Meaning - Paul Portner

SET THEORY

Background

Semanticists often make use of notions from set theory as they attempt to provide a model of meaning. Sets are collections of objects, for example: the set of all red things, the set of all dogs, the set of all things that either are blue or like pizza. We can name a set by indicating its members, either with a list using braces "{" and "}"

{Shelby, Hobo} = the set with two members, Shelby and Hobo.

or with a description. Often we use a special brace-notation for describing sets:

{x : x is red} = the set of all red things. We read this "the set of all x's such that x is red".

We indicate that something is a member (or "element") of a set with the symbol ∈. (Note that *Shelby* refers to my dog.)

Shelby∈{Shelby, Hobo}
Shelby∈{x : x is a dog}

Sets can stand in various simple relations to each other. For example:

a. Two sets are the same if they have the same members.

Example: {Shelby, Hobo} = {x : x is a dog owned by a member of my family}

b. Set A is a subset of set B if every member of A is a member of B.

The subset relation is indicated with the symbol ⊆.

Example: The set consisting of Shelby and Hobo is a subset of the set of dogs.
{Shelby, Hobo} ⊆ {x : x is a dog}

c. Two sets are disjoint if they don't have any members in common.

Example: {Shelby, Hobo} is disjoint from the set of cats.

Sets can be built out of other sets:

d. The intersection of two sets A and B is the set consisting of those things which are in A and which are also in B.

Intersection is indicated by the symbol $\cap$.

Example: {1, 2, 3, 4}$\cap${x : x is a number greater than 2} = {3, 4}.

> e. The union of two sets A and B is the set consisting of those things which are in A plus those things which are in B.

Union is indicated by the symbol $\cup$.

Example: {1, 3, 5, 7}$\cup${2, 3, 4, 5} = {1, 2, 3, 4, 5, 7}

There is a special set, the empty set, with no members. It can be written as {}.

Sets can have other sets as members. For example, the set {Hobo, {Shelby}} has two members. Only one is a dog, Hobo. The other is a set, {Shelby}. Therefore, in symbols we can write {Shelby}$\in${Hobo, {Shelby}} and Hobo$\in${Hobo, {Shelby}}. The following is not true: {Hobo}$\in${Hobo, {Shelby}}.

Exercises:

What sets are indicated by the following?

(i) {x : x is a member of the US Congress}$\cap${x : x is from New York}
(ii) {}$\cap${x : x is a dog}
(iii) {}$\cup${x : x is a dog}
(iv) ({x : x is a dog}$\cup${x : x is a cat})$\cap${x : x is from New York}
(Note the placement of parentheses, indicating how the sets are grouped.)
(v) {x : x is a dog}$\cup$({x : x is a cat}$\cap${x : x is from New York})

Are the following true or false?

(vi) {x : x is a fish}$\subseteq${x : x breathes water}
(vii) {x : x is a fish}$\cap${x : x breathes water}$\subseteq${x : x breathes water}
(viii) {x : x breathes water}$\subseteq${x : x is a fish}$\cap${x : x breathes water}
(ix) {}$\subseteq${Shelby, Hobo}
(x) Shelby$\subseteq${Shelby}
(xi) Shelby$\in${Shelby}
(xii) {x : x breathes water}$\in${y : {z : z is a fish}$\subseteq$y}

Using Set Theory in Semantics

Sets allow us to formalize many ideas in semantics. For example, in the discussion of modality in the text we saw that propositions can be thought of as sets of possible worlds. Thinking of propositions as sets of possible worlds also lets us use set theory to explain fundamental semantic relationships like synonymy and entailment. Let's look at entailment: *Shelby is a dog* entails *Shelby is an animal.* In terms of sets, this is so because the set of worlds in which *Shelby is a dog* is true is a subset of those in which *Shelby is an animal.* In symbols

$\{w : \text{Shelby is a dog in } w\} \subseteq \{w: \text{Shelby is an animal in } w\}$

Exercise: How can we define synonymy, tautology, and contradiction in terms of sets?

Sets also help us develop precise theories of compositional semantics. For example, we can easily define the semantics of conjunction, predication, intersective modification, and quantification.

Conjunction

Let A be the set of worlds in which *Shelby is a dog* is true, and let B be the set of worlds in which *Teetoo is a cat* is true. We can combine these two sentences with *and*: *Shelby is a dog and teetoo is a cat.* This sentence is true in the worlds $A \cap B$.

