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Study questions are provided below for the first 13 chapters of the text Gravity and Magnetic Exploration: Principles, Practices, and Applications. These questions serve as a study guide for the reader and a resource for instructors.

Figure and table numbers cited in the Study Questions refer to those in the text except for figures identified with bold italicized labels (e.g., **Figure** 1.20), which refer to the figures in these Study Questions.

## 1.1 Introduction

1.1) The fields of the Earth that are exploited in gravity and magnetic exploration are referred to as planetary fields. (a) What is the origin of the use of the term planetary? (b) What are its implications to geophysical exploration?

1.2) Most gravity and magnetic exploration of the Earth is confined to investigations of the lithosphere. (a) What is the lithosphere and how does its thickness vary over the Earth? (b) What is the major factor in establishing the thickness of the lithosphere?

1.3) (a) How does Earth's density vary from its surface to its core? (b) What factors determine the density variations with depth in the Earth?

1.4) The range of density variations is significantly less than the range of magnetic polarization variations in the lithosphere. What is the significance of this difference to the associated gravity and magnetic anomalies?

1.5) The gravity method involves investigation of small variations in the

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total gravity field of the Earth caused by mass changes in the lithosphere. Assuming that the gravity field of the Earth at the surface of the Earth is  $9.81 \text{ m/s}^2$ , what percentage of this field is typically involved in changes due to mass variations in the lithosphere?

1.6) (a) What is the significance of the dipolar nature of magnetism in contrast to the monopolar gravity field in the study of the subsurface Earth by the respective fields? (b) How does this affect the change in amplitude of each field with distance? (c) What are the implications of this difference on the choice of methods of exploration?

1.7) (a) Explain the source of the negative magnetic anomaly associated with the positive magnetic anomaly observed over a positively magnetized subsurface source shown in Figure 1.3 of the text. (b) Assuming that the subsurface source in Figure 1.3 is less magnetic than the surrounding Earth, redraw (b) the vectorial relationships shown in Figure 1.3, and (c) the total field magnetic anomaly. (d) How would the anomaly change if the ambient magnetic field is vertical rather than inclined as shown in Figure 1.3? (e) How would the gravity anomaly differ from the magnetic anomaly shown in Figure 1.3 assuming the anomalous source has a greater density than that of the surrounding rocks?

1.8) Contrast the early history of the gravity and magnetic methods in terms of planetary versus exploration investigations.

1.9) Discuss the principal components of the phases that are essential in the application of (a) the gravity method and (b) the magnetic method to exploration of the Earth.

1.10) Errors which are inherent in geophysical data propagate as various mathematical processes are performed. For a set of 17 observations with the mean value 2.06 and standard deviation 0.32 and another set of 39 observations with a mean of 4.54 and standard deviation of 0.26, calculate the (a) sum, (b) difference, (c) product, and (d) quotient of these data including their combined standard deviation.

#### 1.2 The Gravity Method

2.1) The gravity method, which measures the variations in the acceleration of gravity caused by horizontal changes in the density, and thus the mass

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differential of the subsurface Earth, is often used in the early stages of an exploration program followed by the use of methods with higher, more diagnostic power, in resolving the nature of the subsurface. (a) Why is this the case? (b) Give examples of the role of the gravity method in this regard?

2.2) (a) What is the difference between weight and mass? (b) When are they equivalent? (c) What are their respective units in the SIu system?

2.3) (a) What is the basis for Newton's universal law of gravitation? (b) Why is this law referred to as universal?

2.4) Gravity force is always positive; however, in geophysical exploration many gravity anomalies are negative such as those associated with salt diapirs in the Gulf Coast of the US. Explain.

2.5) A gravity anomaly is determined to be 10 mGal. What is its value in (a) cm/s<sup>2</sup>, (b) m/s<sup>2</sup>, (c)  $\mu$ Gal, and (d) the gravity unit?

2.6) (a) What is the geoid? (b) How does it differ from the ellipsoid that best represents the surface of the Earth? (c) What is the correlation of the geoidal undulations with both surface and subsurface features of the Earth?

2.7) Is the geoid a surface of equal gravitational acceleration? Explain.

2.8) The International Gravity Formula gives the gravity of the Earth on what surface?

2.9) The mean equatorial radius of the Earth is 6378.4 km and the polar radius is 6356.9 km. As a result, (a) what is the difference in normal gravity between the equator and the pole? (b) Why does this difference in radius not account for the total observed gravitational acceleration of the Earth between the equator and the pole? Explain.

2.10) The normal change in gravity on the best-fitting Earth ellipsoid varies with latitude, but not longitude. Explain.

2.11) (a) Calculate the difference between centrifugal acceleration on the surface of the Earth at the equator and at the pole assuming the equatorial radius and polar radius are respectively 6378.4 and 6356.9 km. (b) What percentage in the change in gravity at sea level between the equator and

pole is accounted for by the variation in centrifugal force?

2.12) In addition to spatial variations in gravity over the Earth, temporal changes occur. What are their (a) sources and (b) percentage of the total field?

2.13) Relative rather than absolute gravity measurements are the norm in gravity exploration. Why is this the case?

2.14) Artificial satellites of the Earth are in a position of dynamic gravitational equilibrium between the gravitational force of the Earth and centrifugal force of the orbiting satellite. Despite this fact, gravity measurements have a significant role in mapping the gravity field of the Earth. Explain this paradox.

2.15) How does the gravity on the surface of the Moon compare with that on the Earth considering that the Moon's radius and mass are roughly a one-quarter and one-eightieth of those of the Earth, respectively.

2.16) What was the significance to gravitational studies of Cavendish's relatively accurate measurement of the universal gravitational constant in the latter part of the eighteenth century? Explain.

2.17) The fundamental concepts of isostasy originated in India. Explain why?

2.18) Airborne gravity and gradiometer measurements of sufficient accuracy for geophysical exploration were slow to develop in comparison with static land gravity surveys. Explain why.

2.19) Gravity measurements to map subsurface geology require observations over an area. Why is this so?

2.20) Gravity measurements require an extensive data processing procedure to map and isolate specific gravity anomalies. Explain the reasons for this.

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## 1.3 Gravity Potential Theory

3.1) The gravity method is commonly referred to as a potential field method. What are the (a) origin and (b) implications of that term?

3.2) (a) What is gravitational potential? What is its relationship to and how does it differ from (b) gravitational force, (c) gravitational acceleration, and (d) work? (e) What are the units of gravitational potential?

3.3) Discuss the difference between and applications to geophysical exploration of (a) gravity force, (b) vectors, and (c) tensors?

3.4) What is the difference between (a) Newtonian potential and (b) logarithmic potential?

3.5) The gravitational potential on the sea-level surface is uniform - i.e., it is an equipotential surface; however, sea-level is not a surface of uniform gravitational force. Explain.

3.6) Why is it desirable to compute the gravitational potential due to a mass source rather than the gravitational force?

3.7) Show that the gravitational force is inversely proportional to the distance between the corresponding equipotential surfaces.

3.8) Gravitational force is a conservative field. What are the implications of this characteristic for the gravitational force and equipotential surfaces?

3.9) What assumption must be made regarding the potential at an infinite distance from the source of the gravitational potential and why must this assumption be made?

3.10) The potential of **F** is  $U = (x^2 + y^2)^{-1}$ . (a) Find **F**. Describe the (b) field lines and (c) equipotential surfaces of **F**. (d) Demonstrate by integrating around the perimeter of a rectangle in the (x, y)-plane that **F** is conservative.

3.11) Specifically, which components of the gravitational field are measured by (a) a gravimeter and (b) a gravity gradiometer? (c) How are the tensor components of the gravitational field measured and what are their

units?

3.12) What are the advantages and disadvantages of measuring and using gravitational (a) force vectors and (b) tensors in the interpretation of subsurface masses?

3.13) Prepare a cross-section of the gravity anomaly measured by a gravimeter, as well as the horizontal and vertical vectors and their tensors, along a principal profile of a horizontal cylindrical excess mass extending to infinity in the y direction.

3.14) (a) Calculate the gravity anomaly profile over a vertical cylinder with depth (z) to the horizontal top [see sources #5 and #6 in Figure 3.8] of 100 m, radius of 50 m, density contrast is 500 kg/m<sup>3</sup>, and infinite depth extent using the source #6 equation of Table 3.2. Calculate the anomaly out to horizontal distance from the axis of the cylinder where the anomaly value is roughly 10% of the maximum magnitude, using the appropriate equations given in Table 3.2. (b) Compare this result with the results from a vertical line mass with the same specifications.

3.15) Repeat Study Question #3.14, but limit the depth extent to 1 km and compare the results.

3.16) By integrating the vertical component of the gravity effects, find the gravity potentials for the (a) sphere (#2), (b) horizontal cylinder (#4), and (c) vertical cylinder (#6.A) in Table 3.2, as well as the (d) vertical sheet (#11), (e) small-offset vertical fault (#13), and (f) large-offset vertical fault (#14) in Table 3.3.

3.17) (a) Derive the gravity anomaly effect of a horizontal slab of material of thickness ( $\Delta t$ ) with a density contrast ( $\Delta \sigma$ ) at a position on top of the slab and at an elevation  $10 \times \Delta t$ . (b) Discuss the results and their implications to gravity anomaly calculations and interpretation.

3.18) (a) Using Gauss' law (Equation 3.98) derive the expression for the total mass of a subsurface mass that has a density contrast ( $\Delta \sigma$ ) with the surrounding rock from the gravity anomaly of the source. (b) What are the limitations in the use of this expression in actual practice?

3.19) Consider the gravitational field  $\mathbf{F}_{\mathbf{g}}$  due to a unit point mass m.

Let v be any volume containing the mass and  $\hat{\mathbf{e}}_{\mathbf{n}}$  be the unit vector normal at each point on the surface s that bounds the volume v. (a) Evaluate (showing all steps) the surface integral  $N = \int_{s} \int (\hat{\mathbf{e}}_{\mathbf{n}} \cdot \mathbf{F}_{\mathbf{g}}) \partial s$ , where N is the number of lines of force passing out through the surface s (Figure 3.16) i.e., N is the flux of  $\mathbf{F}_{\mathbf{g}}$  through s. (b) What is the significance of this result?

3.20) How does Gauss' law lead to Poisson's and Laplace's equations and to the conclusion that gravity anomalies are inherently ambiguous?

3.21) For a limestone formation of constant thickness within a sequence of horizontal sandstones and shales, draw schematically the gravity anomaly profile over this limestone layer if (a) it extends over a very large horizontal distance and is offset by (b) a vertical fault, (c) a normal fault that dips at  $45^{\circ}$ , and (d) a strike-slip fault.

3.22) Repeat Study Question #3.21 assuming the sedimentary formations including the limestone unit dip from left to right at  $20^{\circ}$  from the horizontal.

3.23) (a) Derive Poisson's theorem that relates the magnetic potential, V(r), of an anomaly source of constant magnetization,  $(\Delta J)$ , and density,  $(\Delta \sigma)$ , to its gravity potential. (b) What are the assumptions of this theorem are necessary to satisfy this theorem? (c) What are the implications of this theorem for the relationship between gravity and magnetic anomalies derived from the same subsurface source?

3.24) (a) What is the gravitational potential at a point a distance z above a homogeneous disk on the axis of the disk? Assume a surface density of  $\Delta\sigma$ ? (b) What is the gravitational acceleration in the direction of the axis of the disk due to the disk at the same point?

3.25) The mass of Mars is approximately 10.8% of the Earth's mass and its radius is 3,394 km. What is the gravity (a) at the surface of Mars and (b) at a satellite altitude of 300 km?

