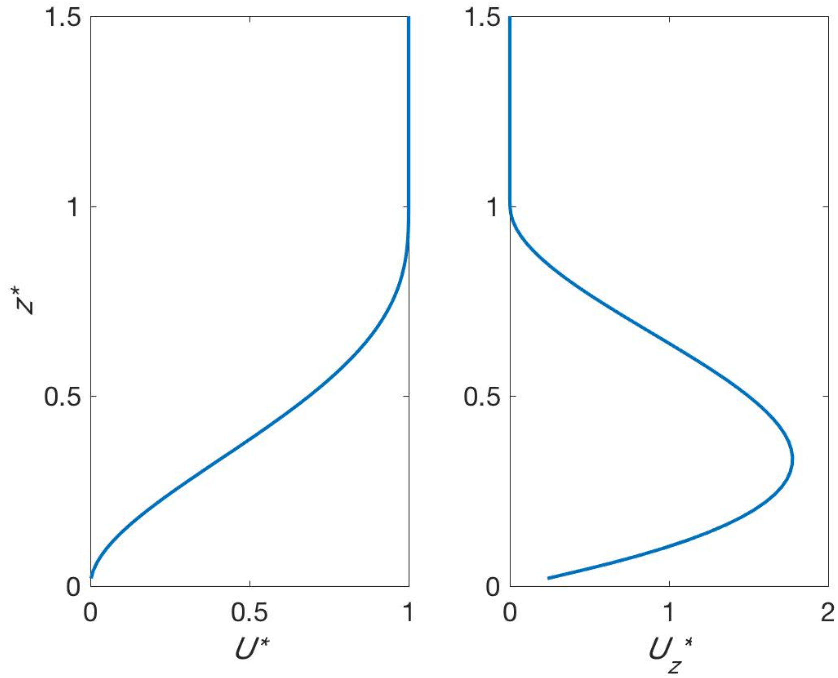

Figure 1: Profiles of (a) velocity and (b) shear for the separating boundary layer.

17: Instability of a separating boundary layer

A priori unstable? The profile has an inflection point which represents a shear maximum. Instability will occur if not prevented by viscosity, stratification (not a factor here) or boundaries.

Here is our script:

```
% OC680, 2018, script Hmwk6_proj1
% Separating BL problem
% W. Smyth, Feb. 2018
clear
close all
fs=18;

% define parameter values
Lz=3.0;
del=.02;
z=[del:del:Lz-del]';
N=length(z);
U=z.^2.*(6-8*z+3*z.^2);
U(z>1)=1;
ks=[0:.05:2.1];
Res=10.^linspace(6,1,30);
bc='rf';

%
```

