Fundamentals of MIMO Technology and Spatial Multiplexing

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Outline

- Introduction
- Efficiency of MIMO Data Transmission with Spatial Multiplexing
- Basic Forms of MIMO Transmission and Reception
- MIMO Receivers – Complexity Reduction
- Examples of MIMO Systems
- Additional Discussion and Remarks
- Acknowledgement

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Introduction

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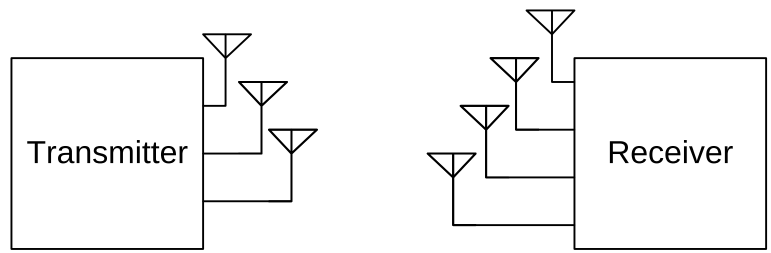
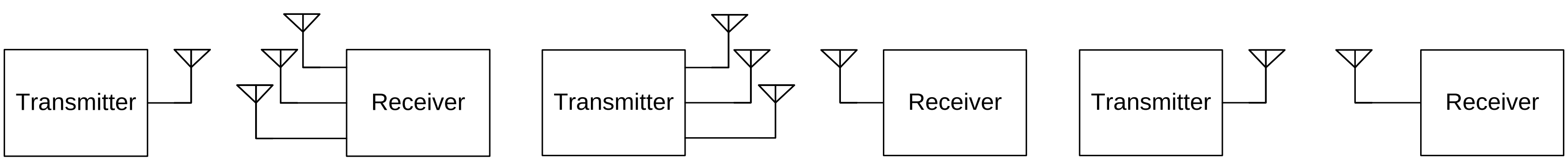
What is MIMO

- MIMO: Multiple Input/Multiple Output Systems



  - Doesn't have to be wireless
- Degenerated forms:



SIMO MISO SISO

- MIMO can provide multiple independent channels (spatial multiplexing).
  - SIMO and MISO only provide diversity over a single channel

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History of MIMO

- Early Results
  - A.R. Kaye and D.A. George (1970)
    - First Description of MIMO Channels
  - Branderburg and Wyner (1974)
    - Capacity Analysis
  - W. van Etten (1975, 1976)
    - Receiver Structures
  - Arogyaswami Paulraj and Thomas Kailath (1993, 1994)
    - Concept of Spatial Multiplexing
  - Telatar (1995)
    - MIMO Channel Capacity

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History of MIMO (cont.)

- Major Re-Introduction of MIMO Concepts and Systems
  - Greg Raleigh and Gerard J. Foschini (1996)
    - Described a practical system with "Layered Space-Time Architecture" and detailed analysis
  - Bell Labs announcement on BLAST MIMO technology (1998)
    - First demonstrated spatial multiplexing in laboratory prototype
    - Caught industry's attention on the MIMO technology
    - Started new development wave
    - The title is a little vague to cause some misinterpretation
      - Caused more interest due to the misunderstanding
      - Nothing was wrong in the text

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History of MIMO (cont.)

- Practical Development
  - Iospan Wireless Inc. developed the first commercial system in that used MIMO with OFDM (2001)
  - Airgo Networks developed an IEEE 802.11n precursor implementation based on MIMO-OFDM (2005)
  - MIMO-OFDM in IEEE 802.11n standard (development from 2006)
  - MIMO-OFDM in IEEE 802.16e (WiMax) standard (development from 2006)
  - MIMO-OFDM in 3GPP HSDPA, LTE standards (development from 2008)

Key Features of MIMO

- Providing Spatial Diversity
  - Not unique, SIMO and MISO can also provide spatial diversty
    - SIMO – Receiver diversity
    - MISO – Transmitter diversity
- Providing Spatial Multiplexing
  - Unique to MIMO
  - MIMO system can create N, up to min(N_t, N_r), independent parallel channels
  - Essential for achieving high spectrum efficiency (capacity > 1bit/Hz) at high SNR
  - Spatial Multiplexing may not be advantageous in low SNR environment

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Efficiency of MIMO Data Transmission with Spatial Multiplexing

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Spectral Efficiency of Single Channel Transmission

- System Capacity from Information Theory
  - Shannon's unconstrained, bandwidth-limited channel capacity for the ideal AWGN channel with arbitrary inputs is

$$C = \log_2(1 + \mathrm{SNR})$$

    - C is in unit of bit/(second*Hz) *per use* (I.e. per channel)
    - At low spectral efficiency, i.e., low SNR: $C \approx \mathrm{SNR}/\ln 2$ (linear)
    - At high spectral efficiency, i.e, C and SNR are large:

    $C \approx \log_2(\mathrm{SNR})$ (~ 3 dB, or double SNR, per bit)
    - Example:
      - C = 1 => SNR = 1 (0 dB);
      - C = 5 => SNR = 31 (14.9 dB);
      - C = 10 => SNR = 1023 (30.1 dB);
      - C = 15 => SNR = 32767 (45.2 dB: difficult to achieve!);
  - Conclusion: difficult to achieve very high spectral efficiency transmission with a single channel.

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MIMO for High Spectral Efficiency Transmission