3.26) Consider a mountain that can be represented by a sphere of radius R with mass m half submerged in an Earth whose density  $\sigma$  is twice the density of the mountain. Assume that R is much less than the Earth's radius so that the Earth's surface can be regarded as flat in the neighborhood of the mountain. If the mountain were not present, the gravitational field intensity

near the Earth's surface would be  $g_o$ . (a) Show that the difference between  $g_o$  and the actual value g at the top of the mountain is  $[(mG/R^2)(2-\sqrt{2})]$  where  $m = (4/3)[\pi\sigma]R^3$ . (b) If the top of the mountain is eroded to a surface parallel to the Earth's surface in a short time compared with the time required for the mountain to achieve equilibrium again, show that the difference between  $g_o$  and the actual value g at the Earth's surface at the center of the eroded mountain is  $[(-3/4)(mG/R^2)]$ .

3.27) For the spherical shell, verify that its gravitational (a) potential and (b) intensity are the same as if the shell's mass were concentrated at its center. (c) How does gravity change with depth from the Earth's surface to its center?

3.28) Converting the potential into generalized coordinates facilitates applying the potential in any orthogonal system of curvilinear coordinates [e.g. Cartesian, polar, parabolic, elliptic, cylindrical, spherical, oblate spheroidal, prolate spheroidal, ellipsoidal, toroidal, etc.]. For example the generalized coordinates  $(q_1, q_2, q_3)$  may be defined in terms of Cartesian coordinates (x, y, z) by proportionality factors  $(h_1, h_2, h_3)$  as

$$h_1^2 = \left(\frac{\partial x}{\partial q_1}\right)^2 + \left(\frac{\partial y}{\partial q_1}\right)^2 + \left(\frac{\partial z}{\partial q_1}\right)^2 \tag{1.1}$$

and similarly for  $h_2$  and  $h_3$ . Accordingly, the elements of length in generalized coordinates are

$$\partial x = h_1 \partial q_1, \ \partial y = h_2 \partial q_2, \ \text{and} \ \partial z = h_3 \partial q_3.$$
 (1.2)

Similarly, the elements of area in generalized coordinates are

$$\partial y \partial z = h_2 h_3 \partial q_2 \partial q_3, \ \partial x \partial z = h_1 h_3 \partial q_1 \partial q_3, \text{ and } \partial x \partial y = h_1 h_2 \partial q_1 \partial q_2,$$
(1.3)

and the elemental volume is

$$\partial v = h_1 h_2 h_3 \partial q_1 \partial q_2 \partial q_3. \tag{1.4}$$

Using the above results, show that the gravity field components in generalized coordinates are given by (a)  $F_{q_1} = \frac{1}{h_1} \left( \frac{\partial U}{\partial q_1} \right)$ , (b)  $F_{q_2} = \frac{1}{h_2} \left( \frac{\partial U}{\partial q_2} \right)$ , and (c)  $F_{q_3} = \frac{1}{h_3} \left( \frac{\partial U}{\partial q_3} \right)$ , and (d) the Laplacian of U is

$$\nabla^2 U = \frac{1}{h_1 h_2 h_3} \left[ \frac{\partial}{\partial q_1} \left( \frac{h_2 h_3}{h_1} \frac{\partial U}{\partial q_1} \right) + \frac{\partial}{\partial q_2} \left( \frac{h_1 h_3}{h_2} \frac{\partial U}{\partial q_2} \right) + \frac{\partial}{\partial q_3} \left( \frac{h_1 h_2}{h_3} \frac{\partial U}{\partial q_3} \right) \right].$$
(1.5)

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Express the gravity effects in generalized coordinates of the (e) sphere (#2), (f) horizontal cylinder (#4), and (g) vertical cylinder (#6.A) in Table 3.2.

## 1.4 Density of Earth Materials

4.1) What are the major factors that control the densities of (a) igneous, (b) metamorphic, and (c) sedimentary rocks and sediments?

4.2) Is density diagnostic of most rock types? Explain.

4.3) What are the common limitations to using densities from published tabulations?

4.4) The Adams-Williamson Equation (Equation 4.7) is useful in determining the change in density with depth in the Earth, but it has several limitations. Explain the nature of these limitations and their effect on estimating densities.

4.5) The value of the gravitational acceleration on the Earth is roughly  $9.81 \text{ m/s}^2$  and the mean radius is approximately 6,371 km. Therefore, (a) what is the mean density of the Earth and how does this compare with the density of surface materials of the Earth and that of the Earth's lithosphere? (b) What are the implications of this result?

4.6) Draw a profile of the density of the lithosphere in the continents from the surface to the asthenosphere, including an envelope of the range of densities over the depth profile.

4.7) (a) How do the densities of crystalline rocks change with depth from the surface of the Earth and (b) what is the approximate depth at which densities approach constant values due to increasing lithostatic pressure? (c) What factors cause this depth to vary?

4.8) (a) Contrast the densities of the lithosphere in the oceans and continents and (b) discuss the factors that are significant in causing the differences between these regions.

4.9) A density of 2,670 kg/m<sup>3</sup> is traditionally used as a mean value of the rocks of the continents above sea level. (a) What is the basis of this value

and (b) what is its validity?

4.10) Densities of metamorphosed rocks show a great variability, but generally increase in density with increasing metamorphic grade. What factors are involved in this relationship?

4.11) The increase in density of sedimentary rocks with depth varies considerably among sedimentary basins. Explain the cause of these variations.

4.12) Rocks change in density slowly with temperature. (a) Why is this so and (b) under what geologic conditions is temperature likely to be a significant factor in causing density variations?

4.13) Discuss the advantages and limitations of the various methods that are used to determine the density of rocks.

4.14) (a) Explain how gravity measurements can be used to determine the density of surface rocks. (b) What are the advantages and disadvantages of these methods over other techniques for measuring density?

4.15) What are the major limitations to measuring the density of rocks adjacent to drill holes that are surveyed with drillhole gravimeters?

4.16) Sedimentary rock densities used in petroleum exploration are commonly determined from seismic *p*-wave velocities obtained from seismic reflection studies. However, the relationship between laboratory-measured velocity and density shows a dispersion even within the rocks of a sedimentary basin. What are likely causes of this variability?

4.17) A horizontal sandstone formation serves as an aquifer. The analysis of samples from the formation indicates that it has a grain density of 2,650 kg/m<sup>3</sup> and an average porosity of 15%. What is the density of the formation (a) when it is water saturated and (b) when it is dry? (c) What is the minimum thickness of the aquifer, assuming the formation covers an extensive area and is bounded by a shale formation that has a density of 2,200 kg/m<sup>3</sup> if a 1.0 mGal gravity anomaly is observed over the margin of the aquifer?

4.18) A gravity survey conducted in a region with rolling surface topography shows a positive correlation between the hills of the region and minor positive gravity anomalies. What are the possible sources of these positive



Figure 1.1 Bouguer gravity anomaly profiles (solid lines) over a local topographic feature (dashed line) reduced for the feature's gravity effects using 10 values of topographic density ( $\sigma$  in g/cm<sup>3</sup>). See **Study Question 4.20**.

anomalies and how would you determine the likely the cause of this positive correlation?

4.19) Adjacent gravity surveys with a common straight boundary have used different surface densities in correcting for the mass variations in their similar surface topography. What is the manifestation in the combined Bouguer gravity anomaly map of these surveys and what is the likely interpretation of this manifestation if the density difference in the calculation of the gravity anomalies is not recognized?

4.20) Consider the gravity observations (open circles) collected along a profile across the topography depicted in *Figure 1.1*. The Bouguer gravity anomaly effects of the topography were computed using different values of density  $\sigma$ . From these results shown in the upper portion of *Figure 1.1*, (a) what is the density of the topographic feature and why? What do the reduced profiles indicate about the related topographic corrections that are (b) positively and (c) negatively correlated with the topography?

4.21) A drillhole gravimeter is to be used to measure the density of a series

of horizontal sedimentary formations to a precision of  $5 \text{ kg/m}^3$ . Assuming that the gravity observations can be made to a precision of 0.005 mGal, what is the minimum thickness formation that can be measured?

4.22) Is it possible to control the range of investigation of density variations from a drill hole that is surveyed with a drillhole gravimeter? Explain.

4.23) Explain the physical principle of the gamma-gamma density logging device? What factors control the accuracy of these measurements?

#### 1.5 Gravity Data Acquisition

5.1) Accuracy and the horizontal spatial resolution of gravity anomalies are both important components to be considered in the acquisition of gravity. (a) What is the normal range in the accuracy of gravity measurements for geophysical exploration and (b) how is the accuracy requirement determined for a specific survey? (c) How is the accuracy of measurements related to the resolution in both static and dynamic gravity observations?

5.2) (a) Why are shipborne and airborne gravity measurements inherently less accurate than a static (land) measurement, and (b) why are satellite measurements even less accurate? (c) Furthermore, why is the resolution of airborne measurements less than that of land or shipborne measurements?

5.3) Specifically, (a) what is measured by a gravimeter and (b) what is the relationship of these measurements to the Earth's ellipsoid and geoid?

5.4) Why are relative gravity measurements made with a gravimeter (gravity meter) inherently more accurate than absolute gravity measurements?

5.5) Most gravimeters measure gravity indirectly by observing the change in the strain of a spring with the change in weight of a constant mass. This strain can be amplified in several ways so that an accurate observation can be attained. How is this achieved in most modern gravimeters? Explain its operation.

5.6) Currently, most gravimeters utilize the zero-length spring concept to achieve high sensitivity. Explain (a) what a zero-length spring is, (b) how it

is constructed, and (c) how it achieves high sensitivity?

5.7) Zero-length spring gravimeters use either a quartz or a metal spring. Explain the pros and cons of these two spring types in gravimeters.

5.8) The torque needed to bring a mass to a null position in a gravimeter must be calibrated in terms of gravitational acceleration. What procedures are used to calibrate the gravimeter? Explain.

5.9) Gravimeters are subject to error due to ambient temperature and atmospheric pressure variations. Explain how these effects are mitigated.

5.10) The high sensitivity required in many exploration gravity surveys leads to the need to consider a broad range of possible error sources inherent to gravity instrumentation, as well as those derived from external environmental variations. Identify these and explain their significance to gravity data acquisition.

5.11) Explain the principle and advantages of the vibrating spring gravimeter.

5.12) Gravimeters are routinely used in the dynamic environments of ships and aircraft despite problems with extraneous horizontal and vertical accelerations and the difficulty of keeping the gravimeter vertical, all of which decrease the observation accuracy. Explain how these problems are minimized and how extraneous accelerations are separated from the static gravity due to subsurface variations in mass.

5.13) The electromagnetic accelerometer is one of the more important methods of measuring gravity in the dynamic environment of ships and aircraft. Explain the (a) principle and (b) the attributes of this type of gravimeter.

5.14) A significant factor compromising the accuracy of moving gravity measurements is the effect of the changing Coriolis acceleration that is the centrifugal force due to the horizontal movement of the observation platform over the rotating Earth's surface, which is referred to as the Eötvös effect. Explain this effect and its importance to achieving high accuracy dynamic gravity measurements. What controls the sign of this effect and why?

5.15) (a) What are tares in gravity measurements? (b) What are they caused by? (c) How is their effect overcome?

5.16) The measurement of gravity gradients and their associated tensors in both static and dynamic modes are of considerable interest in exploration. (a) How are these measurements made? (b) Explain their usefulness in gravity interpretation?

5.17) Absolute gravity measurements using free-fall methodology have specialized uses in exploration geophysics. Explain their application.

5.18) Global positioning satellite (GPS) systems have had a profound impact in increasing the efficiency and accuracy of gravity acquisition. (a) Explain how and why GPS has had this effect. (b) Vertical positioning with GPS provides the altitude with respect to what datum? (c) What is the significance of this to the reduction of gravity data?

