A First Course in Digital Communications

Ha H. Nguyen and E. Shwedyk

CAMBRIDGE UNIVERSITY PRESS

February 2009

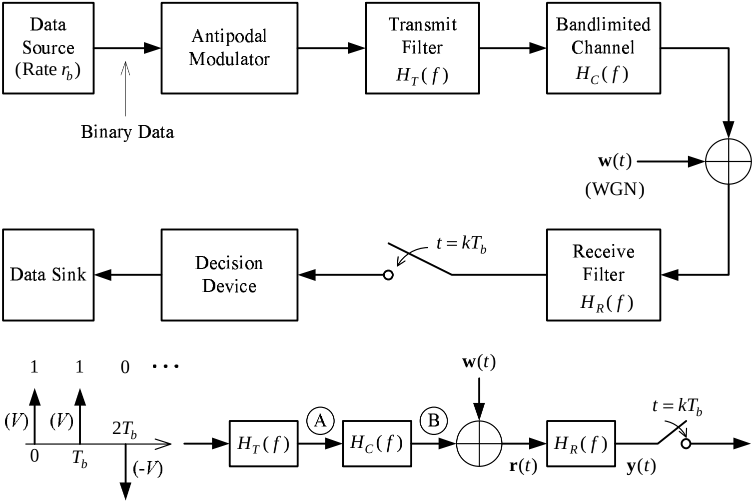
Introduction

- Have considered only the detection of signals transmitted over channels of infinite bandwidth.
- Bandlimited channels are common: telephone channel, or even fiber optics, etc.
- Bandlimitation depends not only on the channel media but also on the source, specifically the source rate, R_s (symbols/sec).
- Band limitation can also be imposed on a communication system by regulatory requirements.
- The general effect of band limitation on a transmitted signal of finite time duration is to *disperse* (or *spread*) it out $\Rightarrow$ Signal transmitted in a particular time slot interferes with signals in other time slots $\Rightarrow$ *inter-symbol interference*(ISI).
- Shall consider the demodulation of signals which are not only corrupted by additive, white Gaussian noise but also by ISI.

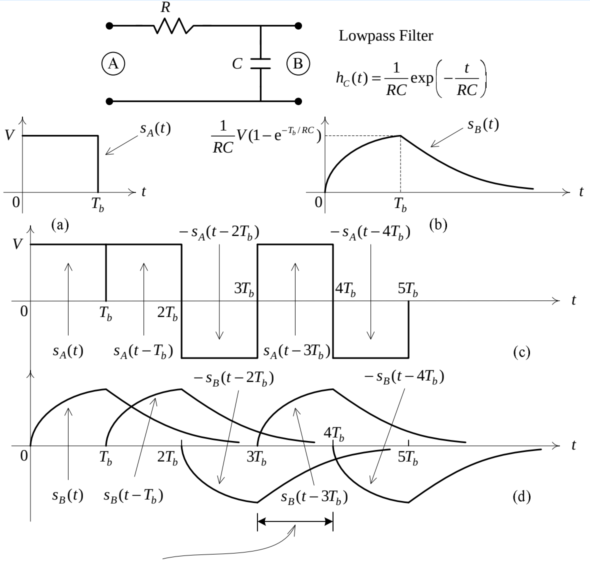
Major Approaches to Deal with ISI

1. Force the ISI effect to zero $\Rightarrow$ Nyquist's first criterion.
2. Allow some ISI but in a controlled manner $\Rightarrow$ Partial response signaling.
3. Live with the presence of ISI and design the best demodulation for the situation $\Rightarrow$ Maximum likelihood sequence estimation (Viterbi algorithm).

Communication System Model



ISI Example



During this interval, $\mathbf{r}(t) = \mathbf{b}_0 s_B(t) + \mathbf{b}_1 s_B(t-T_b) + \mathbf{b}_2 s_B(t-2T_b) + \mathbf{b}_3 s_B(t-3T_b) + \mathbf{w}(t)$

Nyquist Criterion for Zero ISI

$$\mathbf{y}(t) = \sum_{k=-\infty}^{\infty} \mathbf{b}_k s_R(t - kT_b) + \mathbf{w}_o(t),$$

where $s_R(t) = h_T(t) * h_C(t) * h_R(t)$ is the overall response of the system due to a unit impulse at the input.

$$\mathbf{b}_k = \begin{cases} V & \text{if the } k\text{th bit is "1"} \\ -V & \text{if the } k\text{th bit is "0"} \end{cases}.$$

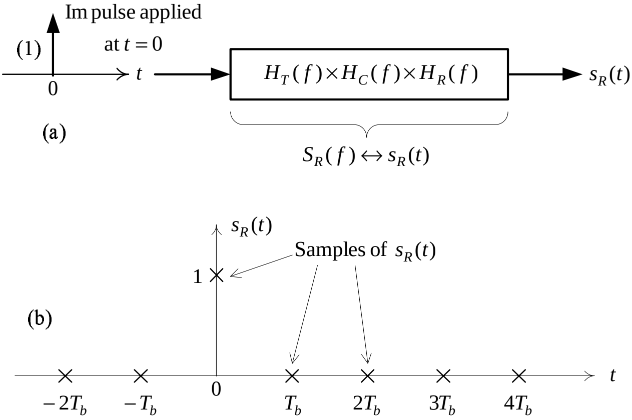
Normalize $s_R(0) = 1$. Look at sampling time $t = mT_b$:

$$\mathbf{y}(mT_b) = \mathbf{b}_m + \underbrace{\sum_{\substack{k=-\infty \\ k\neq m}}^{\infty} \mathbf{b}_k s_R(mT_b - kT_b)}_{\text{ISI term}} + \mathbf{w}_o(mT_b).$$