- MIMO and Spatial Multiplexing
  - In proper environments, a MIMO system can create multiple independent parallel channels from Spatial Multiplexing
    - The number (N) of such channels is ≤ min(N_{Tx}, N_{Rx})
  - In the ideal case, these channels have the same magnitudes and SNRs
    - To fairly compare with non-MIMO case, we assume total transmitted power are the same (the same total interference power to other cells) in both cases, i.e., each channel has an SNR_{MIMO} = SNR/N.
  - The total Spectral Efficiency is

$$C_{Total} = \sum_{n=0}^{N-1} \log_2(1+SNR_i) = N\log_2(1+SNR/N)$$

- At low SNR $(SNR \Rightarrow 0),\ C_{Total} \Rightarrow \frac{N \times SNR/N}{\ln 2} = SNR/\ln 2$

since $\ln(1+x) \approx x \ \ for\ x \to 0$

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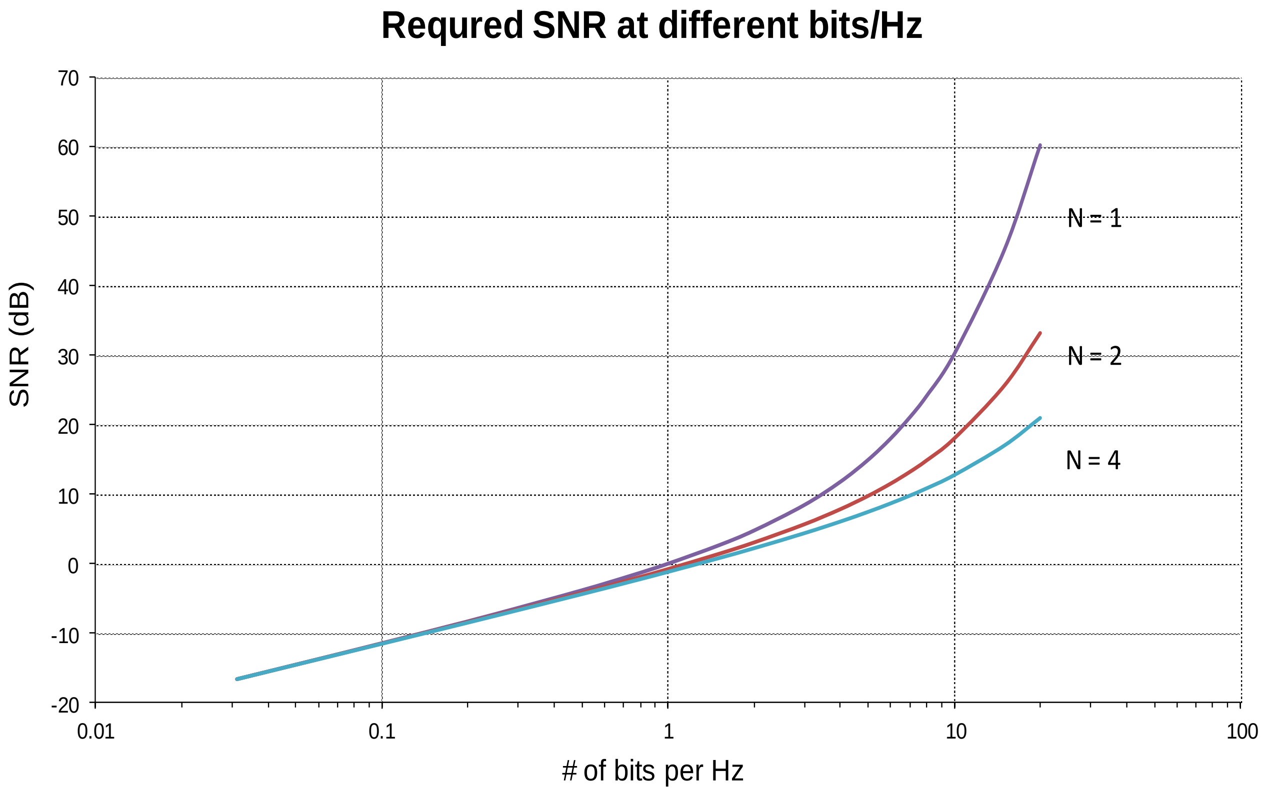
Spectral Efficiency with Spatial Multiplexing (N = 1, 2, and 4)



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Conclusions on Transmission Spectral Efficiency using MIMO

- In high spectral efficiency region (high SNR), 3 dB SNR increase is needed for the increase of each bit per Hz.
  - It's difficult to achieve high spectral efficiency if only a single channel is used
- With spatial multiplexing, MIMO can create multiple parallel channels to achieve high efficiency transmission
  - It will be most efficient in a rich scattering environment, where the magnitudes of these channels have small dispersion
- With constant transmitted power (constant interference), MIMO does not improve the efficiency at low SNR comparing to the single channel case
  - MIMO is much more effective in high SNR regions

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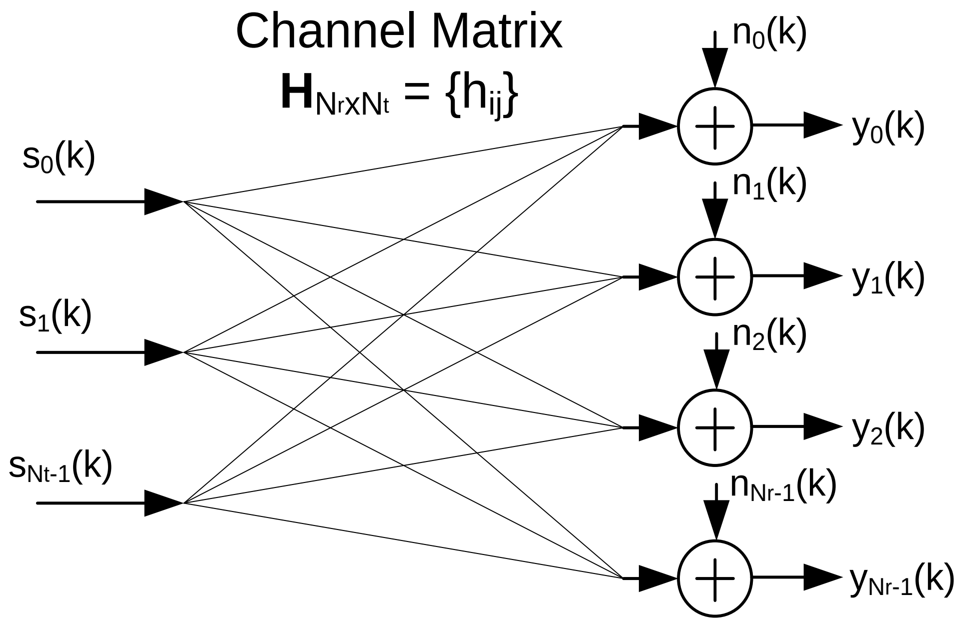
Basic Forms of MIMO Transmission and reception