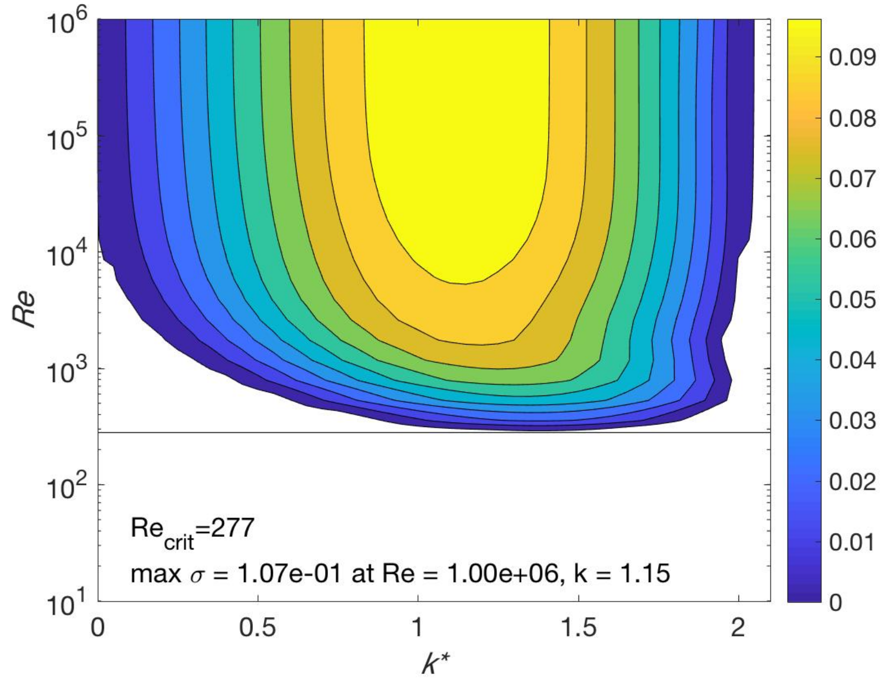

Figure 2: Growth rate versus wavenumber and Reynolds number for the separating boundary layer.

```
% plot background profile U*
figure
subplot(1,2,1)
plot(U,z,'linewidth',2)
xlabel('U*','fontsize',fs,'fontangle','italic')
ylabel('z*','fontsize',fs,'fontangle','italic')
ylim([0 1.5])
set(gca,'fontsize',fs-2)

subplot(1,2,2)
plot(ddz(z)*U,z,'linewidth',2)
xlabel('U_z*','fontsize',fs,'fontangle','italic')
ylim([0 1.5])
set(gca,'fontsize',fs-2)

%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% loop to compute sig(k,Re) (takes a while!)

% loop over k
nk=length(ks)
nRe=length(Res)
for i=1:nk
    k=ks(i);
```

```
    % loop over Re
    for j=1:nRe
        nu=1/Res(j);
        [s]=VSF(z,U,nu,k,0,bc);
        sig(i,j)=real(s(1));
    end
    disp([num2str(i/nk) ' done']) % track progress
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
% Find the critical Re by fitting a quadratic to sig(Re) and finding the
% zero.
[ms,is]=max(sig,[],1);
[mms,js]=max(ms);
k_mx=ks(is(js));
Re_mx=Res(js);
sss=max(real(sig));
jjj=(max(find(sss>max(sss)/1000)))
p=polyfit(log10(Res(jjj-2:jjj)),sss(jjj-2:jjj),2)
Re_root=10.^roots(p)
Re_err=abs(Re_root-Res(jjj));
Re_crit=Re_root(Re_err==min(Re_err))

% plot results
figure
contourf(ks,Res,sig',max(sig(:))*[0:.1:.9]);
hold on; plot(xlim,280*[1 1],'k')
set(gca,'yscale','log')
xlabel('k*','fontsize',fs+2,'fontangle','italic')
ylabel('Re','fontsize',fs+2,'fontangle','italic')
set(gca,'fontsize',fs-2)
%title('Growth rates for separating boundary layer','fontsize',16)
lab=sprintf('Re_{crit}=%.0f',Re_crit)
text(.05,.12,lab,'units','normal','fontsize',fs-2)
lab=sprintf('max \\sigma = %.2e at Re = %.2e, k = %.2f',mms,Re_mx,k_mx)
text(.05,.05,lab,'units','normal','fontsize',fs-2)
colorbar

return
```

If you use fine enough resolution in Re, the minimum value for instability is 280. It helps to interpolate, as shown in the code above. If you got a much smaller result, e.g. 140, you may have the upper and lower boundaries reversed in your 4th derivative matrix.

At the critical Reynolds number, the frozen flow approximation is absolutely not valid, since the growth rate is zero and therefore cannot possibly be "fast" in comparison to the viscous spreading of the background flow.

The growth rate decreases monotonically with decreasing Reynolds number, so this flow is stabilized by