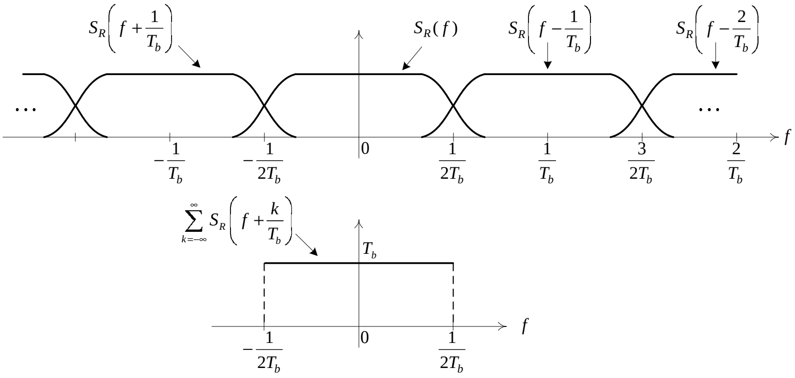
What are the conditions on the overall transfer function $S_R(f) = H_T(f)H_C(f)H_R(f)$ which would make ISI term zero?

Time-Domain Nyquist's Criterion for Zero ISI

The samples of $s_R(t)$ due to an impulse should be 1 at $t = 0$ and zero at all other sampling times kT_b $(k \neq 0)$.

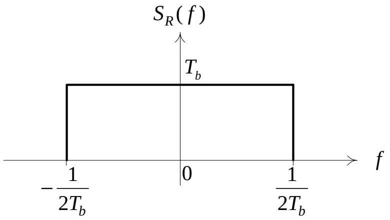


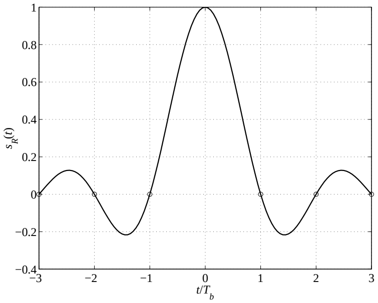
Frequency-Domain Nyquist's Criterion for Zero ISI



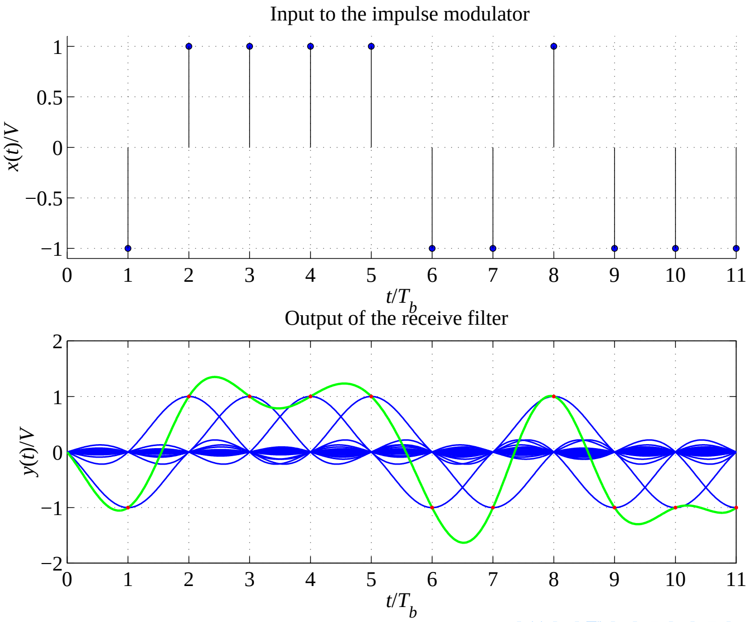
- If $W < \frac{1}{2T_b}$ $\Rightarrow$ ISI terms cannot be made zero.
- If $W = \frac{1}{2T_b}$ $\Rightarrow$ $S_R(f) = T_b$ over $f \le |\frac{1}{2T_b}|$, $s_R(t) = \frac{\sin(\pi t/T_b)}{(\pi t/T_b)}$.
- If $W > \frac{1}{2T_b}$ $\Rightarrow$ Infinite number of $S_R(f)$ to achieve zero ISI.

Pulse Shaping when $W = \frac{1}{2T_b}$





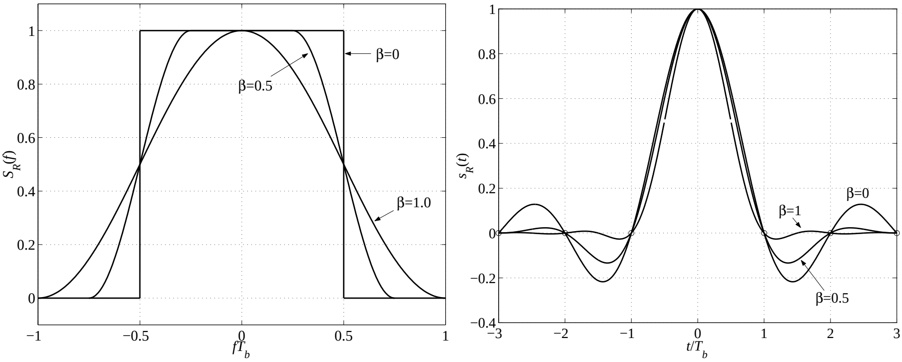
$s_R(t)$ decays as $1/t \Rightarrow$ if the sampler is not perfectly synchronized in time, considerable ISI can be encountered.