High Level Description of a MIMO System

- Block Diagram of a MIMO System



$\mathbf{s}_i(k)$ – Mod. Symbols

- Matrix Representation

$$\mathbf{y}_{N_r}(k) = \mathbf{H}_{N_r \times N_t} \mathbf{x}_{N_t}(k) + \mathbf{n}_{N_r}(k), \;\; \mathbf{x}_{N_t}(k) = \left(\mathbf{s}_0(k) \ldots \mathbf{s}_{N_t-1}(k)\right)^T$$

Channel Matrix and Its Estimation

- To utilize the potential of MIMO, it is essential to accurately know the channel matrix $\mathbf{H}_{N_r \times N_t}$ (or simply $\mathbf{H}$).
- The channel matrix is usually estimated using known symbols embedded in the transmitted data sequence
  - Called reference symbols or pilots
- Here we assume the elements of $\mathbf{H}$ are scalars
- The estimation is done in the receiver. Thus it is know there.
- To fully utilize the spatial multiplexing, it is desirable to feedback the parameters of $\mathbf{H}$ to the transmitter
  - Require to using part of the reverse channel

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Channel Known to the Transmitter: Precoding

- Using Singular Value Decomposition, the matrix H can be diagnalized by orthogonal matrices U and V such that:

$$\mathbf{H}_{N_r\times N_t} = \mathbf{U}_{N_r\times N_r}\Lambda_{N_r\times N_t}\mathbf{V}_{N_t\times N_t} \qquad \Lambda_{N_r\times N_t} = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \lambda_N & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

λ_i's are the singular values of $\mathbf{H}$

- Thus, $\mathbf{y}_{N_r}(k) = \mathbf{U}_{N_r\times N_r}\Lambda_{N_r\times N_t}\mathbf{V}_{N_t\times N_t}\mathbf{x}_{N_t}(k) + \mathbf{n}_{N_r}(k)$
- By transmitting $\tilde{\mathbf{x}}_{N_t}(k) = \mathbf{V}^H_{N_t\times N_t}\mathbf{x}_{N_t}(k)$ instead of $\mathbf{x}_{N_t}(k)$ and pre-multiply the receive signal vector by $\mathbf{U}^H_{N_r\times N_r}$

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Precoding (cont.)

- The transformed received signal vector becomes:

$$\tilde{\mathbf{y}}_{N_r}(k) = \mathbf{U}^H_{N_r\times N_r}\mathbf{U}_{N_r\times N_r}\Lambda_{N_r\times N_t}\mathbf{V}_{N_t\times N_t}\mathbf{V}^H_{N_t\times N_t}\mathbf{x}_{N_t}(k) + \mathbf{U}^H_{N_r\times N_r}\mathbf{n}_{N_r}(k)$$
$$= \Lambda_{N_r\times N_t}\mathbf{x}_{N_t}(k) + \tilde{\mathbf{n}}_{N_r}(k)$$

- The i-the elements of $\tilde{\mathbf{y}}_{N_r}(k)$: $\tilde{y}_i(k) = \lambda_i s_i(k) + \tilde{n}_i(k)$, i.e., N parallel scalar channels!
- V is orthonormal – No change in noise statistics
- Water-filling weighting – Achieving channel total capacity
- Precoding require the transmitter has the knowledge of the channel, which may or may not be easy to achieve
  - Approximate knowledge of the channel also helps
- System capacity can be achieve by water-filling weighting each channel (associated with λ_i).

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Linear MIMO Receivers

- Consider the basic MIMO system equation:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \qquad \text{(indices dropped for clarity)}$$

- We first consider the zero forcing form
  - Premultiply $\mathbf{y}$ by $\mathbf{H}^H$ we have

$$\breve{\mathbf{y}} \cong \mathbf{H}^H \mathbf{y} = \mathbf{R}\mathbf{x} + \breve{\mathbf{n}}$$

where $\mathbf{R} = \mathbf{H}^H\mathbf{H}$ is an N_t x N_t square matrix

  - Premultiply $\breve{\mathbf{y}}$ by the (psudo)inverse of $\mathbf{R}$, $\mathbf{x}$ is recovered as

$$\hat{\mathbf{x}} = \mathbf{R}^{-1}\mathbf{H}^H\mathbf{y} = \mathbf{x} + \mathbf{R}^{-1}\breve{\mathbf{n}}$$

  - When $\mathbf{n} = \mathbf{0}$, $\hat{\mathbf{x}} = \mathbf{x}$
- This is the zero forcing solution of Linear MIMO receiver

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Linear MIMO Receivers (cont.)

- Derivation of MMSE MIMO receiver
  - Consider again $\mathbf{y} = \mathbf{Hx} + \mathbf{n}$
  - To determine coefficient matrix $\mathbf{A}$, such that, the estimate of $\mathbf{x}$, $\hat{\mathbf{x}}_{MMSE} = \mathbf{Ay}$ minimize the norm of the error vector, i.e., minimize

$$\varepsilon = E\left[|e|^2\right] \cong E\left[|\mathbf{Ay} - \mathbf{x}|^H\right]$$

  - $\mathbf{A}$ can be determined by taking derivative of ε with respect to $\mathbf{A}$ or using orthogonal principle, i.e.:

$$E\left[\mathbf{ey}^H\right] = E\left[(\mathbf{Ay} - \mathbf{x})\mathbf{y}^H\right] = \mathbf{A}E\left[\mathbf{yy}^H\right] - E\left[\mathbf{xy}^H\right] = 0$$

  - Thus $\mathbf{A}$ satisfies:

$$\mathbf{A} = \left(E\left[\mathbf{xy}^H\right]\right)\left(E\left[\mathbf{yy}^H\right]\right)^{-1} \cong \mathbf{H}^H\left(\mathbf{HH}^H + \sigma^2\mathbf{I}\right)^{-1}$$

where we assume $E\left[\mathbf{xx}^H\right] = \mathbf{I}, E\left[\mathbf{nx}^H\right] = \mathbf{0}$ and the noise is white.

  - Finally

$$\hat{\mathbf{x}}_{MMSE} = \mathbf{Ay} = \mathbf{H}^H\left(\mathbf{HH}^H + \sigma^2\mathbf{I}\right)^{-1}\mathbf{y}, \ \left((\mathbf{HH}^H + \sigma^2\mathbf{I}) \sim N_r \times N_r\right)$$

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Linear MIMO Receivers (cont.)

