c) and the discussion of chemical-reaction noise in the chapter.

Problem 24.10

Can you guess why the power consumption per neuron ( $\sim 0.66$  nW) is much higher than the average power consumption per cell ( $\sim 1$  pW)?