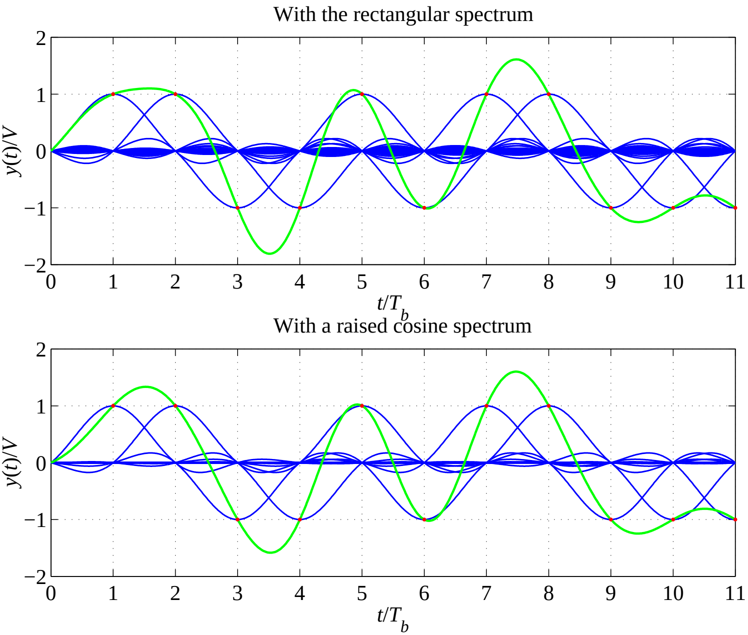

Input to the impulse modulator
x(t)/V
1
0.5
0
−0.5
−1
0 1 2 3 4 5 6 7 8 9 10 11
t/T_b
Output of the receive filter
y(t)/V
2
1
0
−1
−2
0 1 2 3 4 5 6 7 8 9 10 11
t/T_b

Raised Cosine Pulse Shaping

$$S_R(f) = S_{\mathrm{RC}}(f) = \begin{cases} T_b, & |f| \le \frac{1-\beta}{2T_b} \\ T_b \cos^2\left[\frac{\pi T_b}{2\beta}\left(|f| - \frac{1-\beta}{2T_b}\right)\right], & \frac{1-\beta}{2T_b} \le |f| \le \frac{1+\beta}{2T_b} \\ 0, & |f| \ge \frac{1+\beta}{2T_b} \end{cases}.$$

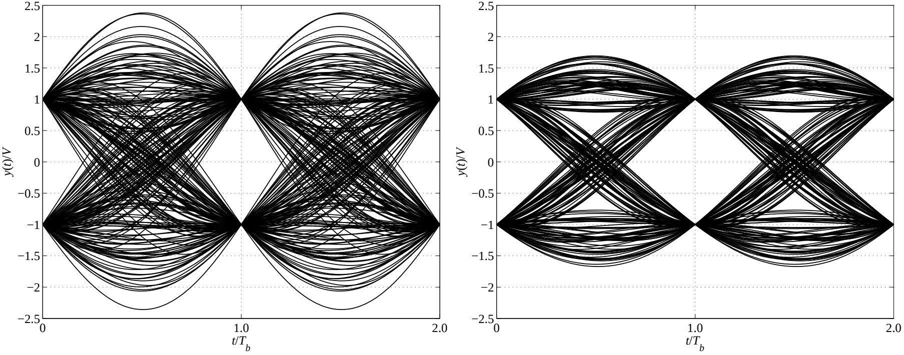
$$s_R(t) = s_{\mathrm{RC}}(t) = \frac{\sin(\pi t/T_b)}{(\pi t/T_b)} \frac{\cos(\pi\beta t/T_b)}{1 - 4\beta^2 t^2/T_b^2} = \mathrm{sinc}(t/T_b)\frac{\cos(\pi\beta t/T_b)}{1 - 4\beta^2 t^2/T_b^2}.$$




With the rectangular spectrum
y(t)/V
2
1
0
−1
−2
0 1 2 3 4 5 6 7 8 9 10 11
t/T_b
With a raised cosine spectrum
y(t)/V
2
1
0
−1
−2
0 1 2 3 4 5 6 7 8 9 10 11
t/T_b

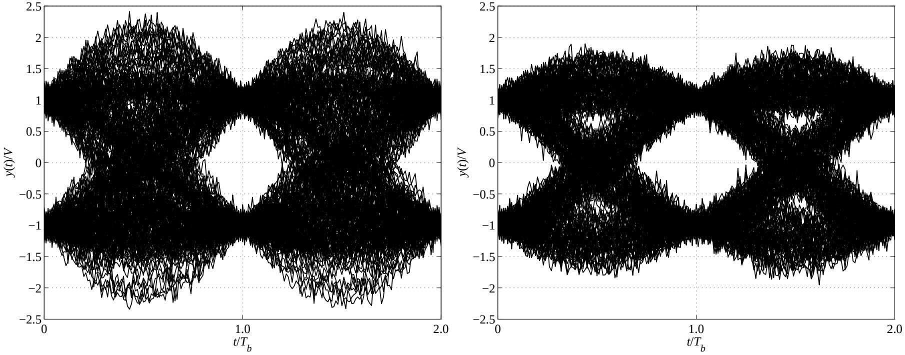
Eye Diagrams

To observe and measure (qualitatively) the effect of ISI.



Left: Ideal lowpass filter, Right: A raised-cosine filter with $\beta = 0.35$.

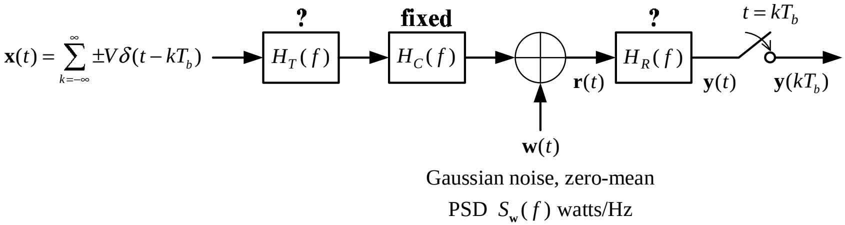
Eye Diagrams with SNR$= V^2/\sigma_{\mathbf{w}}^2 = 20$ dB



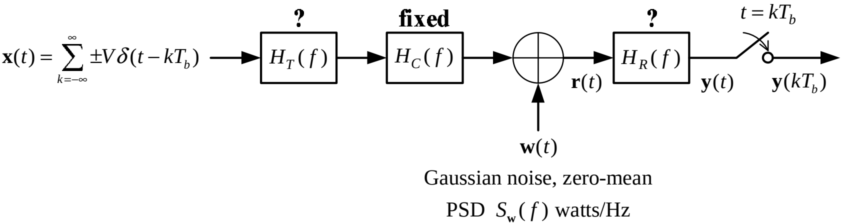
Left: Ideal lowpass filter, Right: A raised-cosine filter with $\beta = 0.35$.