5.19) Both passive and active measurements are made of the Earth's gravity field are made from Earth-orbiting satellites. Explain (a) the principles of these methods and (b) their application and use in studying the subsurface Earth.

5.20) Temporal variations in the Earth's gravity field are receiving increasing attention for mapping local and regional changes in fluid distribution and movement within and on the Earth. What special concerns are associated with the successful use of gravity for these applications?

5.21) (a) What factors should be considered in designing land gravity surveys and (b) how do these factors affect the results of the survey?

5.22) Station or observation interval is one of the most important components of gravity survey design. (a) What considerations are involved in selecting this interval and (b) what are the trade-offs between station spacing and the accuracy of the observations in dynamic (marine and airborne) surveys?

5.23) A gravity investigation has been proposed that will search for subsurface structures which can be approximated by two-dimensional horizontal cylinders. The orientation of these structures in this area is uncertain. The center line of the approximating cylinders will be no deeper than 150 m, their radius will be no smaller than 30 m, the length will be no less than 1,000 m, and the density contrast between the structures and the surrounding geological formations is no less than 100 kg/m<sup>3</sup>. Design a field observation program to map these structures.

5.24) Topographic maps are unavailable for a region to be investigated with a gravity survey, but estimates of the expected terrains corrections do not exceed 0.03 mGal. The surface material is believed to have a density of 1,800 kg/m<sup>3</sup>. The bedrock topography is irregular giving a maximum positive anomaly of 0.04 mGal and a maximum width of 150 m. The geologic strike in this area, which measures 8,000 m east/west and 7,000 m north/south is uncertain. The latitude effect in this area is 0.0001 mGal/m. The regional gravity anomaly, on the basis of extrapolation from adjacent surveys, is believed to approximate east/west striking sine waves with a period of 1,500 m and a total amplitude of 1.0 mGal. The gravimeter available for the survey has a sensitivity of 0.01 mGal. Set up a complete field program for identifying the presence of the above-mentioned structures and discuss the accuracy with which the survey reductions must be made.

5.25) Find the length of a simple pendulum whose period is exactly 1 sec at a point where g = 32.2 ft/s<sup>2</sup>.

5.26) (a) What is the change  $\partial T$  in the period of a simple pendulum when the acceleration of gravity changes by  $\partial g$ ? (b) What is the fractional change in period  $\partial T/T$  in terms of the fractional change  $\partial g/g$ ? (c) A pendulum clock that keeps correct time at a point where g = 980 gals is found to lose 10 sec/day at a higher altitude. Use the result of part (b)-above to find the value of g at the new location approximating the differentials  $\partial T$  and  $\partial g$  by small finite changes in T and g.

5.27) Pendulum observations in gravity exploration usually involve relative measurements. The period of the pendulum  $T_b$  is measured at a base station of known gravity  $g_b$ . The period at a new location  $T_n$  is then measured. (a) What is the acceleration of gravity  $(g_n)$  at the new location? (b) What is the equivalent period of this system?

5.28) Suppose that a gravimeter is built using a simple weight on a spring. If it is decided that a change in spring extension can be measured to 0.001 mm and that variations in g need to be measured to 0.1 mGal, what is the

required total elongation of the spring?

5.29) A falling body measurement of absolute gravity is made. Suppose the time is measured to within  $\pm 10^{-5}$  sec when the body has fallen through a distance of  $10 \pm 10^{-5}$  m. Estimate the error in gravity.

#### 1.6 Gravity Data Processing

6.1) Gravimeters are subject to time variations or drift. (a) What is the origin of drift, (b) how it is determined, and (c) removed from gravity observations?

6.2) How do atmospheric pressure changes affect gravimeter observations and what is the order of magnitude of this effect?

6.3) (a) What are the origin and magnitude of Earth tides? (b) Why are tides amplified over the direct calculation of the gravity effect of the Moon and Sun at their specific locations?

6.4) Calculated and observed Earth tides may vary in both phase and amplitude. Explain potential sources of this variation.

6.5) Why is it important that care be used in selecting the geographic and vertical datums to which a gravity observation site is referenced?

6.6) (a) To what datum is the International Gravity Formula referenced? (b) How does this differ from the datum generally used in determining elevations of gravity stations? (c) What are the implications of the difference between these datums to gravity measurements?

6.7) Assuming that the mean radius of the Earth is 6,371 km, (a) calculate the percentage change in gravity as observed at sea level and at an altitude of 1 km in the Earth's atmosphere above sea level. (b) What is the percentage change, over this altitude range, in the centrifugal force counteracting the gravitational force change? (c) Is the change in gravitational force and centrifugal force with altitude a linear or exponential relationship? Explain.

6.8) In surveys of limited north-south extent, the normal gravity at an observation site is commonly not determined from the latitude of the sta-

tion, but rather from a constant value for the rate of change of gravity with north-south distance. Explain under what conditions this approximation is permissible and provide an illustration of its use.

6.9) Variations in the atmospheric mass effect on the gravity of a station may be important in precision surveys over a broad range of elevations. (a) Why is this the case, and (b) what is the order of magnitude of the atmospheric mass effect?

6.10) Derive the equation for the Bouguer slab, a horizontal slab of infinite radius, thickness of  $\Delta t$ , with a density of  $\Delta \sigma$ , from the equation for the gravity effect of a spherical shell.

6.11) In surface gravity surveys the effect of terrain, that is, departures from the uniformly thick horizontal slab, is always to decrease the observed gravity. (a) Explain why this is so. (b) Does it also hold for subsurface gravity measurement as well as airborne measurements? Explain.

6.12) The observed gravity at a station is 20.05 mGals less than at the base station. The station is located 5,010 m north of the base which is located at  $45^{\circ}$ S latitude. The elevation of the station is 102.6 m above the base station. The terrain effect at the station is 3.02 mGals. Assuming the density of the surface sediments is 1,900 kg/m<sup>3</sup>, what are (a) the simple and (b) the complete Bouguer gravity anomalies at the station relative to the base?

6.13) The observed gravity at a base station is 980.29862 gal and the observed gravity at another station, relative to the base, is -16.21 mGal. The station is located at 39°16.20'N latitude and 86°32.69'W longitude at an elevation of 503.6 m above sea level. The effect of local terrain is 1.07 mGals. The ellipsoid of the Earth is 8.20 m below the geoid at the station location. Assuming that the Earth material between the station and sea level has a density of 2,670 kg/m<sup>3</sup>, calculate (a) the simple and (b) the completer Bouguer anomalies at the station with and without the indirect effect.

6.14) A gravimeter with a scale constant of 1.0987 mGal/scale division is used to measure the vertical gravity acceleration at a station relative to a base station. The base gravimeter reading at 0800 hrs ( $\equiv$  hours) was 200.18 scale divisions and at 1006 hrs the reading was 200.09 scale divisions. At 0855 hrs the station reading was 182.41 scale divisions. The station is lo-

cated 135 m below the base and is 1,000.24 km south of the base in the northern hemisphere. The base is located at  $45^{\circ}$ N latitude and  $85.55^{\circ}$ W longitude. The terrain effect is 0.09 mGals and the density of the formations extending to sea level is 2,000 kg/m<sup>3</sup>. Calculate (a) the simple and (b) the complete Bouguer gravity anomaly at the station relative to the base station.

6.15) Calculate (a) the free-air and (b) the complete Bouguer gravity anomaly at a marine observation at sea level at a location of  $42^{o}13.28'S$ latitude and  $30^{o}00.20'W$  longitude where the ocean depth is 4,603.95 m. The ship on which the observations are being made is heading N60°E at a velocity of 6 km/hr. The observed gravity at the site is 979.6722 gal. What would (c) the free-air and (d) the Bouguer anomalies be for an observation located at the ocean bottom, assuming that there are no local anomalous masses which alter the free-air gradient of gravity?

6.16) Calculate (a) the free-air and (b) the complete Bouguer gravity anomaly at a depth of 300 m in a granite intrusion which has a density of  $2,650 \text{ kg/m}^3$  where the observed gravity is 980.0044 gals. The latitude of the observation is  $36.3267^{\circ}$ N and the longitude is  $92.3353^{\circ}$ W. A hill extending 100 m above the level of the surface vertically above the observation has a gravity effect at the underground site of 2.66 mGal and a nearby valley cut below the level of the surface level above the observation has a gravity effect of 0.92 at the underground station.

6.17) Calculate the free-air and complete Bouguer gravity anomaly relative to a base station at a height of 120 m above the ground surface where the observed gravity is -40.62 mGal at a site located at  $53.2244^{\circ}$ N latitude and  $96.6767^{\circ}$ W longitude. The elevation of the ground surface directly beneath the airborne observation is 205 m above sea level and the density of the material between this location and sea-level is 2,600 kg/m<sup>3</sup>. The aircraft is headed W30°S at a velocity of 102 km/hr. A nearby hill has a gravitational attraction at the airborne site of 1.22 mGal and a nearby valley has a gravity effect at the site of -2.55 mGal relative to the surface directly beneath the airborne observation. Assuming no local gravity anomalies perturb the vertical gradient of gravity, what is (a) the free-air and (b) the complete Bouguer gravity anomaly at the surface?

6.18) Free-air, Bouguer, and isostatic residual gravity anomalies all have important uses in the geological interpretation of gravity data. Explain the major application of (a) the free-air, (b) the Bouguer, and (c) the isostatic residual anomaly and the reasons for their use in this application.

6.19) In high-accuracy gravity surveys two different corrections are commonly made to the data for atmospheric effects. Explain (a) these corrections and (b) their application.

6.20) (a) What assumptions are commonly made in the calculation of the combined elevation and mass effect in Bouguer gravity anomalies, and (b) how valid are these assumptions?

6.21) Explain why terrain respectively above and below the level of a gravity station results in errors to the Bouguer correction of the same sign.

6.22) What are the important criteria used in selecting a method to calculate the effect of terrain in gravity surveying?

6.23) Justify the assumption that the isostatic anomalies calculated by either (a) the Airy-Heiskanen or (b) the Pratt-Hayford theories do not differ in any significant way.

6.24) Give a physical description for the parabolic nature of the curvature correction for a Bouguer slab as a function of station elevation.

6.25) (a) Explain the origin of the indirect effect in gravity anomaly computation and its impact on the geological interpretation of gravity surveys.(b) Where would you expect to find the largest indirect effects?

6.26) Explain the three classes of gravity anomalies and their uses.

6.27) Bouguer anomalies are the principal anomalies used in interpreting land gravity surveys; however, they are not without limitations. Explain (a) the limitations of Bouguer gravity anomalies and (b) their impact on geological interpretation of gravity surveys.

6.28) (a) Define residual and regional gravity anomalies and (b) explain why a specific anomaly may be a residual in one survey and a regional anomaly in another.

6.29) One of the major uses of filtered anomalies is to increase the resolution of anomalies, that is, the identification of specific anomalies. The

second vertical derivative filtered anomaly yields some of the highest resolution results, but in many cases it is desirable to use a lower resolution filter such as the vertical gradient of gravity. Explain why this is the case.

6.30) (a) What are the differences between filtering used to enhance anomalies and that used to identify anomalies? (b) Give examples of each and of how each is used.

6.31) (a) What are the uses of the upward continuation of gravity anomaly fields and (b) what assumptions are made in its application?

6.32) In maritime applications, velocity is commonly expressed in knots where 1 knot = 1.85200 km/hr. (a) Express the Eötvös correction (Equation 6.31) in knots? (b) Is this correction added or subtracted from gravimeter readings taken on a westward moving platform? Why? (c) Determine the fractional error in the Eötvös correction in terms of the uncertainties in  $\theta$ ,  $\alpha$ , and V. (d) Which of these parameters has the most significant effect on the Eötvös correction in practice?