- Discussions of zero forcing (ZF) and MMSE receivers
  - ZF receiver generate an unbiased estimate of Tx symbols
  - It's main problem is noise enhancement
  - MMSE receiver provides better SNR at output
  - The estimate is biased
    - Not a serious problem for PSK Tx symbols
    - Need bias removal for QAM signals
  - It can be show that MMSE receiver converges to as ZF receiver when SNR goes to infinity, i.e.,

$$\lim_{\sigma^2 \to 0} \left\{ \mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H + \sigma^2 \mathbf{I} \right)^{-1} \right\} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}$$

  - Linear receivers do not provide as good performance as non-linear receivers

Maximum Likelihood (ML) Estimator

- Recall: $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$
- The ML estimator determines $\hat{\mathbf{x}}_{ML}$, which minimizes $\left\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}_{ML}\right\|$ where $\hat{\mathbf{x}}_{ML} \in \mathbf{s}^{N_t}$; i.e., its elements are Tx symbols
- This is a constraint LS estimation problem
  - It is nonlinear and its complexity is (NP)-hard
  - In the receivers, $\hat{\mathbf{x}}_{ML} \in \mathbb{C}^{N_t}$, i.e., its elements are complex numbers
- The performance of the ML estimator is much better than the linear receivers. However, it is also much more complex than the linear receivers
  - Complexity on order of K^{Nt} (K – symbol order)

Note: In this presentation, we shall assume $N_r \geq N_t = N$ (the number of independent layers). Practically, this may not always be true.

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QRD ML Estimator

- Prerequisite: QR decomposition (often used in MIMO)
  - Assuming **H** has a rank of N_t, we have: $\mathbf{H} = \mathbf{QR}$

    where **Q** is an $\mathrm{N_r}$ x $\mathrm{N_t}$ orthnormal matrix, **R** is an $\mathrm{N_t}$ x $\mathrm{N_t}$ upper triangular matrix.
- Solution:
  - Since Q is orthnormal, we have:

$$\left\|\mathbf{y}-\mathbf{H}\hat{\mathbf{x}}_{ML}\right\|^2=\left\|\mathbf{y}-\mathbf{QR}\hat{\mathbf{x}}_{ML}\right\|^2=\left\|\mathbf{Q}(\mathbf{Q}^H\mathbf{y}-\mathbf{R}\hat{\mathbf{x}}_{ML})\right\|^2=\left\|\mathbf{Q}^H\mathbf{y}-\mathbf{R}\hat{\mathbf{x}}_{ML}\right\|^2$$

$$\cong\left\|\tilde{\mathbf{y}}-\mathbf{R}\hat{\mathbf{x}}_{ML}\right\|^2=\left\|\begin{pmatrix}\tilde{y}_0\\ \tilde{y}_1\\ \vdots\\ \tilde{y}_{N_t-1}\end{pmatrix}-\begin{pmatrix}r_{00} & r_{01} & \cdots & r_{0N_t-1}\\ 0 & r_{11} & \cdots & r_{1N_t-1}\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \cdots & 0 & r_{N_t-1N_t-1}\end{pmatrix}\begin{pmatrix}\hat{s}_0\\ \hat{s}_1\\ \vdots\\ \hat{s}_{N_t-1}\end{pmatrix}\right\|^2$$

  - Can be viewed as an $\mathrm{N_t}$ layer system

QRD ML Estimator (cont.)

– Each layer has a metric function:

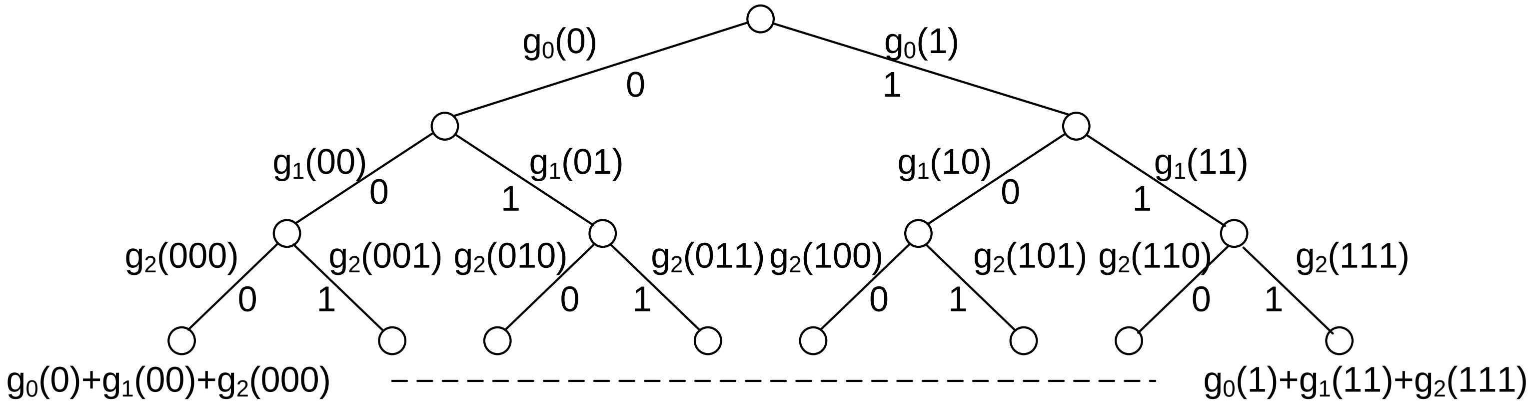
$$g_0(\hat{s}_{N_t-1}) = \left|\tilde{y}_{N_t-1} - r_{N_t-1N_t-1}\hat{s}_{N_t-1}\right|^2, \; g_1(\hat{s}_{N_t-2}, \hat{s}_{N_t-1}) = \left|\tilde{y}_{N_t-2} - r_{N_t-2N_t-2}\hat{s}_{N_t-2} - r_{N_t-1N_t-2}\hat{s}_{N_t-1}\right|^2$$

$$\cdots \; g_{N_t-1}(\hat{s}_0, \hat{s}_1, \ldots \hat{s}_{N_t-1}) = \left|\tilde{y}_0 - r_{00}\hat{s}_0 - r_{01}\hat{s}_1 \cdots - r_{0N_t-1}\hat{s}_{N_t-1}\right|^2$$

– The objective of the estimator is to find a sequence of $\{\hat{s}_i\}$ that minimizes the overall metric

$$g_0(\hat{s}_{N_t-1}) + g_1(\hat{s}_{N_t-2}, \hat{s}_{N_t-1}) \cdots g_{N_t-1}(\hat{s}_0, \hat{s}_1, \ldots \hat{s}_{N_t-1})$$

– Tree search can be used to find the ML solution



QRD ML Estimator (cont.)