Design of Transmitting and Receiving Filters

- Have shown how to design $S_R(f) = H_T(f)H_C(f)H_R(f)$ to achieve zero ISI.
- When $H_C(f)$ is fixed, one still has flexibility in the design of $H_T(f)$ and $H_R(f)$.
- Shall design the filters to minimize the probability of error.

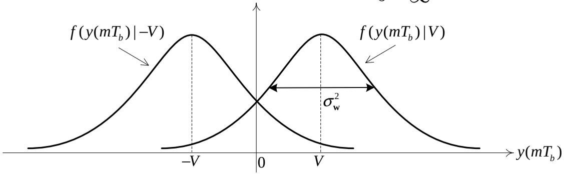


- Noise is assumed to be Gaussian (as usual) but *does not necessarily have to be white.*

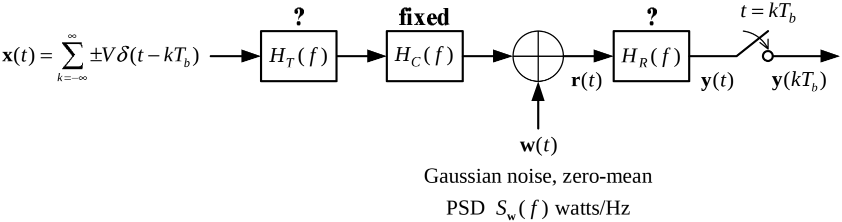


$$\mathbf{y}(mT_b) = \pm V + \mathbf{w}_{\mathrm{o}}(mT_b),$$

where $\mathbf{w}_{\mathrm{o}}(mT_b) \sim \mathcal{N}(0, \sigma^2_{\mathbf{w}})$, with $\sigma^2_{\mathbf{w}} = \int_{-\infty}^{\infty} S_{\mathbf{w}}(f)|H_R(f)|^2 \mathrm{d}f$.



Since $P[\text{error}] = Q\left(\frac{V}{\sigma_{\mathbf{w}}}\right) \Rightarrow$ Need to maximize $\frac{V^2}{\sigma^2_{\mathbf{w}}}$.



Given the transmitted power P_T, the channel's frequency response $H_C(f)$ and the additive noise's power spectral density $S_{\mathbf{w}}(f)$ choose $H_T(f)$ and $H_R(f)$ so that the zero ISI criterion is satisfied and the $\mathrm{SNR} = \frac{V^2}{\sigma_{\mathbf{w}}^2}$ *is maximized.*

1. Compute the average transmitted power: $P_T = \frac{V^2}{T_b}\int_{-\infty}^{\infty}|H_T(f)|^2\mathrm{d}f$ (watts).
2. Write the inverse of the SNR as

$$\begin{aligned}\frac{\sigma_{\mathbf{w}}^2}{V^2} &= \frac{1}{P_T T_b}\left[\int_{-\infty}^{\infty}|H_T(f)|^2\mathrm{d}f\right]\left[\int_{-\infty}^{\infty}S_{\mathbf{w}}(f)|H_R(f)|^2\mathrm{d}f\right]\\ &= \frac{1}{P_T T_b}\left[\int_{-\infty}^{\infty}\frac{|S_R(f)|^2}{|H_C(f)|^2|H_R(f)|^2}\mathrm{d}f\right]\left[\int_{-\infty}^{\infty}S_{\mathbf{w}}(f)|H_R(f)|^2\mathrm{d}f\right].\end{aligned}$$

3. Apply the Cauchy-Schwartz inequality: $\left|\int_{-\infty}^{\infty}A(f)B^*(f)\mathrm{d}f\right|^2 \le \left[\int_{-\infty}^{\infty}|A(f)|^2\mathrm{d}f\right]\left[\int_{-\infty}^{\infty}|B(f)|^2\mathrm{d}f\right]$, which holds with equality if and only if $A(f)=KB(f)$. Identify $|A(f)| = \sqrt{S_{\mathbf{w}}(f)}|H_R(f)|$, $|B(f)| = \frac{|S_R(f)|}{|H_C(f)||H_R(f)|}$. Then

$$|H_R(f)|^2 = \frac{K|S_R(f)|}{\sqrt{S_{\mathbf{w}}(f)}|H_C(f)|},\quad |H_T(f)|^2 = \frac{|S_R(f)|\sqrt{S_{\mathbf{w}}(f)}}{K|H_C(f)|}.$$

Design Under *White* Gaussian Noise

- For white noise (at least flat PSD over the channel bandwidth):

$$
\begin{aligned}
|H_R(f)|^2 &= K_1 \frac{|S_R(f)|}{|H_C(f)|}, \\
|H_T(f)|^2 &= K_2 \frac{|S_R(f)|}{|H_C(f)|} = \frac{K_2}{K_1} |H_R(f)|^2,
\end{aligned}
$$

- $K_1,\ K_2$ set power levels at transmitter and receiver.
- The transmit and receive filters are a *matched filter pair*:

$$
\begin{aligned}
H_R(f) &= |H_R(f)| \mathrm{e}^{j\angle H_R(f)}, \\
H_T(f) &= K|H_R(f)| \mathrm{e}^{j\angle - H_R(f)},
\end{aligned}
$$