6.33) Use the gravity effect of the infinite horizontal cylinder (source #4 in Table 3.2) to show that mass variations integrate in the source domain in the same way as their gravity effects integrate in the signal domain. Specifically, (a) give the gravity effect of a horizontal cylinder of radius  $R_1$  and density  $\sigma_1$  in which a second cylinder of radius  $R_2 = 0.1R_1$  and density  $\sigma_2 = 0.5\sigma_1$  is embedded. Assume that the axis of the second cylinder is located  $45^o$  clockwise from the top at  $0.5R_1$ . (b) If the regional is taken as the gravity effect of the first cylinder and subtracted from the total effect in (a)-above, how is the density derived from the inversion of the residual anomaly on the second cylinder related to  $\sigma_2$ ?

#### 1.7 Gravity Anomaly Interpretation

7.1) (a) The assumption of source two-dimensionality, that is strike infinite source, is desirable in gravity interpretation because of the increased ease of interpretation in comparison with 3D analysis. However, this assumption is often not justifiable for deep sources that have considerable depth extent. Using 2.5D gravity anomaly computational software, calculate gravity anomalies from a square-shaped cross-section source with dimensions of  $1 \times 1$  km with its upper surface at 1 km that is strike infinite and which has a den-

sity contrast of 200 kg/m<sup>3</sup> with the surrounding rock unit, and compare this anomaly with a similar source only 4 km long on either side of the computational profile. (b) Repeat these calculations for a body whose top surface is 5 km and compare the validity of the two-dimensionality assumption for the shallow and deeply buried source with a strike length of 4 km on either side of the profile. (c) Repeat (b) but compare the two-dimensionality when the source is replaced with a body that is 1 km wide and 10 km in depth extent. (d) On the basis of your results describe the effect of depth and depth extent of a source of gravity anomalies on the assumption of two-dimensionality.

7.2) Numerous techniques such as second vertical derivative and highfrequency cut filtering are used to isolate for interpretation anomalies with specific characteristics. However, care is needed in using these anomalies for quantitative analysis by inversion or iterative forward modeling. Explain.

7.3) Figure 7.5(b) illustrates the change in a gravity anomalies associated with a doubling of the volume of a source, where the mass is doubled as well because the density differential is the same in each case. What is the relationship between the energy of the anomaly, that is the area under the anomaly curve, for the two respective anomalies. Confirm this with the values of the energy for the two anomaly profiles shown in the figure?

7.4) A gravity survey conducted to find a buried vertical mine shaft has located a localized negative complete Bouguer gravity anomaly which has an amplitude of 0.25 mGal. This anomaly has been interpreted as due to the shaft. Corroboration for this interpretation has been given by the correlation of the anomaly with a 3 m surface depression. Explain any possible source of this anomaly other than a mine shaft.

7.5) (a) What are the key geologic variables that determine whether a specific subsurface source can be identified by a gravity survey? (b) Which of these are the most important and why in searching for (1) carbonate reefs in a sedimentary basin, (2) caves in limestone bedrock, (3) faults within a granite massif, (4) bedrock valleys in a glaciated terrain where the bedrock is sandstone and the surficial glacial drift is generally from 5 to 50 m thick, and (5) and sulfide ore deposits within the upper 100 m of a metamorphic rock terrane?

7.6) (a) Construct a linear graph of the amplitude and maximum gradient for the four gravity anomalies in Figures 7.8 and 7.9 versus the depth to the

top of the prism source. (b) What are the implications of the differences in these relationships for interpretation of the anomalies?

7.7) (a) Plot the amplitude and maximum gradients of the gravity anomalies in Figure 7.9 versus the half-length of the strike of the sources, assuming that the values at infinity are approached asymptotically. (b) What are the implications of these relationships for making assumptions based on regarding this source as two-dimensional?

7.8) (a) What are the key geophysical variables that are important in interpreting the source of gravity anomalies. (b) Which of these are the most important and why, in searching for (1) carbonate reefs in a sedimentary basin, (2) caves in limestone bedrock, (3) faults within a granite massif, (4) bedrock valleys in a glaciated terrain where the bedrock is sandstone and the surficial glacial drift is generally from 5 to 50 m thick, and (5) and sulfide ore deposits within the upper 100 m of a metamorphic rock terrane?

7.9) The decay rate of the amplitude of the gravity anomaly of a specific geometric shape with distance, the structural index, may vary. Explain the origin of this variation and provide an example.

7.10) The gravity method is to be used to detect caverns in a flat-lying dolomite bed in the near surface of the Earth. These caverns can be approximated by a spherical volume and have a radius of 3 m and a depth to center of 3 to 6 m below the flat surface. Assuming the density of the dolomite is 2,700 kg/m<sup>3</sup> and the caverns are air filled, (a) calculate to type nearest microgal and plot the anticipated minimum and maximum negative gravity anomalies across these caverns. Extend the gravity calculations to a distance where the anomalies are about 5% of the minimum. (b) What station spacing would you recommend for a survey to map the anomalies from the range of caverns anticipated, and what accuracy is required of the elevations of the stations? (c) Explain your results.

7.11) **Figure 1.2** shows the structure contour map for the top of the Silurian Niagara carbonate formation of a region in southeast Michigan. The three structural highs are pinnacle reefs which have a density of roughly 2,700 kg/m<sup>3</sup>. These reefs are primarily enclosed by an evaporite formation which has a density of 2,200 kg/m<sup>3</sup>. *Figure 1.3* is the complete Bouguer gravity anomaly map of the same area shown in *Figure 1.2*. (a) What is the gravity expression of reef 1, 2, and 3? (b) What are the amplitudes of



Figure 1.2 Structure contour map for the top of the Silurian Niagaran formation, St. Clair County, Michigan. The contour interval is 25 feet. See **Study Question 7.11**. Adapted from Keith (1967).

the residual anomalies? (c) What is the possible cause of the lack of gravity anomaly over reef 3? (d) Can reefs 1 and 2 be identified as separate sources on the gravity anomaly map?

7.12) Figure 1.4 shows the simple Bouguer gravity anomaly map of a portion of northeastern Indiana which is underlain by Paleozoic sedimentary rocks and Figure 1.5 is the seventh order polynomial residual gravity anomaly map derived from Figure 1.4 (King, 1974). The bedrock in the area consists of Silurian carbonates which have a density of 2,700 kg/m<sup>3</sup>. The bedrock is overlain by highly variable Pleistocene glacial drift, which has an average density of approximately 2,300 kg/m<sup>3</sup>. Pre-glacial bedrock valleys occur in the area. (a) What is the sign of the gravity anomalies that are anticipated over these valleys? (b) Do you observe these types of anomalies in the Bouguer and residual anomaly maps? (c) Where are the bedrock valleys located? Indicate them on the maps. (d) What is maximum relief of these bedrock valleys, on the basis of the gravity anomalies?

7.13) A vuggy zone with a porosity of 40% is believed to occur below the water table within a limestone which has a density of 2,680 kg/m<sup>3</sup>. Assume



Figure 1.3 Simple Bouguer gravity anomaly map of the area shown in *Figure 1.2*. The contour interval is 0.1 mGal. Note the strong regional gravity gradient from the basement rocks. The location of the reefs is indicated by the shaded pattern. See **Study Question 7.11**. Adapted from Keith (1967).

that the depth to the center of the zone is 10 m and it can be approximated by a thin horizontal square rectangular plate 100 m on the side and 2 m thick. (a) What is the maximum gravity anomaly anticipated from this feature? (b) How does this compare with the anomaly approximated by the Bouguer slab?

7.14) Figure 7.13 shows the variation in specific gravity anomalies as a function of the ratio of the length to the width of vertical and dipping prisms of variable depth extent. Review the anomalies of this figure and describe the effect of strike length on the anomalies as a function of (a) dip, (b) width, and (c) depth extent of the prisms.

7.15) Explain and illustrate why the depth to a concentrated subsurface source such as a reef can be determined more easily from its gravity anomaly than that of a long concentrated source such as an anticline?

7.16) Figure 1.6 is the simple Bouguer gravity anomaly of an isolated



Figure 1.4 Simple Bouguer gravity anomaly map of a portion of northeastern Indiana. The contour interval is 0.5 mGal. See **Study Question 7.12**. Adapted from King (1974).

gravity and magnetic anomaly in northwestern Indiana. The geology of the region consists of Proterozoic crystalline basement overlain by relatively flat-lying Paleozoic sedimentary rocks. A simple linear gradient has been removed from the arbitrary anomaly values to isolate the residual anomaly.



Figure 1.5 Seventh-order residual gravity map derived from the simple Bouguer gravity anomaly map in *Figure 1.4*. The contour interval is 0.25 mGal. The hatched areas are negative residual gravity anomalies. See **Study Question 7.12**. Adapted from King (1974).

(a) Determine the depth to the top of the anomaly source using a variety of graphical depth determination techniques as described in Chapter 7. (b) Explain possible sources of differences in the depths determined by the var-



Figure 1.6 Simple Bouguer gravity anomaly of an isolated anomaly in northwestern Indiana. A simple linear gradient has been removed from the arbitrary anomaly values to isolate the residual anomaly. The contour interval is 1 mGal. See **Study Questions 7.16**, **7.23**, and **7.24**. Adapted from Rudman and Blakely (1965).

ious methods.

7.17) Explain the basis of ideal body theory for determining the maximum

depth to the source of a gravity anomaly.

7.18) What are the sources of error and their relative importance in determining the subsurface limits of a gravity anomaly source from the maximum of the horizontal derivative or the zero vertical derivative of the anomaly?

7.19) (a) Calculate equivalent (within 10%) gravity anomalies from two idealized subsurface sources using geologically reasonable parameters, that is, dimensions, shape, and density contrast, that have depths that vary by at least 200%. (b) What lesson can be learned from this exercise?

7.20) A lava tube is believed to occur within a basalt flow with density 2,950 kg/m<sup>3</sup>. The tubes range in diameter from 1.5 to 3.0 m and centers are at a depth of 3.0 to 6.0 m below the surface. (a) Calculate and plot the gravity anomaly profile perpendicular to the tube, assuming it can be approximated by a horizontal cylinder. Make the calculations to the nearest microgal and prepare four profiles for R = 1.5 and 3.0 m and Z = 3.0 and 6.0 m. Assume the tube is air filled and calculate the anomaly to 10% of the maximum value. (b) What station spacing would you recommend for the survey to map the lava tubes?

7.21) A vertical fault has brought into horizontal juxtaposition a limestone bed with density 2,700 kg/m<sup>3</sup> and a sandstone bed of density 2,550 kg/m<sup>3</sup>. The horizontal density contrast occurs over a 150 m interval and begins 30 m below the surface. (a) Calculate and graph the gravity anomaly profile perpendicular to the fault. Extend the profile to a distance of 600 m on either side of the fault. To map the gravity anomaly of this fault, what (b) station spacing and (c) elevation survey accuracy would you recommend?

7.22) An underground working can be approximated by a two-dimensional slab which is 30 m thick. The working is filled with water and located in a rock of density  $2,500 \text{ kg/m}^3$ . The slab is 150 m wide. Assuming that the minimum gravity anomaly for detection is 0.1 mGal, what is the maximum depth at which this working can be detected?

7.23) By least squares inversion interpret the gravity anomaly shown in *Figure 1.6*. Refer to **Study Question 7.16** for background.

