Applied Quantum Mechanics

Chapter 9 problems

LAST NAME FIRST NAME

Problem 9.1

A new type of nano-wire laser is to be designed with optical emission at $\lambda = 1550\ \mathrm{nm}$. The laser consists of an InGaAs semiconductor active region with refractive index $n_{\mathrm{InGaAs}} = 4$ inside a Fabry-Perot cavity of length L_C. The InGaAs is of width 40 nm and of thickness 40 nm. The sides of the wire are embedded in InP which has a refractive index of $n_{\mathrm{InP}} = 3.22$. One mirror end of the wire is faced with gold and the other end is bonded to sapphire (Al_2O_3) which has refractive index $n_{Al_2O_3} = 1.78$.

(a) Estimate the optical confinement factor of the structure.

(b) Assume that the optical reflectivity of the mirror at the semiconductor gold interface is 0.99. Find the reflectivity of the mirror formed at the semiconductor sapphire interface.

(c) Assume a spontaneous emission coefficient $\beta = 10^{-4}$, optical confinement factor $\Gamma = 0.1$, non-radiative recombination rate $A_{\mathrm{nr}} = 10^{8}\ \mathrm{s}^{-1}$, radiative recombination coefficient $B = 10^{-10}\ \mathrm{cm}^3\ \mathrm{s}^{-1}$, non-linear recombination coefficient $C = 5 \times 10^{-29}\ \mathrm{cm}^6\ \mathrm{s}^{-1}$, a bulk gain model with optical transparency at carrier density $n_0 = 10^{18}\ \mathrm{cm}^{-3}$, optical gain slope coefficient $G_{\mathrm{slope}} = 2 \times 10^{-16}\ \mathrm{cm}^2\ \mathrm{s}^{-1}$, optical gain saturation coefficient $\varepsilon_{\mathrm{bulk}} = 5 \times 10^{-18}\ \mathrm{cm}^3$, and internal optical loss $\alpha_{\mathrm{int}} = 10\ \mathrm{cm}^{-1}$. Calculate the *L-I* characteristics and threshold current for a wire of length $L_C = 10\ \mu\mathrm{m}$, $L_C = 30\ \mu\mathrm{m}$, and $L_C = 100\ \mu\mathrm{m}$.

(d) Discuss the results of your calculations and how you might improve the design to decrease laser threshold current.

Problem 9.2

Driving a laser with a short electrical pulse can generate an optical pulse.

(a) Using the laser diode described in Exercise 9.4, plot the full-width at half-maximum (FWHM) of laser optical pulse output as a function of electrical pulse width. The electrical pulse has a rectangular shape with maximum value $I_{\max} = 50$ mA and minimum value $I_{\min} = 0$ mA and width τ.

(b) What happens to the minimum optical pulse width if $I_{\min}$ is allowed to increase such that $I_{\max} > I_{\min} > 0$ mA ?

(c) Find values of $I_{\min}$, $I_{\max}$, and τ that on average result in emission of one photon per pulse.

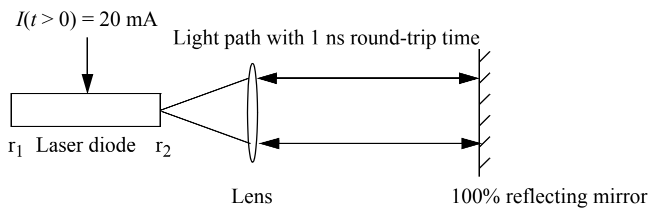
Problem 9.3

(a) Plot the noise-free transient and steady-state behavior for the laser diode of Exercise 9.4 but with each mirror having reflectivity 0.999, active region volume $2 \times 1 \times 0.2\ \mu\mathrm{m}^3$ with cavity length $2\ \mu\mathrm{m}$, and a step current of $100\ \mu\mathrm{A}$.

(b) Repeat (a) only now in the presence of uncorrelated Gaussian photon and carrier number noise, i.e. use Eq. (9.85) and Eq. (9.86), but ignore Eq. (9.87).

(c) Use (b) to numerically find the mean and standard deviation of photon number for times long after the transient associated with the step current being turned on.

Problem 9.4
Modify the computer program used in Exercise 9.4 to demonstrate cavity formation, as illustrated in Fig. 9.18. In your solution you should apply delayed rate equations using the same device parameters as in Exercise 9.4, but with a 1 ns round-trip photon delay from a 100% reflecting external cavity mirror as indicated in the following figure.



Problem 9.5
The probability of s discrete photons in the lasing mode and n electrons in a laser diode is P_{ns}.

(a) Write down an equation for the time evolution of P_{ns}.

(b) Sketch how you expect P_{ns} to evolve as the injection current in a laser diode is increased.

Problem 9.6
In a semiconductor optical gain medium electron scattering times are short compared to the spontaneous emission time and carriers relax to a quasi-equilibrium described by the Fermi-Dirac distribution, f_k. When carrier density, n, is small (and/or temperature, T, is high) such that the chemical potential $\mu \ll E_F$, where E_F is the Fermi energy, then f_k may be approximated by a Maxwell-Boltzmann distribution. Assuming a reduced electron mass m_r such that $\frac{1}{m_r} = \frac{1}{m_e^*} + \frac{1}{m_{hh}^*}$ where m_e^* is the conduction band effective electron mass and m_{hh}^* is the heavy hole effective mass, show that the *total* photon spontaneous emission rate is proportional to n^2 and has temperature dependence $T^{-3/2}$. How is the n^2 dependence modified when n becomes large?