- The maximum output SNR is

$$
\left(\frac{V^2}{\sigma_{\mathbf{w}}^2}\right)_{\max} = P_T T_b \left[\int_{-\infty}^{\infty} \frac{|S_R(f)|\sqrt{S_{\mathbf{w}}(f)}}{|H_C(f)|} \mathrm{d}f\right]^{-2}.
$$

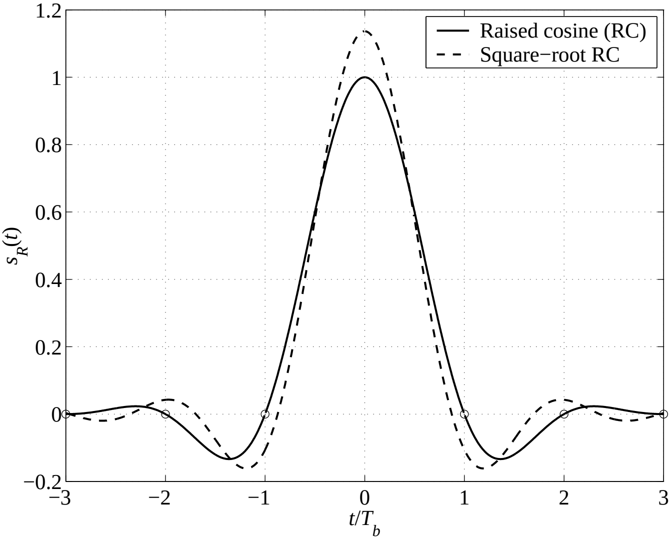
Design Under *White* Gaussian Noise and Ideal Channel

- If the channel is ideal, i.e., $H_C(f) = 1$ for $|f| \leq W$ and $K_1 = K_2$ then $|H_T(f)| = |H_R(f)| = \sqrt{|S_R(f)|}/\sqrt{K_1}$.
- If $S_R(f)$ is a raised-cosine spectrum then both $H_T(f)$ and $H_R(f)$ are *square-root raised-cosine* (SRRC) spectrum:

$$H_T(f) = H_R(f) = \begin{cases} \sqrt{T_b}, & |f| \leq \frac{1-\beta}{2T_b} \\ \sqrt{T_b}\cos\left[\frac{\pi T_b}{2\beta}\left(|f| - \frac{1-\beta}{2T_b}\right)\right], & \frac{1-\beta}{2T_b} \leq |f| \leq \frac{1+\beta}{2T_b} \\ 0, & |f| \geq \frac{1+\beta}{2T_b} \end{cases}.$$

$$\begin{aligned} h_T(t) &= h_R(t) \\ &= s_{\mathsf{SRRC}}(t) = \frac{(4\beta t/T_b)\cos[\pi(1+\beta)t/T_b] + \sin[\pi(1-\beta)t/T_b]}{(\pi t/T_b)[1-(4\beta t/T_b)^2]}. \end{aligned}$$

RC and SRRC Waveforms ($\beta = 0.5$)



Example: Transmission rate $r_b = 3600$ bits/sec, P[bit error] $\leq 10^{-4}$. Channel model: $H_C(f) = 10^{-2}$ for $|f| \leq 2400$ Hz and $H_C(f) = 0$ for $|f| > 2400$ Hz. Noise model: $S_{\mathbf{w}}(f) = 10^{-14}$ watts/Hz, $\forall f$ (white noise).

(a) Since $r_b = 3600$ bits/sec and $W = 2400$ Hz, choose a raised-cosine spectrum with $\beta \frac{r_b}{2} = 600$ or $\beta = \frac{1}{3}$.

$$S_R(f) = \begin{cases} \frac{1}{3600}, & |f| < (1-\beta)\frac{r_b}{2} = 1200 \text{ Hz} \\ \frac{1}{3600}\cos^2\left[\frac{\pi}{2400}(|f| - 1200)\right], & 1200 \text{ Hz} \leq |f| \leq 2400 \text{ Hz} \\ 0, & \text{elsewhere} \end{cases} .$$

(b) $|H_T(f)| = K_1 |S_R(f)|^{1/2}$ and $|H_R(f)| = |S_R(f)|^{1/2}$. Since $|H_T(f)||H_C(f)||H_R(f)| = S_R(f)|$. Evaluated at $f = 0$ gives $\frac{1}{\sqrt{3600}} K_1 (10^{-2}) \frac{1}{\sqrt{3600}} = \frac{1}{3600}$, or $K_1 = 100$.

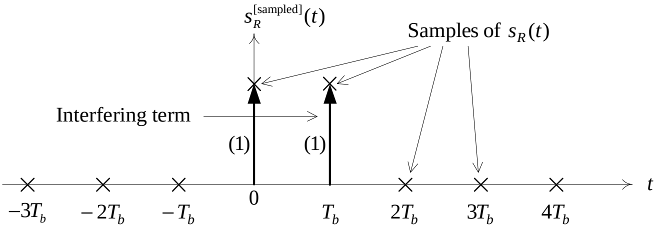
(c) $Q\left(\sqrt{\left(\frac{V^2}{\sigma_{\mathbf{w}}^2}\right)_{\max}}\right) \leq 10^{-4} \quad \Rightarrow \quad \left(\frac{V^2}{\sigma_{\mathbf{w}}^2}\right)_{\max} \geq 14.04 \approx 14.$

$$P_T = \frac{1}{T_b}\left(\frac{V^2}{\sigma_{\mathbf{w}}^2}\right)_{\max}\left[\int_{-\infty}^{\infty} \frac{|S_R(f)|\sqrt{S_{\mathbf{w}}(f)}}{|H_C(f)|} \mathrm{d}f\right]^2 = 3600 \times 14 \times \frac{10^{-14}}{10^{-4}}\left[\int_{-\infty}^{\infty} |S_R(f)| \mathrm{d}f\right]^2 .$$

But $\int_{-\infty}^{\infty} |S_R(f)| \mathrm{d}f = s_R(t)|_{t=0} = 1$. Therefore $P_T = 5$ μwatts.