7.24) Assuming that the maximum permissible density contrast of the vertical cylinder source of the gravity anomaly in *Figure 1.6* is  $350 \text{ kg/m}^3$ ,

what is the maximum depth to the top of the cylinder? Refer to **Study Question 7.16** for byackground.

7.25) For the gravity and magnetic anomaly data of Montana described in USGS Map GP-444 (Davis et al., 1963), (a) what are the causes of the differences in density of the various rock units? The Cenozoic sedimentary deposits are assumed to have an average density of  $2,300 \text{ kg/m}^3$ . Assuming the grain density of the sedimentary deposits is  $2,670 \text{ kg/m}^3$ , what is the porosity of the sedimentary units if the pore space is (b) water filled and (c) air filled? (d) What gravity anomalies are associated with the Cenozoic basins? (e) How do the horizontal gravity gradients compare on the western and eastern margins of Townsend Valley? (f) What is the possible geologic cause of this difference? (g) The interpretative cross-sections of sheet 1 show several faults. Does the gravity show evidence of these individual faults? (h) Compare the use of the gravity and magnetic methods in mapping surface lithologies in this area. (i) Three assumptions were made in the calculations of the gravity effects of the Cenozoic basins. How valid are these assumptions? (i) Are there any inverse relationships between the gravity and magnetic anomalies? Where? (k) Explain the possible source of this relationship.

7.26) Gravity anomalies are caused by density variations or contrasts in the Earths crust and mantle. The nature of these contrasts helps to determine the size and shape of anomalies observed in gravity surveys. The amplitude of a gravity anomaly is directly proportional to the density contrast. Sketch the gravity anomalies associated with the density contrasts in *Figure 1.7*, considering relative amplitudes and shapes and using the dashed line for the zero level of the anomalies.

7.27) In **Figure 1.8**, plot the location and shape of anomalies (b), (c), (d), and (e) in relation to anomaly (a). Assume the sources are spheres (or 2D horizontal cylinders) and infinite slabs (horizontal layers). The numbers represent densities in CGSu. The vertical scale is in arbitrary units. Assume a maximum value of 4 units for anomaly (a).

7.28) To demonstrate simple gravity modeling of 2D structures, (a) compute the maximum amplitude of the gravity anomaly due to an over-pressured shale ridge represented by the 2D structure in **Figure 1.9**, which involves a 2D horizontal cylinder and two 2D prismatic blocks. How does this result compare with the maximum amplitudes obtained from (b) a single horizontal cylinder of appropriate radius and average density, and (c) the use of the



Figure 1.7 Determining gravity anomalies from the density contrasts of spheres or horizontal cylinders. The densities,  $\sigma$ , are given in g/cm<sup>3</sup>. See **Study Question 7.26**.



Figure 1.8 Determining gravity anomalies from the density contrasts of spherical or cylindrical sources and horizontal layers. The numbers respresent densities in  $g/cm^3$ . See **Study Question 7.27**.

2D graticule in *Figure 1.10*? Using the single horizontal cylinder approximation, what is the attenuation factor by which the maximum value of the anomaly is reduced if measurements were made (d) off the structural axis at horizontal distances of 1.5 km and 3.5 km, and (e) at 1.5 km above datum



Figure 1.9 Simple gravity modeling of 2D structures using 2D horizontal cylinder and 2D rectangular prisms. The numbers represent densities in g/cm<sup>3</sup>. See **Study Question 7.28**.

as for example in airborne gravity?

7.29) To illustrate simple 3D gravity modeling, (a) compute the maximum amplitude of the gravity anomaly due to a shale or salt piercement represented by the 3D structure in **Figure 1.11**, which is involves a sphere and two vertical cylinders. (b) How does this result compare to the maximum amplitude from a single sphere of appropriate size and average weighted density? Using the single sphere approximation, what is the attenuation factor by which the maximum value of the anomaly is reduced if measurements were made (c) off the structural axis at horizontal distances of 1.5 km and 3.5 km, and (d) at 1.5 km above datum as for example in airborne gravity?

7.30) A gravity reading of 165 mGal has been observed 300 m below the surface in a vertical mining shaft. Neglecting the small effect of the open shaft, (a) what would the reading be if the gravimeter were on the surface



Figure 1.10 Simple gravity modeling of 2D structures using a 2D graticule. See **SQ.7-28**. The sum of the elements (solenoids) within the crosssectional area of the 2D gravitational source multiplied by the density contrast, vertical distance between the horizontal lines and a constant as indicated in the figure is the gravity anomaly of the source at the apex of the radial lines. Adapted from Hubbert (1948). See **Study Question 7.28**.

directly above its underground position? (b) What would the reading be if the gravimeter were lowered to a depth of 1,000 m directly below its underground position? Assume that the free-air gradient as 0.3086 mGal/m and the Bouguer gradient as 0.1115 mGal/m.

7.31) In favorable cases it is possible to compute the mass anomaly (excess or deficit of mass) that is responsible for a gravity anomaly by Gauss' law. *Figure 1.12* shows the distribution of gravity anomaly values caused by a body of magnetite ore in Precambrian gneiss which can be used to estimate the mass of the ore. (a) Contour the anomaly at a contour interval of 0.05 mGal. (b) Remembering that the gravity values are relative to an arbitrary base and also that the gravity anomaly caused by an isolated mass



Figure 1.11 Simple gravity modeling of 3D structures using a sphere and vertical cylinders to approximate the source's geometry. See **Study Question 7.29**.

approaches zero at large lateral distances from the mass, assign appropriate values to the contours on the basis of the approximately zero anomaly near the edges of the map. (Hint: use 0.40 mGal as the zero anomaly.) (c) Draw a convenient grid system on the map and estimate the average value of the gravity anomaly within each square. All the data are now available to solve for the anomalous mass, but be very careful of units. (d) The anomalous mass is the excess mass over and above normal rock that would occupy the same volume. Assuming that the anomalous mass is magnetite ore, a correction based on the relative densities of ore and rock can be made to estimate the tonnage of ore. Obtain an estimate of tonnage of ore that may be responsible for the gravity anomaly. (Use  $\sigma$  (magnetite) = 4,700 kg/m<sup>3</sup>;  $\sigma$  (gneiss) = 2,650 kg/m<sup>3</sup>) (e) Is the estimate based on Gauss' law likely to be a minimum or a maximum value for the amount of ore? What are the principal limitations in the use of this method of interpretation? For what



Figure 1.12 Gravity stations with residual gravity values, posted in 0.01 mGal, used in the estimation of mass of a magnetite body by Gauss' law, Iron Mountain, Llano County Texas. Adapted from Barnes and Romberg (1943). See **Study Question 7.31**.

types of gravity anomalies should the interpretation work best? How does the solution depend on the depth or shape of the body in theory and practice? (f) In general, the gravity anomaly could be caused by any number of lithologic variations. The geologic facts together with a magnetic anomaly indicate that magnetite is present and the gravity anomaly supplies a quantitative estimate of amount of magnetite. But does the magnetite form an ore body or could it be disseminated? (g) Draw a profile along the length of the anomaly and another one across it at its maximum value. From these profiles and the contours of the anomaly, describe the approximate shape, attitude, relative depths, and the areal extent of the body causing the anomaly (assuming that it is compact ore). (h) Estimate approximately the average depth of the body at the maximum value of the anomaly by the half width formula, using the cross-sectional profile.

## 1.8 The Magnetic Method

8.1) The magnetic method has a broad range of uses in geological exploration from sub-meter to 100s of kilometers scales. (a) Give examples of potential uses of the method over this range of scales and (b) explain why the method has this versatility even though it seldom provides a definitive answer to a geological problem.

8.2) The magnetic method is based on observing the perturbation of the terrestrial magnetic field due to subsurface horizontal variations in the magnetic polarization. (a) What are the principal components of the magnetic field measured in magnetic exploration and (b) what are their representative magnitudes.

8.3) Explain the concept of magnetic poles and their relationship to magnetization which is used as the fundamental property of matter in magnetic exploration of the Earth.

8.4) The magnetic method is based on a dipolar force field rather than the monopolar force field of the gravity method. Explain and describe the impact of the dipolar nature of the magnetic field in magnetic exploration.

8.5) The force at a location in space caused by the magnetization of an adjacent material cannot be measured directly; rather, the resulting magnetic field is measured. (a) What is the difference between the magnetic force and the magnetic field caused by a magnetized body? (b) Give the SIu and EMu of the magnetic field?

8.6) (a) What is the basis of Coulomb's law? (b) What is its role in magnetic exploration? (c) How does it compare with the universal law of gravitation?

8.7) The magnetic method, being a potential field method based on observations of a force field of the Earth, has numerous similarities to the gravity method. However, there are significant differences which are advantageous to the magnetic method. (a) Identify these differences and (b) explain their

impact on the magnetic method of exploration.

8.8) Distinguish among (a) dipole moment, (b) volume magnetization, (c) mass magnetization, (d) volume magnetic susceptibility, (e) mass magnetic susceptibility, and (f) magnetic permeability.

8.9) Assuming that the geomagnetic field can be modeled as the field of a dipole aligned along the axis of the Earth, (a) determine the dipole magnetic moment of the Earth, taking the radius of the Earth to be approximately 6, 360 km and the measured magnetic field at the pole along the axis of the Earth to be 60,000 nT. The magnetic permeability of space is  $\mu_o = 4\pi \times 10^{-7}$  kg m A<sup>-2</sup> s<sup>-2</sup>. As a result, (b) what is the magnitude of the geomagnetic field on the surface of the Earth at the magnetic equator, and (c) at an elevation of 400 km above the Earth's surface at both the magnetic equator and pole?

8.10) Assuming that the terrestrial magnetic field is due to a dipole aligned along the axis of rotation of the Earth, what is the angle of inclination of the field at  $60^{\circ}$ ,  $45^{\circ}$ ,  $30^{\circ}$ , and  $15^{\circ}$  of latitude?

8.11) The magnetic field of the Earth is commonly represented by lines of force. (a) How are these lines portrayed with respect to the direction of the magnetic field and the amplitude of the magnetic field. Upon encountering an interface between two media with dissimilar magnetic permeabilities, (b) explain the effect on the lines of magnetic force. (c) What are the implications of this effect?

8.12) Significant time variations in the geomagnetic filed are observed at a range of periods. Describe their (a) origin, (b) periods, (c) amplitudes, and (d) impact on observations of magnetic anomalies originating from within the Earth.

8.13) The International Geomagnetic Reference Field (IGRF) is used to estimate the terrestrial magnetic field over a 5-year span of time. However, the field over this duration of time, the Definitive Geomagnetic Reference Field (DGRF), is specified from the measurements of the actual field over this duration. (a) Why is it advantageous to define the actual field over the 5-year epoch rather than use the IGRF? (b) How is this field (the DGRF) used in preparing magnetic anomaly maps for geological exploration? 8.14) Most measurements of the Earth's magnetic field for the purpose of mapping magnetic variations in the lithosphere now use scalar magnetometers rather than vector magnetometers. (a) What are the reasons for this? Use of the scalar field in magnetic anomaly mapping requires (b) what assumption? (c) What limits does this assumption place on the accuracy of magnetic anomalies of the total magnetic field? (d) Give examples of the maximum error in the anomaly, assuming the anomalous total magnetic field is 100 nT, 1,000 nT, and 10,000 nT in an ambient terrestrial field of 60,000 nT. (e) What is the significance of these results to the interpretation of magnetic anomalies?

8.15) (a) What critical difference between magnetic and gravity fields has a profound impact on implementing the magnetic method? (b) Give examples.