- QRD Complexity increases exponentially with the length of Tx data vector and the Tx data symbol size
  - e.g. for $N_t = 4$ and QAM64 (K=64), $64+64^2+64^3+64^4 = 17{,}043{,}520$ metrics need to be computed.
  - Practically impossible for large alphabet size (K) and long vector
- The result is the ML estimate of the Tx data vector.
- In a coded system it is necessary to generate soft bit metrics in order to achieve better system performance
  - Generating soft bit metrics (LLRs) for decoding:

$$LLR = \ln\left[\sum_{\mathbf{x}:s_k(\mathbf{x})=1} \exp\left(-\frac{1}{\sigma^2}\left\|\mathbf{y}-\mathbf{H}\mathbf{x}\right\|^2\right)\right] - \ln\left[\sum_{\mathbf{x}:s_k(\mathbf{x})=-1} \exp\left(-\frac{1}{\sigma^2}\left\|\mathbf{y}-\mathbf{H}\mathbf{x}\right\|^2\right)\right]$$

    - The values in exp() were calculated above
    - It's very computationally intense ~ $2\text{x}\log_2(K)\text{x}K^{Nt}$

MIMO Receivers – Complexity Reduction

General Discussion

- MIMO linear receivers (ZF and MMSE) are simple to implement. However, their performance are relatively poor comparing to ML receiver
- Practical MIMO receivers has been developed to bridge the gap between these two extremes including:
  - Zero-Forcing receiver with decision feedback
  - QRD ML receiver with Sphere Detector (QRD SD)
  - M-algorithm based QRC ML receiver (QRD-M)
  - Lattice reduction
- There are many variations of these implementations
- The simplified QRD methods are direct consequences of corresponding results from investigations of simplification of MLSE equalizers in 1970-80s

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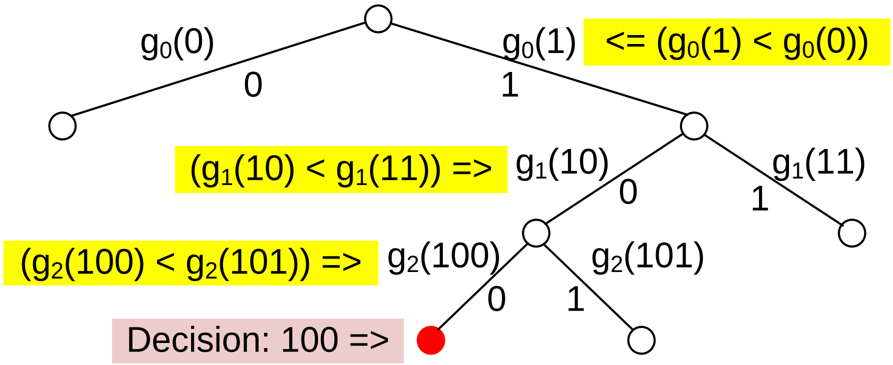
Zero-Forcing with Decision Feedback (ZF-DF)

- Zero-Forcing receiver can also be expressed in the QRD form, namely

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} = \mathbf{QRx} + \mathbf{n} \Rightarrow \mathbf{Q}^H\mathbf{y} \cong \tilde{\mathbf{y}} = \mathbf{Rx} + \tilde{\mathbf{n}}$$

i.e., $\hat{\mathbf{x}} = \mathbf{R}^{-1}\tilde{\mathbf{y}}$, to be solved using backward substitution

  - A decision is made in each step of backward substitution;
  - The decision is used in next step



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QRD-M

- In the ZF-DF, one decision is made in each stage (one survivor path)
- The performance can be improved if more than one (M) survivor paths with smallest cumulative metrics are saved
- The M survivors are extended to MK paths (K = symbol order) to next stage.
- The M paths with smallest cumulative metrics are saved
- Repeat the above two steps until detection is completed
- QRD-M has fixed computational complexity
- Performance depends on M
  - The larger M, the better performance, but the higher complexity

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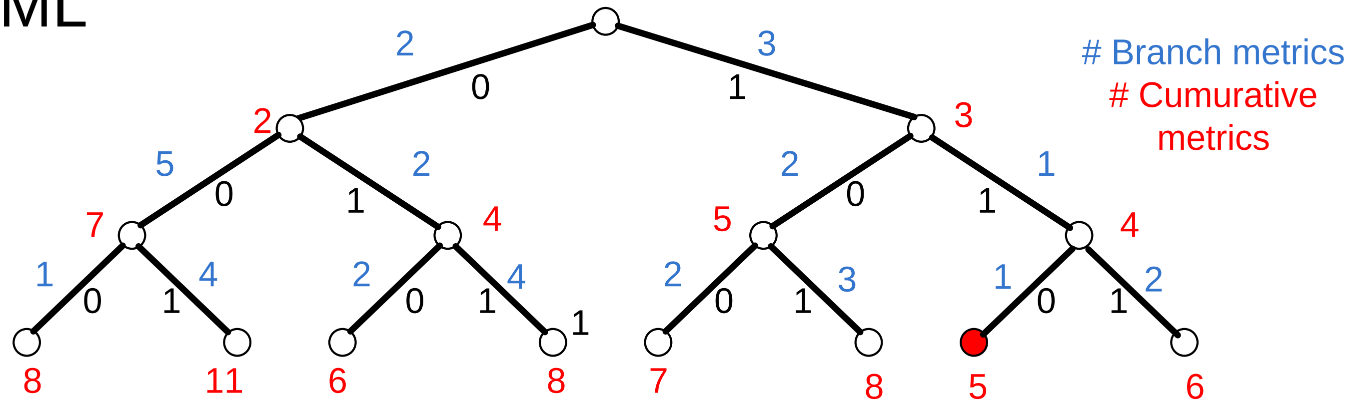
QRD-SD (Sphere Detector)