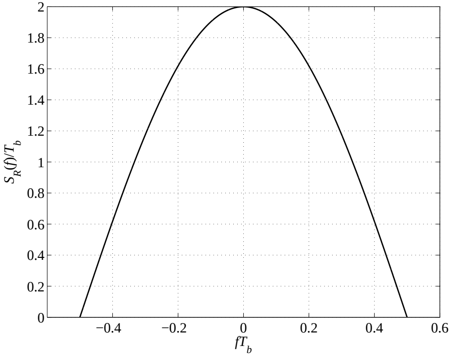
Duobinary Modulation

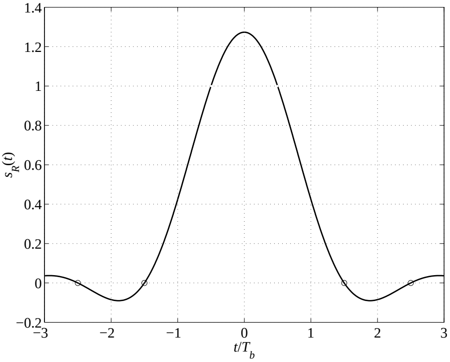
- If bandwidth is very limited and one cannot afford to use more than $\frac{1}{2T_b}$ Hz, the only way to achieve zero ISI is to have flat $S_R(f)$, which is practically difficult to implement.
- An alternative is to allow a certain amount of ISI but in a controlled manner $\Rightarrow$ *Duobinary* modulation.
- Shall restrict the ISI to only one term, namely that due to the previous symbol.



Overall System Response of Duobinary Modulation

$$S_R(f) = \begin{cases} 2T_b \cos(\pi f T_b), & -\frac{1}{2T_b} \le f \le \frac{1}{2T_b} \\ 0, & \text{elsewhere} \end{cases} \xleftrightarrow{\mathcal{F}} s_R(t) = \frac{\cos(\pi t/T_b)}{\pi\left[\frac{1}{4} - (\frac{t}{T_b})^2\right]}.$$

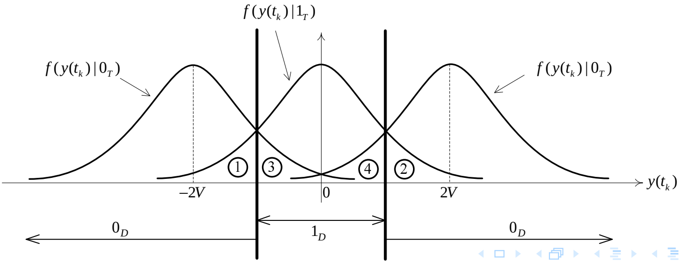




Using Precoder in Duobinary Modulation

$$\begin{aligned}\mathbf{y}(t_k) &= \mathbf{V}_k + \mathbf{V}_{k-1} + \mathbf{w}_\text{o}(t_k),\ \text{where } t_k = kT_b - \frac{T_b}{2},\ k = 0, \pm 1, \pm 2, \ldots \\ &= \begin{cases} 2V + \mathbf{w}_\text{o}(t_k), & \text{if bits } k \text{ and } (k-1) \text{ are both "1"} \\ 0 + \mathbf{w}_\text{o}(t_k), & \text{if bits } k \text{ and } (k-1) \text{ are different} \\ -2V + \mathbf{w}_\text{o}(t_k), & \text{if bits } k \text{ and } (k-1) \text{ are both "0"} \end{cases}.\end{aligned}$$

$$\begin{cases} \mathbf{b}_k = 0 \text{ then } \mathbf{d}_k = \mathbf{d}_{k-1} \Rightarrow \mathbf{y}(t_k) = \pm 2V + \mathbf{w}_\text{out}(t_k) \\ \mathbf{b}_k = 1 \text{ then } \mathbf{d}_k = \bar{\mathbf{d}}_{k-1} \Rightarrow \mathbf{y}(t_k) = 0 + \mathbf{w}_\text{out}(t_k) \end{cases}$$



- $P[\mathbf{y}(t_k) = -2V] = P[\mathbf{y}(t_k) = 2V] = \frac{1}{4}$; $P[\mathbf{y}(t_k) = 0] = \frac{1}{2}$.

$$P[\text{error}] \approx \frac{1}{4}\ \text{area④} + \frac{1}{2}[\text{area①} + \text{area②}] + \frac{1}{4}\ \text{area③} = \frac{3}{2} Q\left(\frac{V}{\sigma_{\mathbf{w}}}\right),$$

- Let $H_C(f) = 1$ over $|f| \leq \frac{1}{2T_b}$ and consider AWGN with PSD $\frac{N_0}{2}$. Then

$$\left(\frac{V^2}{\sigma_{\mathbf{w}}^2}\right)_{\max} = P_T T_b \left[\sqrt{N_0/2} \int_{-\frac{1}{2T_b}}^{\frac{1}{2T_b}} 2T_b \cos(\pi f T_b) \mathrm{d}f\right]^{-2} = (P_T T_b)\left(\frac{2}{N_0}\right)\left(\frac{\pi}{4}\right)^2$$

$$P[\text{error}]_{\text{duobinary}} = \frac{3}{2} Q\left(\frac{\pi}{4}\sqrt{\frac{2P_T T_b}{N_0}}\right).$$

- For binary PAM with zero ISI $\left(\frac{V^2}{\sigma_{\mathbf{w}}^2}\right)_{\max}$ is

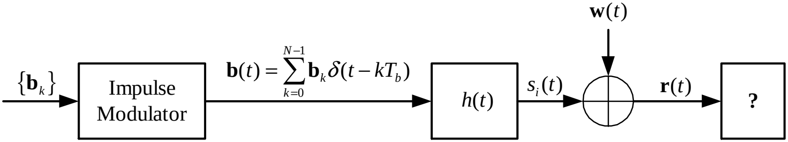
$$P_T T_b \left[\int_{-\infty}^{\infty} \frac{|S_R(f)|\sqrt{S_{\mathbf{w}}(f)}}{|H_C(f)|} \mathrm{d}f\right]^{-2} = P_T T_b \left[\sqrt{N_0/2} \int_{-\frac{1}{2T_b}}^{\frac{1}{2T_b}} |S_R(f)| \mathrm{d}f\right]^{-2} = P_T T_b \left(\frac{2}{N_0}\right)$$

$$P[\text{error}]_{\text{binary}} = Q\left[\sqrt{\frac{2P_T T_b}{N_0}}\right].$$

- Duobinary modulation requires an addition of $\left(\frac{4}{\pi}\right)^2$ or 2.1 dB to achieve the same error probability as the zero-ISI modulation.