8.16) Referring to the IGRF model located at http://www.ngdc.noaa. gov/geomagmodels/struts/calcGridIGRF, evaluate the IGRF of 5 October 2011 on the Earth's surface at 1°-intervals for worldwide plots of the geomagnetic field's (a) x-component of intensity with superposed inclination and declination, (b) y-component of intensity with superposed inclination and declination, (c) z-component of intensity with superposed inclination and declination, (d) h-component of intensity with superposed inclination and declination, and (e) the total intensity with superposed inclination and declination. For each map, use color-fill with color bar to represent intensity, and annotated black and white contours to map inclination and declination, respectively. (f) Why are no declination and x- and y-component intensities available poleward of 80°N and 80°S latitudes?

8.17) For the composite aeromagnetic anomaly map of Ohio in Figure 8.8, (a) evaluate the IGRF total intensity values at the center of each component survey at its respective survey date. (b) What would this map look like if the main field had not been removed? (c) How would this affect the use of these anomalies in subsurface exploration?

8.18) Evaluate the IGRF of 5 October 2011 at Houston, Texas for intensity variations at (a) the elevations of 0, 1, 5, 10, 20, 50, 100, 200, and 400 km.(b) Estimate the intensity gradient (in nT/m) at these elevations. (c) What are the implications of these results for magnetic surveying?

8.19) (a) Evaluate the IGRF at the Earth's surface for temporal variations of intensity from 1900 to 2010 at 10-year intervals. (b) Compare the average

annual rate of change from 1900 to 2010 to the change between 2000 and 2010. Explain these results. (c) Plot and compare the migration paths of the geomagnetic poles from 1900 to 2010.

8.20) Redo Figure 1.3 in the southern geomagnetic hemisphere. Compare the magnetic anomalies from equivalent sources at equivalent magnetic inclinations and declinations in the southern and northern hemispheres.

# 1.9 Magnetic Potential Theory

9.1) (a) What is magnetic potential? (b) What is its relationship to and (c) how does it differ from magnetic force? (d) What are the units of magnetic potential?

9.2) What are the advantages and disadvantages of measuring and using magnetic field (a) vectors and (b) tensors in the interpretation of subsurface anomalous masses?

9.3) The magnetic anomalies from simple idealized geometric forms which can be calculated with closed-form equations are commonly restricted to vertical polarization. Explain the reason for this limitation.

9.4) The value of the total field anomaly is obtained by assuming that the measured field is directionally coincident with the normal magnetic field. This scalar difference is based on the assumption that the anomalous field is equivalent to its projection onto the normal field as shown in Figure 8.14. Explain how this total field is calculated for an anomalous source?

9.5) (a) Explain why the residual magnetic anomaly field of a spatially extensive region should have essentially equivalent negative and positive energies as measured by the summation of the anomaly volumes. (b) How can this be used in determining an optimum regional anomaly elimination procedure?

9.6) (a) What assumptions are made in calculating the magnetic anomaly from a two-dimensional source of arbitrary cross-section? (b) How do these differ from the assumptions made in the calculation of the anomaly from an arbitrarily shaped three-dimensional source? 9.7) Derive an expression for the magnetic field exterior to a uniformly magnetized spherical shell and at the center of the shell.

9.8) What distribution of magnetization within a sphere results in no exterior field?

9.9) A uniformly magnetized slab with parallel bounding surfaces has no resulting magnetic field. Show this to be true using Poisson's equation.

9.10) Under what conditions does the total magnetic field anomaly satisfy Laplace's equation?

9.11) (a) Show that the maximum and minimum values of the induced total magnetic field anomaly of a 3D source lay along geomagnetic declination. (b) Can the ratio of these values (maximum and minimum) be used to infer geomagnetic declination?

9.12) Consider a spherical source of diameter 1 km with its top at 0.5 km below sea level and centered below a  $16 \times 16$  array of observations spaced at 1 km intervals at an altitude of 0.5 km above sea level. Assume that the CGSu volume magnetic susceptibility of 0.001 for the spherical source is the result of magnetization induced by the IGRF of 5 October 2011. Compute and plot the total magnetic field effects for the spherical source centered on (a)  $40^{\circ}$ N and  $-80^{\circ}$ W, (b)  $-12^{\circ}$ S and  $-60^{\circ}$ W, and (c)  $-40^{\circ}$ S and  $-70^{\circ}$ W. For each map list the amplitude range AR = (min; max) of the minimum and maximum values, the amplitude mean AM, the amplitude standard deviation ASD, as well as the inclination I', declination D', and intensity J' in SIu of the induced magnetization.

9.13) From the results of **Study Question 9.12**, (a) how can the anomaly pattern be used to quantify the inclination and declination of the applied magnetic field? (b) How do these total magnetic field effects compare with the gravity effects of the spherical source at the three locations assuming the source's density is 2.67 g/cm<sup>3</sup>?

9.14) For the spherical source of **Study Question 9.12** at (40°N,  $-80^{\circ}$ W), assume that it also has a remanent magnetization with intensity  $J'_{rem} = 0.5 \times J'$ , inclination  $I'_{rem} = 20^{\circ}$ N, and declination  $D'_{rem} = 70^{\circ}$ E. (a) Compute and plot the remanent magnetic field effect. (b) What is the total magnetization? (c) Compute and plot the total magnetic intensity of the

source, as in **Study Question 9.12**, from the total magnetization. (d) How is the resulting anomaly in (c) related to the anomaly due only the induced magnetization calculated in **Study Question 9.12**?

9.15) Convert the vertical magnetic anomaly expressions of the (a) sphere (#2), (b) the horizontal cylinder (#4), and (c) the vertical cylinder (#5) in Table 9.1 into total magnetic field anomalies at (I, I') inclinations and (D, D') declinations.

9.16) Using the generalized coordinates described in **Study Question 3.28**, develop expressions for the magnetic field components (a)  $B_{q_1}$ , (b)  $B_{q_2}$ , (c)  $B_{q_3}$ , and (d) the total magnetic field.

9.17) Give the total magnetic field effects in generalized coordinates for the (a) sphere (#2), (b) horizontal cylinder (#4), and (c) vertical cylinder (#5) of Table 9.1.

# 1.10 Magnetization of Earth Materials

10.1) Explain why it is difficult to estimate the magnetization of rocks by visual inspection.

10.2) Explain the difference in origin and range of values of (a) the induced and (b) the remanent magnetization of rocks.

10.3) Distinguish between (a) magnetic susceptibility and (b) magnetic permeability, (c) magnetization, and (d) magnetic moment.

10.4) (a) Why is it important to consider the strength of the magnetizing field when measuring or specifying the magnetic susceptibility of a material? For example, (b) why are the magnetic susceptibilities of Earth materials used in magnetic interpretation referred to as weak field susceptibilities?

10.5) Diamagnetism, which is present in all atoms, is the dominant form of magnetism in only a few commonly occurring minerals, and the magnetic susceptibility of these minerals is very low. Nonetheless, in special conditions their effect causes magnetic anomalies of interest in geological exploration. Explain (a) diamagnetism and (b) its use in geological exploration. 10.6) Distinguish between (a) ferromagnetism and (b) ferrimagnetism. (c) Which is most important as a source of magnetism in terrestrial rocks, and why?

10.7) The Koenigsberger ratio, the ratio of the magnitude of remanent and induced magnetization in rocks, varies greatly dependent on the nature and origin of the rock. (a) Explain why mafic volcanic rocks, such as basalt, commonly have a high Koenigsberger ratio, typically in excess of one, whereas plutonic, coarse grained, igneous rocks seldom have ratios which reach one. (b) What is the source of the remanent magnetization in volcanic rocks, and (c) how can this magnetization be destroyed in rocks?

10.8) The mineral magnetite,  $Fe_3O_4$ , is the primary carrier of magnetization in terrestrial rocks. Explain why the magnetic susceptibility of this mineral varies.

10.9) Rank the minerals hornblende, pyrite, ulvöspinel, native iron, magnetite, halite, pyrrhotite, maghemite, hematite, quartz, and ilmenite in order of decreasing magnetic susceptibility.

10.0) Locally pyrrhotite-bearing rocks may be important contributors to magnetization; however, the significance of these rocks as a source of rock magnetization is limited in depth in the crust compared with magnetite-bearing rocks. Explain.

10.11) Most methods of magnetic interpretation are based on the assumption that the magnetization is induced by the ambient terrestrial magnetic field. Discuss the basis of this assumption and its validity.

10.12) Demagnetization may alter the magnetic susceptibility of magnetite. Give the cases when this effect is most important and explain why this is so.

10.13) Metamorphism may have a profound effect on the magnetization of rocks, resulting in both increased and decreased magnetizations. Explain the possible effects of metamorphism on rocks and their sources.

10.14) Sedimentary rocks and sediments are commonly assumed to have negligible magnetic susceptibility. However, there is ample evidence that many of these have sufficient magnetization and variability in their mag-

netization to permit mapping of such units with high-sensitivity magnetic surveying. Explain the origin of this magnetization and its variability.

10.15) Long-wavelength magnetic anomalies indicate that the average magnetization of the continental magnetic lithosphere is greater than that indicated by the magnetization of surface rocks, suggesting that the lower crust has a greater magnetization than the upper crust. What are possible sources of this greater magnetization?

10.16) What is the evidence for the assumption that magnetization of the Earth in continental regions is limited to the crust?

10.17) Although Mars lost its magnetic field owing to cooling of the interior of the planet over 4 billion years ago, magnetic mapping shows extensive regions of intense magnetization on Mars; the intensities of which exceed generally observed magnetizations on the Earth. (a) What is the likely source of this magnetization of Mars? (b) Explain.

10.18) The magnetization of soils reflects their parentage and environmental conditions. Explain the variability of the magnetization of soils both (a) horizontally and (b) vertically.

10.19) Compare the magnetic effects of two archaeological soil pits, A and B. Assume the pits can each be approximated by a buried sphere of roughly 0.5 m in radius, but pit A is filled with soil containing a 2.21% weight concentration of iron oxides, whereas pit B holds soils with the weight concentration of 0.76%. For both soils, assume an archaeological conversion factor of 11%, magnetization induced by a vertical geomagnetic field of intensity 57,550 nT, and a bulk density of 1.3 g/cm<sup>3</sup>. Compute and compare the peak anomaly amplitudes for the pits centered at depths of (a) 1.0 m, (b) 1.5 m, and (c) 2.0 m below the magnetometer. (d) What are the implications of these results for detecting the pits by magnetic surveying? (e) How will the magnetic viscosity of the soils affect these results?

10.20) Suppose a backhoe trench, excavated to explore an archaeological site, is back filled with the excavated soil. A magnetic survey across the freshly buried trench reveals a central amplitude anomaly of -15 nT. A repeat survey 5 years later reveals the central amplitude is now -13 nT. Assuming that the change in amplitude is due to the magnetic viscosity of the soil, with a viscosity constant of 3%, how long will it take for the trench

anomaly to disappear into the soil's  $\pm 1$  nT magnetic noise envelope?

10.21) What are the common limitations to using magnetic susceptibilities from published tabulations?

#### 1.11 Magnetic Data Acquisition

11.1) Magnetic observations up to the late 1940s were primarily made with instruments that measured a vector component of the magnetic field based on the oscillation or displacement of a magnet. These were rapidly replaced at that time with electronic magnetometers. (a) Explain the fundamental reasons for this. (b) What advantages did the electronic magnetometers bring to magnetic observations?

11.2) The first practical electronic magnetometer for measuring magnetic fields of interest to geophysical exploration was the flux-gate magnetometer, which was introduced in the 1940s. However, this magnetometer measured the total magnetic field rather than vertical magnetic field, which had been the primary measurement of the magnetic field up to that time. Explain the reason for this change in the component that was measured.