- In QRD-SD, the paths with cumulative metrics smaller than a predetermined threshold are saved at each stage
- The survivors are extended to PK paths (P – number of survived paths, K – symbol order) to next stage.
- Again, the paths with cumulative metrics smaller than a predetermined threshold are saved
- Repeat the above two steps until detection is completed

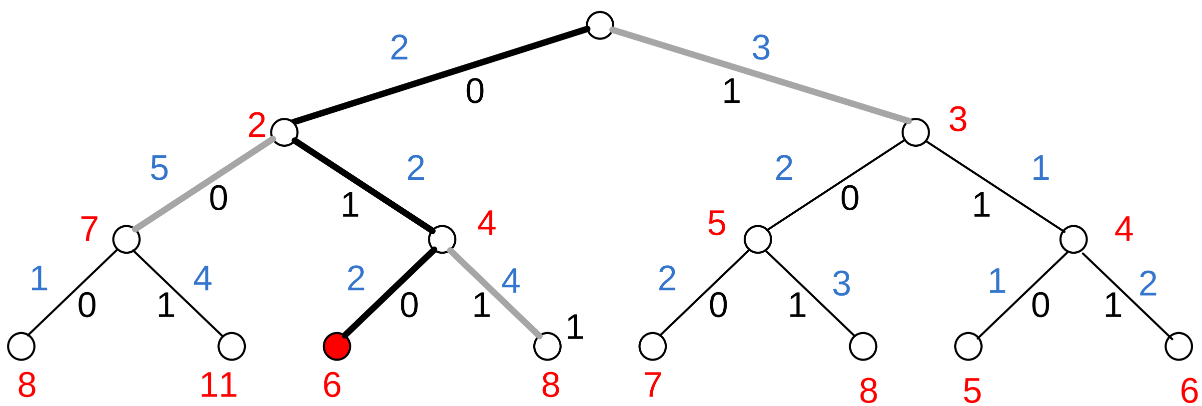
  (The end nodes of the survived paths are in a sphere around the received signal points)
- The computational complexity of QRD-SD is not fixed
- The performance and complexity depend on the threshold
  - The higher threshold, the better performance but also the higher complexity

QRD Estimator Examples

- QRD-ML

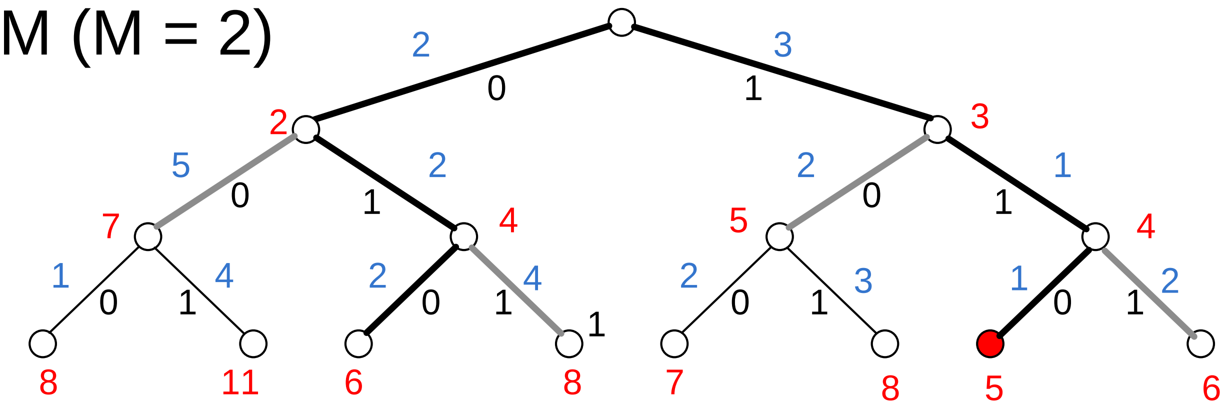


- ZF-DF

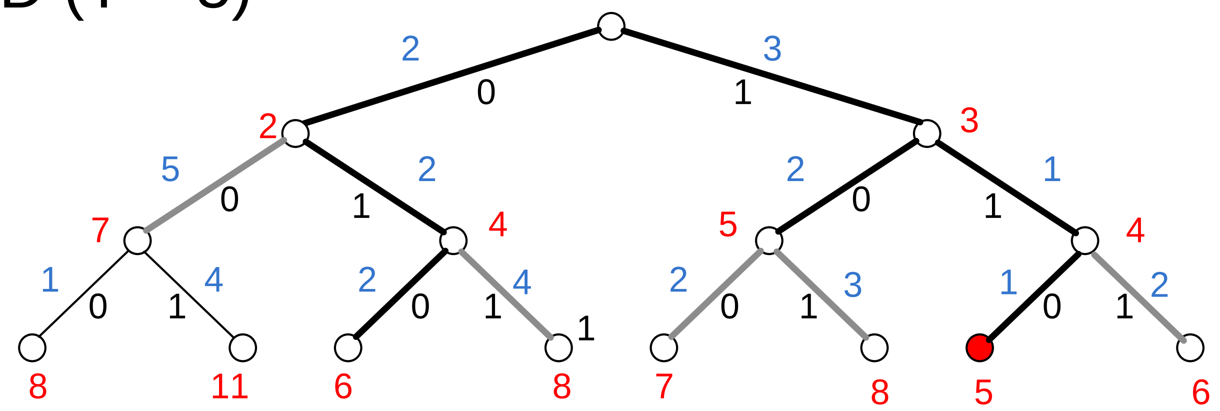


QRD Estimator Examples (cont.)