Maximum Likelihood Sequence Estimation (MLSE)

- Do not attempt to eliminate the ISI but rather takes it into account in the demodulator.
- The criterion is to minimize the sequence error probability.
- Consider a sequence of N equally likely bits transmitted over a bandlimited channel where the transmission begins at $t = 0$ and ends at $t = NT_b$, with T_b the bit interval.
- $h(t)$ is the impulse response of the overall chain: modulator/transmitter filter/channel, assumed to be nonzero over $[0, LT_b] \Rightarrow$ The number of ISI terms is L.



Modulator/Transmitter Filter/Channel

- The receiver sees one of the $M = 2^N$ possible signals, $s_i(t) = \sum_{k=0}^{N-1} b_{i,k} h(t - kT_b)$, $i = 1, 2, \ldots, M = 2^N$, corrupted by $\mathbf{w}(t)$: This as a humongous M-ary demodulation problem in AWGN.
- The decision rule is:

Compute:

$$\gamma_i = \frac{2}{N_0} \int_{-\infty}^{\infty} r(t) s_i(t) \mathrm{d}t - \frac{1}{N_0} \int_{-\infty}^{\infty} s_i^2(t) \mathrm{d}t, \quad i = 1, 2, \ldots, M$$

and choose the *largest*.

- Substituting $s_i(t) = \sum_{k=0}^{N-1} b_{i,k} h(t - kT_b)$, the decision rule is

Compute:

$$\gamma_i = \frac{2}{N_0} \sum_{k=0}^{N-1} b_{i,k} \int_{-\infty}^{\infty} r(t) h(t - kT_b) \mathrm{d}t -$$

$$\frac{1}{N_0} \sum_{k=0}^{N-1} \sum_{j=0}^{N-1} b_{i,k} b_{i,j} \int_{-\infty}^{\infty} h(t - kT_b) h(t - jT_b) \mathrm{d}t$$

and choose the *largest*.

- Define $h_{k-j} = \int_{-\infty}^{\infty} h(t - kT_b)h(t - jT_b)\mathrm{d}t$ and $r_k = \int_{-\infty}^{\infty} r(t)h(t - kT_b)\mathrm{d}t$. Then the decision rule is

Compute:
$$\gamma_i = \frac{2}{N_0}\sum_{k=0}^{N-1} b_{i,k}r_k - \frac{1}{N_0}\sum_{k=0}^{N-1}\sum_{j=0}^{N-1} b_{i,k}b_{i,j}h_{k-j}, \ \ i = 1, 2, \ldots, M$$
and choose the *largest*.

- The path metric γ_i can be expressed as

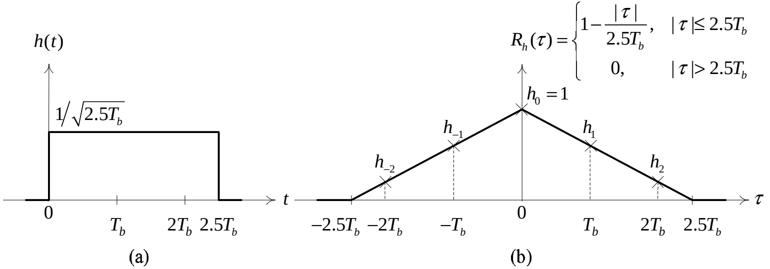
$$\gamma_i = \frac{2}{N_0}\sum_{k=0}^{N-1} b_{i,k}r_k - \frac{1}{N_0}\sum_{k=0}^{N-1} b_{i,k}^2 h_0 - \frac{2}{N_0}\sum_{k=0}^{N-1} b_{i,k}\sum_{j=1}^{k} b_{i,k-j}h_j.$$

- Note that $b_{i,k} = \pm 1$ and $\frac{b_{i,k}^2 h_0}{2} = \frac{h_0}{2}$ is a constant. Also since $h(t) = 0$ for $t \geq LT_b$, which means $h_j = 0$ for $j \geq L$, the path metric becomes

$$\gamma_i = \frac{2}{N_0} \sum_{k=0}^{N-1} \underbrace{\left\{ b_{i,k} r_k - b_{i,k} \sum_{j=1}^{L-1} b_{i,k-j} h_j \right\}}_{\text{branch metric}}.$$

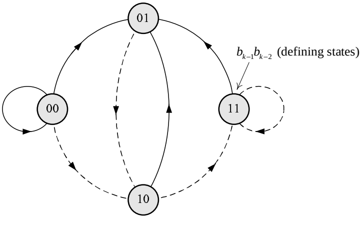
- *Branch metric* depends on: (i) the present output of the matched filter, r_k; (ii) the present value of the considered bit pattern, $b_{i,k}$; (iii) the previous $L-1$ values of the considered bit pattern $b_{i,k-1}, b_{i,k-2}, \ldots, b_{i,k-(L-1)}$.
- The system has memory, namely the ISI terms $\Rightarrow$ Can use a finite state diagram and the trellis to represent the transmitted signals.
- States are defined by the previous $L-1$ bits.
- Determination of the best path through the trellis can be accomplished most efficiently with the *Viterbi algorithm*.