11.3) The flux-gate magnetometer has largely been replaced in airborne measurements with the resonance magnetometer. (a) Why is this the case? (b) Nonetheless, the flux-gate magnetometer has found an important use in satellite measurements of the magnetic field. Explain.

11.4) Resonance magnetometers measure the absolute value of the magnetic field rather than the relative change in the scalar field that is measured by the flux-gate magnetometer. Explain the cause and relative advantage of measuring the absolute value of the field.

11.5) The resonance magnetometers - proton precession, alkali vapor, and Overhauser - differ in their orientation requirements with respect to the magnetic field. Explain these differences and also explain which type is at an advantage as a result of this requirement?

11.6) Explain the working principle of the (a) flux-gate and (b) the resonance magnetometers.

11.7) The SQUID magnetometer has a significantly greater sensitivity than the resonance magnetometers, but it is used in only special applications in geophysical exploration. Explain.

11.8) Measurements of the gradients of the magnetic field have significant advantages over scalar measurements of the field in the interpretation of magnetic data. (a) Explain these advantages. Magnetic gradients can be observed directly or calculated from measurements of the scalar field. (b) What are the relative advantages and disadvantages of these two methods of obtaining magnetic gradients? (c) What assumptions must be made in both cases?

11.9) Both vector and scalar magnetic components are used in geophysical exploration, although the vast majority of exploration surveys measure the scalar field. (a) Why is this the case? (b) What advantages do vector measurements offer over scalar measurements?

11.10) The development of magnetometers capable of making measurements for geophysical exploration from airborne platforms provided many advantages over land-based observations. (a) Explain these advantages. (b) Do airborne measurements have any disadvantages over land-based observations? Explain.

11.11) What are the controlling issues for the choices that need to be made for the following quantities in organizing and planning an airborne magnetic survey: (a) aircraft, (b) magnetometer, (c) flight-line orientation, (d) flight-line altitude, (e) flight-line spacing, and (f) tie-line spacing?

11.12) (a) What are the relative merits of performing airborne magnetic surveys using constant elevation versus constant altitude above the surface of the Earth? (b) How does the surface terrain, the depth to magnetic rocks below the surface, and the objective of the survey enter into the selection of the survey flight altitude?

11.13) How has the availability of GPS surveying measurements, which permit locations to be determined in real time to a precision approaching a meter, improved the measurement of magnetic fields from airborne platforms?

11.14) A major factor in determining the precision of airborne magnetic

measurements is the ability to remove the magnetic noise due to the aircraft and its flight through the Earth's magnetic field. (a) What are the sources of this noise and (b) how are these effects largely eliminated in magnetic surveying?

11.15) To achieve a precision of 0.1 nT in airborne magnetic measurements, what is limit of the envelope (a) vertically and (b) horizontally that must be flown taking into account the vertical and horizontal variations in the geomagnetic field?

11.16) A land surface magnetic survey is to be conducted to investigate the location of near-surface ferrous objects, employing measurements of the total magnetic field and the vertical and horizontal gradients obtained from the difference between two magnetometer measurements. Assuming that the ferrous objects can be approximated by spheres as small as 10 cm in radius with a magnetic susceptibility of 100 SIu and located at depths between 1 and 2 meters below the surface, determine (a) the appropriate level of the magnetic measurements above the ground surface and (b) the appropriate distance between the magnetic measurements to measure the vertical and horizontal magnetic gradients needed to map the magnetic anomalies from these sources.

#### 1.12 Magnetic Data Processing

12.1) The reduction of magnetic measurements from surface and near surface surveys including most aeromagnetic surveys is considerably less complicated in most situations than is the comparable reduction of gravity measurements. (a) Explain the reasons for this. (b) In what situations is this not true? Explain.

12.2) Natural and man-made near-surface sources may give rise to magnetic effects which perturb surface and near-surface magnetic survey measurements, including those from low-level aeromagnetic surveys. These effects may distort or even hide magnetic anomalies from geological sources of interest in magnetic surveying. (a) Explain the origin of these near-surface sources. (b) How can they be minimized, either by measurement procedures or processing?

12.3) Variations in the terrestrial magnetic field over a survey region are more difficult to determine and remove from survey measurements than is

the case for the planetary gravity field. (a) Why is this so? (b) How is the terrestrial magnetic field determined and removed from magnetic surveys?

12.4) Differentiate between the determination and use of the International Geomagnetic Reference Field and the Definitive Geomagnetic Reference Field.

12.5) What is a typical range of values for the secular variation of the terrestrial magnetic field? How is this variation specified in the International and Definitive Geomagnetic Reference Fields?

12.6) In what conditions can the terrestrial magnetic field be considered a first-order surface for the reduction of magnetic measurements? Explain.

12.7) The International Geomagnetic Reference Field was not implemented until magnetic measurements from Earth-orbiting satellites were available. Explain why the availability of these measurements made determination of this reference field possible.

12.8) The terrestrial geomagnetic reference fields are generally truncated at approximately degree 13. (a) Why? (b) Why is the truncation degree open to controversy? (c) What is the effect on lithospheric magnetic anomalies of varying the truncation degree?

12.9) (a) What is the approximate vertical gradient of the terrestrial magnetic field at the surface of the Earth? (b) What are the implications of this value for the reduction of magnetic measurements to anomalies?

12.10) A high-sensitivity aeromagnetic survey with a target precision of 0.3 nT is being made at a mean terrain clearance of 100 m, in a terrain with an elevation range from 600 to 950 m. What is the limit of the variability in terrain clearance of the aircraft needed to achieve the target precision without making an elevation correction to the magnetic observations?

12.11) Do all magnetic terrain corrections of surface magnetic surveys all of the same sign as the terrain corrections in gravity surveys? Explain.

12.12) Magnetic measurements subject to terrain effects are sometimes low-pass filtered using an upward-continuation filter to minimize the terrain effects. (a) What is the basis for this approach? (b) What is the inherent danger for magnetic interpretation in using this approach? (c) Suggest a procedure to determine the optimum upward-continuation elevation for eliminating terrain effects while not perturbing geological interpretation?

12.13) Magnetic gradient measurements are advantageous because they require minimum reduction to make them useful in magnetic interpretation. Explain this statement.

12.14) What are the major sources of temporal variations that perturb magnetic measurements made in geophysical exploration, and the range of their amplitude, over periods of less than a day?

12.15) What are the auroral zones, what are their origin, where are they located and why, and what is their impact on magnetic measurements made for geological exploration.

12.16) Explain the role of the Sun in causing temporal variations in the terrestrial magnetic field.

12.17) (a) How are time-varying magnetic fields that may cause errors in magnetic observations used in geological exploration monitored and removed? (b) What are the relative advantages and disadvantages of these methods?

12.18) (a) Why is it necessary to level the observations of airborne magnetic surveys and how is this accomplished? (b) What is the effect of an inadequate leveling on the interpretation of magnetic anomalies? Give examples.

12.19) Is it possible to filter magnetic anomalies to isolate sources in a specified depth range? Explain.

12.20) Explain how wavenumber correlation filtering can be used to remove unwanted components of the magnetic field which are not derived from static sources within the lithosphere.

12.21) Reduction-to-the-pole filtering is widely used in magnetic processing to enhance interpretation of magnetic anomalies. (a) Explain how interpretation is improved by this procedure. (b) What are potential problems in this procedure and how can they be avoided or minimized?

12.22) Explain the role of Poisson's relation in processing of magnetic data to enhance the interpretation of magnetic anomalies.

12.23) Equivalent-source methods have found a useful role in processing of magnetic measurements to improve magnetic anomaly interpretation. Explain this method and how it is useful in processing magnetic anomaly data.

12.24) (a) Compare the impact of various wavelength filters as shown in Figures 12.10, 12.11, and 12.12 on the interpretation of magnetic anomalies. (b) What is the disadvantage of using the anomalies shown on these maps in iterative forward modeling to interpret the magnetic data?

12.25) Use the total field magnetic effect of the infinite horizontal cylinder from **Study Question 9.15(b)** to show that magnetization variations integrate in the source domain in the same way as their magnetic effects integrate in the signal domain. Specifically, (a) write down the total field magnetic effect of a horizontal cylinder of radius  $R_1$  and magnetization  $J_1$  in which a second cylinder of radius  $R_2 = 0.1R_1$  and magnetization  $J_2 = 2 \times J_1$  is embedded. Assume that the axis of the second cylinder is located 225° clockwise from the top at  $0.5R_1$ . (b) If the regional anomaly is taken as the magnetic effect of the first cylinder and subtracted from the total effect in (a), how is the magnetization derived from the inversion of the residual anomaly on the second cylinder related to  $J_2$ ?

# 1.13 Magnetic Anomaly Interpretation

13.1) The assumption of source two-dimensionality, that is, strike infinite source, is desirable in magnetic interpretation because of the increased ease of interpretation in comparison with 3D analysis. However, this assumption is often not justifiable for deep sources that have considerable depth extent. Using 2.5D magnetic anomaly computational software, (a) calculate magnetic anomalies from a square-shaped cross-section source which is strike infinite with dimensions of  $1 \times 1$  km with its upper surface at 1 km and which has a magnetic susceptibility contrast of 0.03 SIu with the surrounding rock unit. (b) Compare this anomaly with a similar source only 4 km long on either side of the computational profile. (c) Repeat these calculations for a body whose top surface is 5 km and (d) compare the validity of

the two-dimensionality assumption for the shallow and deeply buried source with a strike length of 4 km on either side of the profile. (e) Repeat (b) and (c) but compare the two-dimensionality for the source with a body that is 1 km wide and 10 km in depth extent. (f) On the basis of these results describe the effect of depth and depth extent of a source of magnetic anomalies on the assumption of two-dimensionality and compare these results with those obtained for similar gravity calculations in **Study Question 7.1**).

13.2) (a) What are the key geologic variables that determine whether a specific subsurface source can be identified by a magnetic survey? Which of these are the most important and why, in searching for (b) sulfide ore deposits at a geomagnetic inclination of  $70^{\circ}$  within the upper 100 m of a metamorphic rock terrane that is less magnetic than the ore deposits, (c) a fault which vertically displaces the basement of a sedimentary basin located at  $60^{\circ}$  inclination, (d) an igneous intrusive consisting of a steeply dipping dike and associated series of sills which intrude into the near horizontal layering of the sedimentary rocks in a basin located at  $45^{\circ}$  inclination, (e) a north-south striking magnetic dike which intrudes into a basement complex in northern South America where the inclination of the terrestrial field is nearly horizontal, (f) an east-west striking magnetized dike which intrudes into a basement complex in northern South America where the inclination of the terrestrial field is nearly horizontal, and (g) a magnetic drum in a landfill in the central United States.

13.3) (a) What are the key geophysical variables that are important in interpreting the source of magnetic anomalies. Which of these are the most important and why in searching for (b) sulfide ore deposits at a geomagnetic inclination of  $70^{\circ}$  within the upper 100 m of a metamorphic rock terrane that is less magnetic than the ore deposits, (c) a fault which vertically displaces the basement of a sedimentary basin located at  $60^{\circ}$  inclination, (d) an igneous intrusive consisting of a steeply dipping dike and associated series of sills which intrude into the near horizontal layering of the sedimentary rocks in a basin located at  $45^{\circ}$  inclination, (e) a north-south striking magnetic dike which intrudes into a basement complex in northern South America where the inclination of the terrestrial field is nearly horizontal, (f) an eastwest striking magnetized dike which intrudes into a basement complex into a basement complex in northern South America where the inclination of the terrestrial field is nearly horizontal, and (g) a magnetic drum in a landfill in the central United States.

