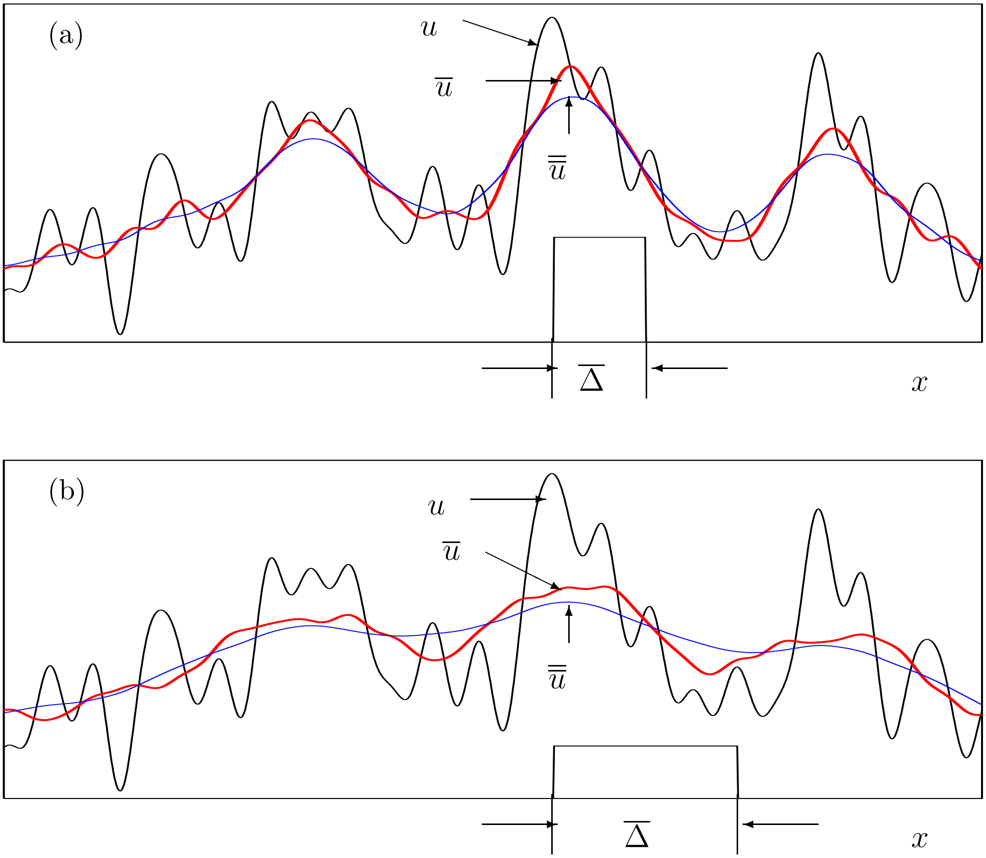

Figure 3: Filtered functions $\overline{u}$ and $\overline{\overline{u}}$ obtained from $u(x)$ by applying a box filter: (a) narrow filter, (b) wide filter.

$$\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial \left(\overline{u_i}\,\overline{u_j}\right)}{\partial x_j} + \frac{\partial \overline{\Pi}}{\partial x_i} = \frac{\partial (\nu\ 2\overline{S}_{ij})}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \tag{8}$$

where $\overline{S}_{ij}$ and $\overline{\Pi}$ are defined analogously to the unfiltered case. The term

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j \tag{9}$$

represents the impact of the unresolved velocity components on the resolved ones and has to be modelled. In mathematical terms it arises from the nonlinearity of the convection term, which does not commute with the linear filtering operation.

An important property of $\overline{u_i}$ is that it depends on time. Hence, an LES necessarily is an unsteady computation. Furthermore, $\overline{u_i}$ always depends on all three space-dimensions (except for very special cases). Symmetries of the boundary conditions generally produce the same symmetries for the RANS

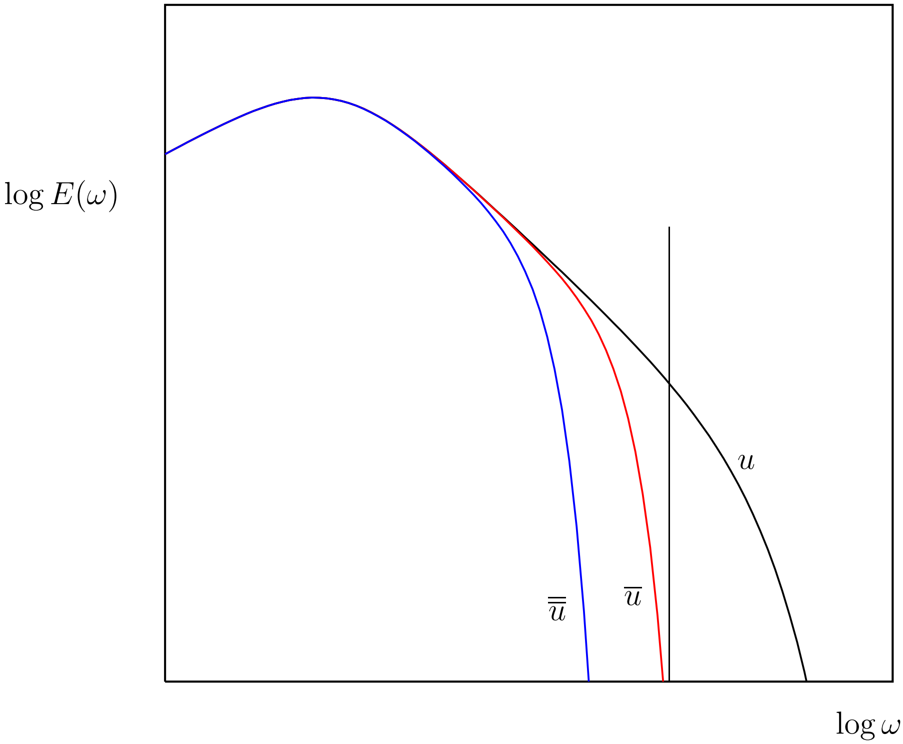

Figure 4: Effect of filtering on the spectrum. Here the box filter employed in Figure 3 is used as well, but the curves are similar for other filters such as the Gauss Filter. u' and $\overline{u'}$ are illustrated by the area between the curves for u and $\overline{u}$, and $\overline{u}$ and $\overline{\overline{u}}$, respectively. The vertical line is related to the Fourier cutoff filter on the same grid.

variable $\langle u_i \rangle$, e.g. vanishing dependence on a homogeneous direction. However, due to the very nature of turbulence, this does not hold for $\overline{u_i}$ since the instantaneous turbulent motion is always three-dimensional. The fact that a three-dimensional unsteady flow is to be computed makes LES a computationally demanding approach. We finally note that for any filter, the term in (9) vanishes in the limit $\overline{\Delta} \to 0$, since then $\overline{u} \to u$ according to (4), and all scales are resolved so that the LES turns into a DNS.

2.3 Variable filter size

It should be mentioned here that filtering as defined by (4) is not easily compatible with boundary conditions. For instance, applying a box filter of constant size $\overline{\Delta}$ yields $\overline{u} \neq 0$ within a distance $\overline{\Delta}/2$ from the computational domain and raises the question of how to impose boundary conditions for $\overline{u}$. This problem is removed by supposing G to be x-dependent and locally asymmetric. However, if $G(x - x')$ is generalized to some $G(x, x')$, or if the prolongation of u