Example

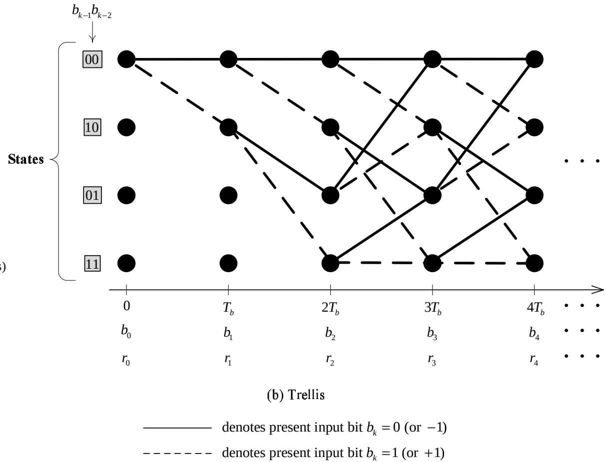


- There are two ISI terms, which are due to $h_1 = 0.6$ and $h_2 = 0.2 \Rightarrow L = 3$ or the memory is $L - 1 = 2$ bits in length.
- The branch metric term is $b_{i,k}r_k - 0.6b_{i,k}b_{i,k-1} - 0.2b_{i,k}b_{i,k-2}$.
- The inputs $b_{i,k-1}$, $b_{i,k-2}$ represent the system memory and hence are states of the state diagram.

State and Trellis Diagrams



(a) State diagram



(b) Trellis

Starting state is chosen to be 00. Before $t = 0$ everything is zero.

Consider $\mathrm{SNR} = E_b/\sigma^2 = 16$ dB (if $E_b = 1$ joule, then $\sigma = 0.158$). The sample output is $r_k = y_k + w_k$, where $y_k = b_k + 0.6b_{k-1} + 0.2b_{k-2}$.

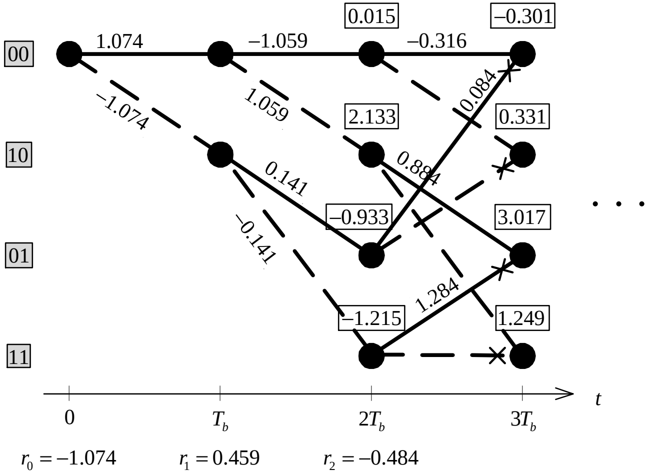
k	0	1	2	3	4	5	6
b_k	0	1	0	1	0	0	1
b_k	-1	$+1$	-1	$+1$	-1	-1	$+1$
y_k	-1.0	0.4	-0.6	0.6	-0.6	-1.4	0.2
w_k	-0.074	0.059	0.116	0.336	-0.216	-0.163	0.165
r_k	-1.074	0.459	-0.484	0.936	-0.816	-1.563	0.365
k	7	8	9	10	11	12	13
b_k	1	0	1	0	0	0	0
b_k	$+1$	-1	$+1$	-1	-1	-1	-1
y_k	1.4	-0.2	0.6	-0.6	-1.4	-1.8	-1.8
w_k	-0.062	0.220	0.050	0.247	0.113	0.311	0.080
r_k	1.338	0.020	0.650	-0.353	-1.287	-1.489	-1.720
k	14	15	16	17	18	19	
b_k	1	1	0	0	1	0	
b_k	$+1$	$+1$	-1	-1	$+1$	-1	
y_k	0.2	1.4	-0.2	-1.4	0.2	-0.6	
w_k	-0.296	-0.054	-0.181	-0.034	0.189	-0.177	
r_k	-0.096	1.346	-0.381	-1.434	0.389	-0.777	

Computations of Branch Metrics

$$\boxed{b_k r_k - b_k \sum_{j=1}^{2} b_{k-j} h_j = b_k r_k - 0.6 b_k b_{k-1} - 0.2 b_k b_{k-2}}$$

b_{k-2}	b_{k-1}	b_k	Branch Metric
$0\ (-1)$	$0\ (-1)$	$0\ (-1)$	$-r_k - 0.6 - 0.2 = -r_k - 0.8$
$0\ (-1)$	$0\ (-1)$	$1\ (+1)$	$+r_k + 0.6 + 0.2 = +r_k + 0.8$
$0\ (-1)$	$1\ (+1)$	$0\ (-1)$	$-r_k + 0.6 - 0.2 = -r_k + 0.4$
$0\ (-1)$	$1\ (+1)$	$1\ (+1)$	$+r_k - 0.6 + 0.2 = +r_k - 0.4$
$1\ (+1)$	$0\ (-1)$	$0\ (-1)$	$-r_k - 0.6 + 0.2 = -r_k - 0.4$
$1\ (+1)$	$0\ (-1)$	$1\ (+1)$	$+r_k + 0.6 - 0.2 = +r_k + 0.4$
$1\ (+1)$	$1\ (+1)$	$0\ (-1)$	$-r_k + 0.6 + 0.2 = -r_k + 0.8$
$1\ (+1)$	$1\ (+1)$	$1\ (+1)$	$+r_k - 0.6 - 0.2 = +r_k - 0.8$

Branch metrics and (*partial*) path metrics for all possible paths for the first 3 bit transmissions.



Pruned Trellis Up To $k = 12$