13.4) Figure 1.13(a) shows the total magnetic intensity anomaly of



Figure 1.13 (a) Total magnetic intensity anomaly in northwestern Indiana. The contour interval is 150 nT. The geomagnetic field inclination is  $72^{\circ}$ N, and the survey elevation is 0.3 km (1,000 ft) above the ground surface. (b) The second vertical derivative of the total magnetic intensity anomaly shown in map (a). The contour interval is 100 nT/1.5 km<sup>2</sup>. Adapted from Rudman and Blakely (1965). See **Study Question 13.4**.

an isolated gravity and magnetic anomaly in northwestern Indiana (with  $D = 1^{\circ}$ E and  $I = 72^{\circ}$ N) that was observed at an elevation of approximately 0.3 km (1000 ft) above the essentially flat ground surface. The contour interval is 150 nT. *Figure 1.13(b)* shows the calculated second vertical derivative map of *Figure 1.13(a)*. The contour interval is 100 nT/1.5 km<sup>2</sup>. The geology of the region consists of Proterozoic crystalline basement overlain by relatively flat-lying Paleozoic sedimentary rocks. (a) Determine the depth to the top of the anomaly source using a variety of graphical depth determination techniques as described in Chapter 13. (b) Explain possible sources of the differences in the depth determined by the various methods. (c) What is the approximate configuration of the source? (d) Show a planar view of the source on the basement surface of the aeromagnetic map.

13.5) (a) Calculate equivalent (within 10%) total magnetic intensity anomalies from two idealized subsurface sources located at the same location using geologically reasonable parameters - dimensions, shape, and magnetization contrast - that have depths that vary by at least 200%. (b) What lessons can be learned from this exercise?

13.6) Using the Equation 13.39 for the total magnetic intensity anomaly

from a two-dimensional tabular body uniformly magnetized by induction, (a) give the symmetry relations on either side of the center of the body for both the angle and log term of the equation. (b) Show why all north-south trending vertical dike anomalies are symmetrical. (c) Show why steeply dipping dike anomalies in high magnetic latitudes are symmetrical. (d) Show why a north-south tabular body in low-magnetic latitudes will produce no anomaly. (e) Derive the equation for the magnetic relief over an infinite vertical contact between two rock masses extending to great depths and extending over large horizontal distances at high-magnetic latitudes. (f) Considering a vertical dike striking east-west, at what magnetic latitude is the anomaly made up only of the log term of the equation and as such what is the shape of the anomaly?

13.7) An igneous sill that can be approximated by a thin horizontal sheet located at a depth of roughly 500 m creates a total magnetic anomaly with a total relief of 20 nT at the margin of the sill. Assume that the direction of magnetization is vertical, that the sill has a magnetite volume contrast with the surrounding sedimentary rocks of 1%, and the magnetic susceptibility of the magnetite is 0.03 SIu. Determine using Figure 13.61 the approximate thickness of the sill. The intensity of the total terrestrial magnetic field is 55,000 nT.

13.8) Near Reykjavik, Iceland an air-filled lava tube just outside is believed to occur within a surface basalt flow with mean magnetite content of 3% by volume. Assume the lava tube strikes N-S and is 9 feet in radius with its central axis located at a depth of 21 feet from the horizontal surface. Using a magnetization intensity of the Earth's normal magnetic field of 5 October, 2001 on the surrounding basalt, calculate and plot along a profile perpendicular to the axis of the tube and extending 90 feet from it the (a) total intensity, (b) horizontal intensity, and (c) vertical intensity for polarization inclinations of 90°, 45°, 0°, -45°, and -90°. (d) How do these results compare to the induced effects of the lava tube? (e) Using the plotted values calculated for the I = I' = 90° inclination, determine the depth to the anomalous feature by the Peter's half-slope method. (f) Explain the possible causes for the difference between this value and the specified depth used in the calculation of the anomalies.

13.9) Near Toledo, Ohio, a vertical granitic dike with 1% magnetite by volume has intruded mafic gneiss with 3% volume magnetite. The east-west striking dike is 120 feet wide and extends to a depth of 24,000 feet from

its top at a depth of 240 feet below the level observation surface. Assuming the components of the Earth's normal magnetic field of 5 October, 2001, calculate and plot a strike-perpendicular profile to a distance of 840 feet from its center the (a) total intensity, (b) horizontal intensity, and (c) vertical intensity for polarization inclinations of  $90^{\circ}$ ,  $45^{\circ}$ ,  $0^{\circ}$ ,  $-45^{\circ}$ , and  $-90^{\circ}$ . (d) How do these results compare to the induced magnetic effects of the dike? (e) Using the plotted values for the  $90^{\circ}$  inclination, determine the depth to the anomalous feature by the Peter's half-slope method and compare it with the known depth. Hint: consider the source dimensions in establishing the depth as indicated in Figure 13.26.

13.10) Consider a N-S striking vertical contact between a sedimentary sequence which has no magnetite and granite which has 2.5% magnetite by volume. The top of the vertical contact is 120 feet below the level surface and the bottom is at 300 feet. Assuming the Earth's normal magnetic field is 60,000 nT with declination N30°W and inclination  $60^{\circ}$ N, calculate and plot along a strike-perpendicular profile the (a) total intensity, (b) horizontal intensity, and (c) vertical intensity for polarization inclinations of  $90^{\circ}$ ,  $45^{\circ}$ ,  $0^{\circ}$ ,  $-45^{\circ}$ , and  $-90^{\circ}$ . (d) How do these results compare with the induced magnetic effects of the dike? (e) Using the plotted values for the  $90^{\circ}$  inclination, determine the depth to the anomalous feature by the Peter's half-slope method. Hint: consider the source dimensions in establishing the depth as indicated in Figure 13.26.

13.11) As an exercise in qualitative map interpretation, classify the magnetic anomalies in *Figure 1.14* and *Figure 1.15* for (a) anomalies possibly caused by a change in basement rock composition (intra-basement anomalies), (b) minor anomalies that might be caused by local rises of basement surface (supra-basement anomalies), (c) anomalies of intermediate character that might reflect large rises, but probably are caused, at least in part, by intra-basement contrasts, and (d) linear anomaly trends (magnetic linearments) originating from basement faulting or contacts between different magnetic units.

13.12) As an exercise in qualitative profile interpretation, consider the magnetic anomaly profile in **Figure 1.16**. Draw a cross-section of the basement surface. Draw the surface relative to the point where the basement is shallowest (or deepest) using an arbitrary scale.

13.13) As an exercise in quantitative interpretation of magnetic profile



Figure 1.14 Total magnetic intensity map; geomagnetic inclination 67°. The contour interval is 25 nT. See **Study Question 13.11**.

data, estimate the depth to the magnetic source of the total magnetic intensity (TMI) anomaly profile in *Figure 1.17* by the (a) straight slope, (b) the half-slope, and (c) the Sokolov methods.

13.14) As a check on the depth estimate from **Study Question 13.13**, (a) compute the depth from half-slopes using the horizontal gradient (HG) of the first horizontal derivative (FHD) profle of the total magnetic intensity (TMI) shown in *Figure 1.18*. (b) Is it easier to compute half-slope depth from the horizontal gradient profile or from the total magnetic intensity profile? Explain. (c) What are the limitations of using FHD profiles?

13.15) The vertical gradient (VG) or first vertical derivative (FVD) of the total magnetic intensity (TMI) anomaly of the anomaly shown in *Figure 1.17* is shown in *Figure 1.19*. (a) Using the maximum values of the TMI and VG anomalies (at the peak) and the depths estimated in **Study Questions 13.13** or **13.14**, determine the value of the structural index, N, using the Euler deconvolution equation. (b) What is the nature or type of source

Study Questions



Figure 1.15 Total magnetic intensity map; geomagnetic inclination  $75^{\circ}$ . The contour interval is 10 nT. See **Study Question 13.11**.

of the magnetic anomaly? Hint: At the peak or trough of the TMI, the HG or FHD has a value of zero.

13.16) This exercise illustrates upgrading of the graphical methods of depth determination taking into account the results of an initial source depth estimate. Referring to Figure 13.27, (a) obtain a depth estimate directly from the map by the half-slope method. Using the appropriate map scale, measure the horizontal distance (HSL) between the two points of half-maximum slope (HMS) determined based on contour spacing. As a first approximation, determine the depth by dividing HSL by 1.6. (b) Measure the horizontal distance w/2 from the crest of the anomaly or maximum anomaly (MA) to the point of maximum slope (MS) or inflection point. This distance is approximately equal to the half-width of the magnetic source. By referring to the curve for HSL (peter's method) in Figure 13.26, determine a more appropriate and reliable value of the dividing factor using w/2. Divide the HSL by the new factor to yield an updated estimate of the depth. (c) For comparison, obtain a depth estimate directly on the map by measuring the



Figure 1.16 Total magnetic intensity profile over the Canadian Arctic Islands; geomagnetic inclination 85°. See **Study Question 13.12**.



Figure 1.17 Total magnetic intensity anomaly profile; geomagnetic inclination 90°. See **Study Question 13.13**.



Figure 1.18 Total magnetic intensity anomaly and horizontal magnetic gradient anomaly profiles; geomagnetic inclination 90°. See **Study Question 13.14**.

straight slope length (SSL) using a dividing factor of 0.7 or 0.8 (see Figure 13.26). (d) Draw a profile across the anomaly in Figure 13.27 perpendicular to its strike along the northwest-southeast line shown on the map using a suitable scale. Estimate the depth using the HSL (Peter's) profile method, and, as previously, update the depth using the w/2 distance measured on the profile as indicated in Figure 13.27. Do you expect the map method to be less accurate than the profile method? Why?

13.17) Using the method described in **Study Question 13.16**, estimate the depths to the sources of the anomalies mapped in (a) *Figure 1.20*, (b) Figure 1.21, and (c) Figure 1.22.

13.18) Apply Euler deconvolution to verify the value of the structural index, N, for the magnetic anomaly data given in (a) Figure 13.31, (b) Figure 13.32, and (c) Figure 13.33. In each case, use the extreme values at the peak (or troughs) of the TMI profile (that is, where FHD or HG = 0) and the known source depth to verify the correct value of N. Do the same to verify the values of N for the gravity anomaly data in (d) Figure 7.17, (e) Figure



Figure 1.19 The vertical gradient of total magnetic intensity anomaly profile of **Study Question 13.13**, used with **Study Question 13.14** to compute the structural index, N, from the Euler deconvolution equation; geomagnetic inclination 90°. See **Study Question 13.15**.



Figure 1.20 Total magnetic intensity anomaly map with a contour interval of 2 nT; geomagnetic inclination 60°. See **Study Question 13.17**.



Figure 1.21 Total magnetic intensity anomaly map with contour interval of 2 nT; geomagnetic inclination 60°. See Study Question 13.17.

7.18, and (f) Figure 7.19.

13.19) The combination of gravity and magnetic anomaly data significantly constrains anomaly interpretation. Compute the maximum (a) gravity and (b) total magnetic intensity anomalies for the shallow-sphere and deep-basement fault shown in *Figure 1.23*. How do these structures compare in terms of their gravity and magnetic anomaly amplitudes? (c) What implications may be drawn from these results for effectively mapping the gravity and magnetic effects of the structures? (d) Using the chart in Figure 13.60, verify the value of the maximum magnetic effect of the basement fault. (e) What would the displacement on the basement fault have to be at that depth for the fault to have the same maximum magnetic effect as that due to the shallow sphere?



Figure 1.22 Total magnetic intensity anomaly map and principal profile. The contour interval of the map is 10 nT. The geomagnetic inclination is  $90^{\circ}$ . See **Study Question 13.17**.



Figure 1.23 Parameters of a shallow spherical source and deep basement fault for the computation of gravity and total magnetic anomaly profiles. See **Study Question 13.19**.

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