- QRD-M (M = 2)



- QRD-SD (T = 5)



MLD Complexity Reduction for Small N_t

- In [LA07] a method to reduce MLD complexity for small N_t was described, it is summarized as follows:
  (1) Decompose **H** and **x** as: $\mathbf{H} = \left(\mathbf{h}\ \tilde{\mathbf{H}}\right),\ \mathbf{x} = \left(s_1\ \tilde{\mathbf{s}}\right)^t$
  (2) We rewrite: $\left\|\mathbf{y} - \mathbf{H}\mathbf{x}\right\|^2 = \left\|\mathbf{y} - \tilde{\mathbf{H}}\tilde{\mathbf{s}} - \mathbf{h}s_1\right\|^2$
  (3) Determine M^{Nt-1} $\hat{s}_{1,\tilde{\mathbf{s}}}$ that minimize $\left\|\mathbf{y} - \tilde{\mathbf{H}}\tilde{\mathbf{s}} - \mathbf{h}s_1\right\|^2$ for each distinctive $\tilde{\mathbf{s}}$ as:
    a) $(\mathbf{h}^H\mathbf{h})^{-1}\mathbf{h}^H\left(\mathbf{y} - \tilde{\mathbf{H}}\tilde{\mathbf{s}} - \mathbf{h}s_1\right) = (\mathbf{h}^H\mathbf{h})^{-1}\mathbf{h}^H(\mathbf{y} - \tilde{\mathbf{H}}\tilde{\mathbf{s}}) - s_1$
    b) Find the closest constellation point $\hat{s}_1$ to $(\mathbf{h}^H\mathbf{h})^{-1}\mathbf{h}^H(\mathbf{y} - \tilde{\mathbf{H}}\tilde{\mathbf{s}})$ by slicing as the estimate of s_1.
    c) Find the smallest one in the M^{Nt-1} metrics with the corresponding $\hat{s}_1$ and $\tilde{\mathbf{s}}$
  – The complexity is proportional to M^{Nt-1} instead of M^{Nt}

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MLD Complexity Reduction for Small N_t (cont.)

- Interference Suppression
  - If the interferences from adjacent cells are uncorrelated between receiver antennas, MRC is optimal
  - If the interference are strong and correlated between antennas, decorrelation (whitening) can improve receiver performance
    - It is also called *linear interference cancellation*
  - The received signal vector from all antennas can be expressed as: $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$

where z is the vector consists of noise/interference. Its correlation matrix $\mathbf{R}_{zz}$ is defined as

$$E\left[\mathbf{z}\mathbf{z}^H\right] = \mathbf{R}_{zz} = \mathbf{R}_{zz}^{1/2}\mathbf{R}_{zz}^{H/2},\ \mathbf{R}_{zz}^{-1} = \mathbf{R}_{zz}^{-H/2}\mathbf{R}_{zz}^{-1/2}$$

  - We define:

$$\mathbf{y}' = \mathbf{R}^{-1/2}\mathbf{y} = \mathbf{R}^{-1/2}\mathbf{H}\mathbf{x} + \mathbf{R}^{-1/2}\mathbf{z} = \mathbf{H}'\mathbf{x} + \mathbf{z}'$$

where $E\left[\mathbf{z}'\mathbf{z}'^H\right] = \mathbf{R}^{-1/2}\mathbf{R}\mathbf{R}^{-H/2} = \mathbf{I}$, i.e. uncorrelated

MLD Complexity Reduction for Small N_t (cont.)

- Interference Suppression (cont.)
  - The original equation for minimization can be rewrite as

$$\left\|\mathbf{y}'-\tilde{\mathbf{H}}'\tilde{\mathbf{s}}-\mathbf{h}'s_1\right\|^2=\left(\mathbf{y}'-\tilde{\mathbf{H}}'\tilde{\mathbf{s}}-\mathbf{h}'s_1\right)^H\left(\mathbf{y}'-\tilde{\mathbf{H}}'\tilde{\mathbf{s}}-\mathbf{h}'s_1\right)$$

$$=(\mathbf{h}'^H\mathbf{h}')^{-1}\mathbf{h}'^H\left(\mathbf{y}'-\tilde{\mathbf{H}}'\tilde{\mathbf{s}}-\mathbf{h}'s_1\right)=(\mathbf{h}'^H\mathbf{h}')^{-1}\mathbf{h}'^H(\mathbf{y}'-\tilde{\mathbf{H}}'\tilde{\mathbf{s}})-s_1$$

$$=(\mathbf{h}^H\mathbf{R}^{-1}\mathbf{h})^{-1}\mathbf{h}^H\mathbf{R}^{-1}(\mathbf{y}-\tilde{\mathbf{H}}\tilde{\mathbf{s}})-s_1$$

  - It is very similar to the original equation with little increase of complexity
    - When N_t= 2, matrix inversion is trivial.
  - **R** can be estimated parametrically, i.e., by estimating the channel of neighbor cell signal, or non-parametrically, i.e., by directly estimating the noise correlations

MLD Complexity Reduction for Small N_t (cont.)

- This approach is most effective for small N_t and large constellation size, e.g, two antenna with 64QAM
- Bit metrics can be computed similar to full MLD
- The result is optimal (no approximation)
- It was originally proposed in

[AL07] Y. Lomnitz and D. Andelman, “Efficient maximum likelihood detector for MIMO systems with small number of streams,” Electronics Letters, pp. 1212-1214, Vol. 43, No. 22, Oct. 2007.

Soft Decoding Bit Metrics (LLRs)

- Generating soft bit metrics (LLRs) for decoding coded data, has the similar form as the QRD-ML case:

$$LLR = \ln\left[\sum_{\mathbf{x}:s_k(\mathbf{x})=1} \exp\left(-\tfrac{1}{\sigma^2}\left\|\mathbf{y}-\mathbf{Hx}\right\|^2\right)\right] - \ln\left[\sum_{\mathbf{x}:s_k(\mathbf{x})=-1} \exp\left(-\tfrac{1}{\sigma^2}\left\|\mathbf{y}-\mathbf{Hx}\right\|^2\right)\right]$$

  - where the s_k's are only taking from the survived paths
- Further simplification: Dual-Max or Max-Log approximation

$$LLR = \max_{s_k(\mathbf{x})=1}\left(-\tfrac{1}{\sigma^2}\left\|\mathbf{y}-\mathbf{Hx}\right\|^2\right) - \max_{s_k(\mathbf{x})=-1}\left(-\tfrac{1}{\sigma^2}\left\|\mathbf{y}-\mathbf{Hx}\right\|^2\right)$$

  - Also applicable for the QRD-ML case
  - Good approximation at high SNR
- In both cases, if $s_k(\mathbf{x}) = 1$, or $s_k(\mathbf{x}) = -1$ does not exist, it will take the smallest possible value.