In general, suppose we have a sentence S_1 which is true in the set of worlds A and another sentence S_2 which is true in the set of worlds B. Then "S_1 and S_2" is true in the worlds $A \cap B$.

Exercise: Give a similar definition for sentences formed with *or*.

Predication:

Mary refers to Mary and *is tall* describes the set $\{x : x \text{ is tall}\}$. In the sentence *Mary is tall*, we predicate *is tall* of *Mary.* This creates a true sentence if and only if $\text{Mary} \in \{x : x \text{ is tall}\}$.

In general, suppose we have a referring noun phrase NP which refers to some individual A, and we have some predicate VP which describes some set B. Then the sentence "NP VP" is true if and only if $A \in B$.

Intersective modification:

Tall describes the set of tall things and *boy* describes the set of boys. The phrase *tall boy* describes the set $\{x: x \text{ is tall}\} \cap \{x: x \text{ is a boy}\}$.

In general, suppose we have an adjective ADJ which describes a set A and a noun which describes a set B. Then the phrase "ADJ N" describes the set $A \cap B$.

Quantification:

Dog describes the set of dogs and *barks* describes the set of things that bark. We can build a sentence out of these two words plus a quantificational determiner like *every*. The sentence *Every dog barks* is true if and only if the set of dogs is a subset of the set of things that bark.

> In general, suppose we have a restrictor R which describes a set A and a scope S which describes a set B. Then the sentence *Every R S* is true if and only if $R \subseteq S$.
>
> Exercise: explain the meanings of sentences with the following determiners in terms similar to what is done above with *every*: *no, some, three, many.*

What's the point of this?

Many semanticists, in particular formal semanticists, think that studying semantics with sets and other (more difficult) logical and mathematical tools leads to a better understanding of meaning than would be possible without such tools. There are two types of arguments which can be made for this opinion.

First, we have a methodological argument: using set theory and similar tools forces us to be very precise in thinking and building theories about meaning, and precision is always better in science. Of course it doesn't follow that a linguist who doesn't use set theory won't think and theorize in a precise way, but since fuzzy thinking is a hazard for us all, it's better to use techniques which force us to be precise.

And second, we have a linguistic argument: set theory and similar tools are helpful to our understanding of semantics because they reveal deep and important aspects of how the human mind works with meaning. For example, such notions as subset and intersection come up again and again as we try to explain certain aspects of semantics (like conjunction, entailment, and quantification). Perhaps this is so because the human mind really conceives of meaning in terms of sets and uses the set-theory concepts in the process of understanding and producing language.

What do you think of these two arguments? Do you think that the following kinds of people would find attractive and plausible the idea that linguistic meaning should be explained in terms of simple and precise set-theory concepts: A computer programmer? A psychologist? An author of poetry? A sociolinguist? A lawyer?

The meaning of "is"

In 1998, American president Bill Clinton was involved in a scandal about his relationship with an intern, Monica Lewinsky, and whether he lied about this relationship. He had said "There's nothing going on between us", but when it was later made plain that there had been a sexual relationship, he argued that he had been truthful the following way:

> "It depends on what the meaning of the word 'is' is. If the--if he--if 'is' means is and never has been, that is not--that is one thing. If it means there is none, that was a completely true statement....Now, if someone had asked me on that day, are you having any kind of sexual relations with Ms. Lewinsky, that is, asked me a

> question in the present tense, I would have said no. And it would have been completely true."

Clinton was arguing that the present tense of "is" implies that he was only making a claim about the time at which he said "There's nothing going on between us". Since he was not involved in a relationship with her at the time he said "There's nothing going on between us", he told the truth. The fact that there had been a relationship previously was, according to him, irrelevant.

Questions:

(i) Is Clinton's argument correct – had he literally told the truth?

(ii) Assuming that Clinton had indeed told the truth, why did people feel he was being deceptive?

(iii) Using concepts of pragmatics you've learned about, explain the nature of his deceptiveness.

(iv) If he was truthful but deceptive, did he lie? In other words, does the concept of lying have to do with literal truth vs. falsity or with pragmatic straightforwardness vs. deceptiveness? Use other examples of what is or is not a lie to back up your answer.

(v) Should people get in legal trouble for this sort of thing?

(vi) Do politicians deserve to lose support for this sort of thing?

(The factual description of these events and quotes are taken from Timothy Noah, Slate Magazine, Sept. 13, 1998, http://www.slate.com/id/1000162/.)