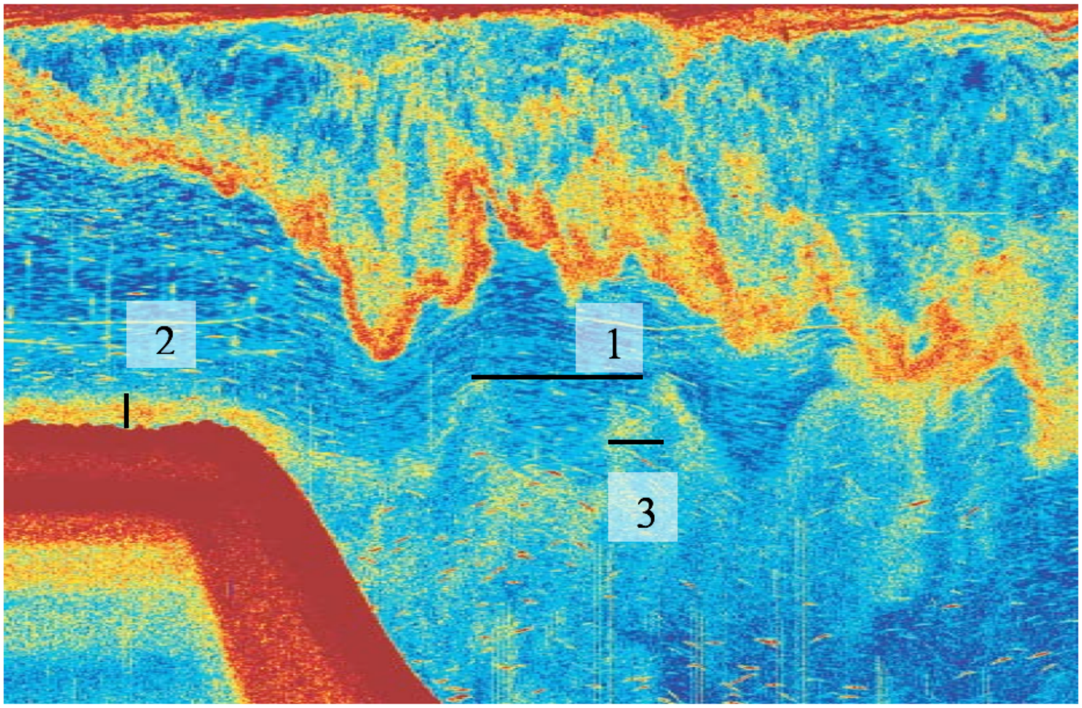

Figure 3: Echosounder image of instabilities observed in sill flow, from Armi & Farmer, 2002, PRSL. Superimposed are estimates of (1) the wavelength of a stratified shear flow instability, (2) the thickness of the separating boundary layer, and (3) the wavelength of what may be a separating boundary layer instability.

viscosity. In other words, the flow is most unstable in the inviscid limit. This indicates that the instability is driven by the inflection point mechanism that applies in inviscid flow (e.g. assignment #2). Viscosity merely modifies the instability by damping its growth rate. This also assures us, via Squire's theorem, that the 2D modes we have been investigating are indeed the most unstable.

The maximum growth rate at Re=10^6 growth rate is 0.107, which is about 0.06 times the maximum absolute shear.

The wavelength of the fastest growing mode is $2\pi/1.15 = 5.5$. The layer thickness, in this scaling, is just 1 (i.e. the length scale is the layer thickness), so the ratio of the wavelength to the layer thickness is 5.5. In the echosounder image, the wavelength of the dominant disturbance downstream of the separation point looks to me to be about 4 times the original thickness of the boundary layer (indicated by lines 1 and 2 on figure 3). Multiplying this by 4 to account for the aspect ratio of the image, I get 16, which is of the same order of magnitude as 6. Another way to make this comparison is to draw a line indicating the predicted wavelength (line 3), so that the reader can compare visually with the scales of the observed waves.

Trying to guess the velocity profile from an echosounder image is an extremely uncertain business, and this is about as close as I would expect to come. Ideally, one would look at ADCP velocity measurements and make a more careful estimate but, for an order of magnitude comparison, this will do.

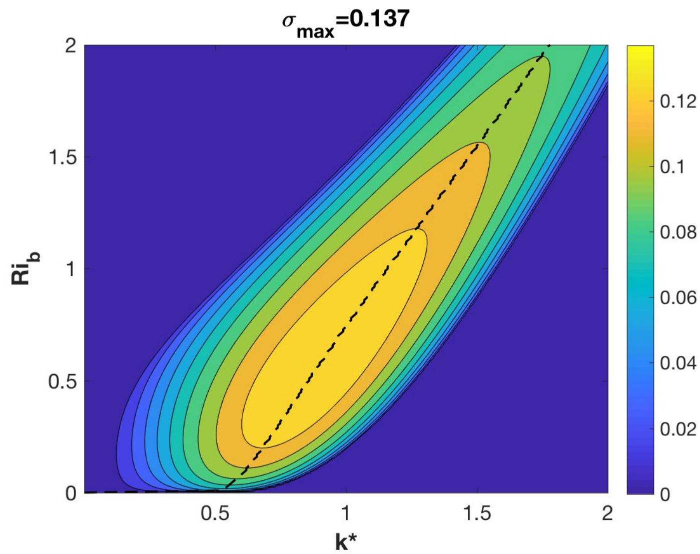

Figure 4: Growth rate versus wavenumber and bulk Richardson number for the half-Holmboe problem.

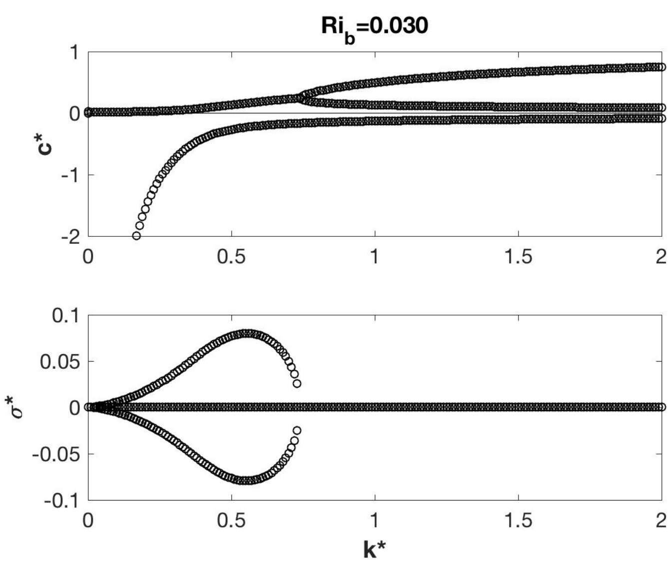

Figure 5: (a) Phase speed and (b) growth rate versus wavenumber for the half-Holmboe problem.