Examples of MIMO Systems

BLAST and Its Variations

- BLAST stands for *Bell Labs Layered Space Time*
- It was first implemented in the lab environment demonstrating the advantage of MIMO (1996 – 1998)
- Later designs were based the same concept with improvements
- D-BLAST (D is for Diagonal) was first developed before 1998
- V-BLAST (V is for Vertical) was soon developed afterwards (1998) to simplify implementation
  - Used in most practical designs
- Modified D-BLAST to take advantage of both

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BLAST (cont.)

- D-BLAST
  - Layered design with the number of layers equal to the number of antennas
  - Each layer coded independently
  - Advantages: Fully utilize spatial and time diversity
  - Disadvantages: Multiple decoders, increased complexity

Time →

Antenna # ↓

1	2	3	4	5	6	1	2	3	4	5	6	1
6	1	2	3	4	5	6	1	2	3	4	5	6
5	6	1	2	3	4	5	6	1	2	3	4	5
5	5	6	1	2	3	4	5	6	1	2	3	4
4	5	5	6	1	2	3	4	5	6	1	2	3
3	4	5	5	6	1	2	3	4	5	6	1	2

BLAST (cont.)

- V-BLAST
  - Can be implemented with one encoder per antenna or with only one encoder and one decoder
  - Encoded data assigned to multiple antennas and over time
  - Advantages: Simplified implementation, good performance in AWGN
  - Disadvantages: Lower time diversity gain
  - Modified D-BLAST: Assign the encoded data (single encoder) diagonally and interleaved in time

Time →

Antenna # ↓

1	7	13	19	25	31		67	73	
2	8	14	20	26	32		68	74	
3	9	15	21	27	33		69	75	
4	10	16	22	28	34	------------------------	70	76	----
5	11	17	23	29	35		71	77	
6	12	18	24	30	36		72	78	

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MIMO in OFDM Based Systems

- MIMO is most widely used in OFDM Systems
  - MIMO is most suitable for single path (flat fading) channels
  - OFDM can be viewed as a collection of multiple parallel single path channels
    - Each subcarrier is a single path system
- Examples of OFDM Systems with MIMO
  - 802.11n/ac
  - LTE
  - WiMax

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MIMO in OFDM Systems (cont.)

- 802.11n/ac
  - Spatial multiplexing with 2, 3, or 4 streams
    - In 802.11, # of streams is the rank of spatial multiplexing
    - 2x is mandatory, 3x and 4x are optional
    - 11ac can have up to 8 streams
  - Modulation: BPSK, QPSK, 16QAM, and 64 QAM
  - FFT size: 64 (20 MHz), 128 (40MHz)
  - Transmitter procedure:
    - Single coded bit sequence mapped into multiple spatial (bit) streams in round-robin fashion
    - Each bit stream goes through interleaving, mod. symbol mapping, space-time coding and iFFT blocks to yield OFDM signal for transmission
  - No special layering processing performed

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MIMO in OFDM Systems (cont.)

- MIMO in LTE Release 8
  - There are 7 downlink "MIMO" modes in LTE Rel. 8
  - Only Modes 3 and 4 can support spatial multiplexing
  - The maximum number of Layers, i.e., the rank of the MIMO spatial multiplexing, is 4
  - The data is organized as resource blocks, which consist of multiple OFDM symbols and multiple subcarriers in each of the OFDM symbols
  - The block of coded bits of each code word are scrambled and mapped to modulation symbols. These modulation symbols of one or two codewords are mapped to one or more layers.
    - The modulation symbols in each layer are mapped to the resource elements – sequentially in frequency bins from one OFDM symbol to next.

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MIMO in OFDM Systems (cont.)

- MIMO in LTE Release 8 (cont.)
  – Precoding in LTE
    - To save feedback information on reverse link, code-books are used for close-loop precoding
    - LTE Codebooks:

Codebook index	Number of layers υ	
	1	2
0	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
1	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$
2	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\j\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1 & 1\\j & -j\end{bmatrix}$
3	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-j\end{bmatrix}$	-

  – TDD-LTE could use optimal precoding due to the link reciprocity

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ADDITIONAL DISCUSSION AND REMARKS

Further Discussion and Remarks

- The most important feature of MIMO technology is that it provides a practical mean to achieve spatial multiplexing, i.e. creates multiple transmission channels.
- With multiple transmission channel, the bits per channel is reduced for the same throughput
  - With fewer bits per channel, the system will keep in the “linear” channel capacity region
  - The SNR per channel is reduced when keeping a constant total transmission power
  - The advantage of statistical multiplexing is mainly in the high SNR/high spectral efficiency region
  - At low SNR region, there’s not much gains
- Practical MIMO system efficiency depends on if the channel is “rich scattering”

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Further Discussion and Remarks (cont.)

- When the number of transmitter and/or receiver antenna is more than the created channel, a MIMO system can also have diversity gain
  - Diversity gain can be obtained by different means and not unique to MIMO
  - MIMO can most effectively utilize the diversity gain together with obtaining statistical multiplexing as shown by Tse et. al.
- MIMO can also be used in multiuser communications (Multiuser MIMO) with the same principle
  - Communication paths are from Tx antenna arrays to antennas on multiple receivers
  - Multiple channels can be created as long as the channels from the Tx antennas to multiple Rx antennas are not correlated

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Further Discussion and Remarks (cont.)

- The MIMO transmitter can be implemented by simply distributing the coded data bits/symbols to multiple antennas
  - No special Tx data channel "layering" arrangement are necessary if so desired
- Precoding in a MIMO system can achieve channel capacity
  - The transmitter will need to know the channel state information (CSI)
  - The channel at the opposite direction is used to sent the CSI information and thus reduce its capacity, unless it is for a TDD system

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Further Discussion and Remarks (cont.)

- Various receiver structures are discussed
  - Linear receivers are easier to implement but the gaps between their performances and the optimum are relatively large
  - Maximum Likelihood (ML) receiver can achieve close to optimal performance
    - Brute force implementation of ML detector is practical impossible in practice except for simple cases
    - Simplified, mostly suboptimal, algorithms have been adopted from corresponding simplified method previously developed for maximum likelihood equalization and other types of receivers
    - QRD is most suitable for ML and approximate ML detectors
  - Lattice reduction methods was developed for improving linear receiver performance but not widely used today, due to its complexity and performance.

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