Answers to Exercises
Chapter 5

Generation of a lava flow cooling model

Table 5.1. Lava flow surface: effective radiation temperature

NOTE: Fractions are in % and temperatures in Kelvin

Case:	**Basaltic 'a'a lava flow**	**Basaltic lava channel**	**Silicic (blocky) lava flow**
f (%)	99.904	--	100
$1 - f$ (%)	0.096	--	0
T_c (or T_1, K)	485.15	1143.15	323.15
T_h (or T_2, K)	1153.15	1391.15	--
T_3 (K)	--	829.15	--
T_4 (K)	--	1386.15	--
f_1 (%)	--	19.9	--
f_2 (%)	--	10.1	--
f_3 (%)	--	55	--
$1 - f_1 - f_2 - f_3$ (%)	--	15	--
T_e (K)	488.71	1112.51	323.15

Table 5.2 Calculation of convective heat transfer coefficient

NOTE: all values are taken from tables in Holman (1992) using the appropriate T_{bound}

Parameter	Basaltic channel	Silicic lava flow
T_{surf} (K)	1112.51	323.15
T_a (K)	298.15	298.15
T_{bound} (K) [= (T_{surf} + T_a) /2]	705.15	310.65
Pr	**0.68**	**0.70**
kinematic viscosity, ν ($m^2\ s^{-1}$)	5.13 x 10^{-5}	2.08 x 10^{-5}
thermal diffusivity, κ ($m^2\ s^{-1}$)	7.51 x 10^{-5}	2.98 x 10^{-5}
Gr	**1.45 x 10^{10}**	**6.18 x 10^{9}**
g ($m\ s^{-2}$)	9.8	9.8
β (K^{-1})	0.0014	0.0032
$T_{surf} - T_a$ (K)	814	25
kinematic viscosity, ν ($m^2\ s^{-1}$)	5.13 x 10^{-5}	2.08 x 10^{-5}
Ra	**9.92 x 10^{9}**	**4.3 x 10^{9}**
Nu	**344**	**260**
H (m)	1.5	1.5
K_{air} ($W\ m^{-1}\ K^{-1}$)	0.05	0.03
h_c ($W\ m^{-2}\ K^{-1}$)	**10.5**	**5.2**

Table 5.3a: Lava flow heat loss: input parameters

Case:	Basaltic 'a'a lava flow	Basaltic lava channel	Silicic (blocky) lava flow
T_{surf} (= T_e, K)	488.71	1112.51	323.15
T_a (K)	298.15	298.15	298.15
T_{core} (°C)	1080	1150	830
T_{core} (K)	1353.15	1423.15	1103.15
T_{base} (K)	298.15	298.15	298.15
T_{bound} (K)	527.5	562.5	402.5
dT (K)	1055	1125	805
k (W m^{-1} K^{-1})	1.43	1.47	1.29
κ (m^2 s^{-1})	4.16 x 10^{-7}	4.28 x 10^{-7}	4.48 x 10^{-7}
dy (m)	0.34	0.34	4.68
dT/dy (K m^{-1})	3138	3302	172

Table 5.3b: Lava flow heat loss – absolute contributions

Case:	Basaltic 'a'a lava flow	Basaltic lava channel	Silicic (blocky) lava flow
q_{rad} (W m^{-2})	2647	82,083	162
q_{conv} (W m^{-2})	5717	24,431	750
q_{cond} (W m^{-2})	4482	4843	222
q_{tot} (W m^{-2})	12,846	111,357	1134

Table 5.3c: Lava flow heat loss – relative contributions

Case:	Basaltic 'a'a lava flow	Basaltic lava channel	Silicic lava flow
q_{rad} (% contribution)	20	74	14
q_{conv} (% contribution)	45	22	66
q_{cond} (% contribution)	35	4	20
q_{tot} (rank)	2	1	3

Exercise 5.4

Starting at the heat balance we have:

$$q_{adv} + q_{cryst} = q_{rad} + q_{conv} + q_{cond} \ ,$$

in which

$$q_{adv} = u\, d\, \rho\, c_p \frac{dT}{dx} \quad \text{and} \quad q_{cryst} = u\, d\, \rho\, c_L \frac{d\phi}{dT}\frac{dT}{dx} \ ,$$

so that

$$u\, d\, \rho\, c_p \frac{dT}{dx} + u\, d\, \rho\, c_L \frac{d\phi}{dT}\frac{dT}{dx} = q_{rad} + q_{conv} + q_{cond} \ ,$$

which simplifies to

$$\frac{dT}{dx} u\, d\, \rho\, (c_p + c_L \frac{d\phi}{dT}) = q_{rad} + q_{conv} + q_{cond} \ .$$

Writing q_{tot} for $q_{rad} + q_{conv} + q_{cond}$ and rearranging allows the cooling rate to be calculated:

$$\frac{dT}{dx} = \frac{q_{tot}}{ud\rho\left[c_p + c_L\left(\frac{d\phi}{dT}\right)\right]}$$

This is the model of Keszthelyi and Self (1998).

But we can also arrive at:

$$u\, d\, \rho\, c_p \frac{dT}{dx} + u\, d\, \rho\, c_L \frac{d\phi}{dT}\frac{dT}{dx} = q_{rad} + q_{conv} + q_{cond} \ ,$$

$$u\, d\, \rho\, c_p \frac{dT}{dx} + u\, d\, \rho\, c_L \frac{d\phi}{dx} = q_{rad} + q_{conv} + q_{cond} \ ,$$

$$u\, d\, \rho\, (c_p \frac{dT}{dx} + c_L \frac{d\phi}{dx}) = q_{rad} + q_{conv} + q_{cond} \ ,$$

so that

$$\frac{dT}{dx} = \frac{q_{tot} - ud\rho c_L \frac{d\phi}{dx}}{ud\rho c_p} \ .$$

Table 5.4: Lava flow cooling rate

Case:	Basaltic 'a'a lava flow	Basaltic lava channel	Silicic (blocky) lava flow
q_{tot} (W m^{-2})	12,846	111,357	1134
u (m s^{-1})	0.05	0.2	0.000145
d (m)	1	1.5	70
ρ (kg m^{-3})	2170	2180	2350
c_p (J kg^{-1} K^{-1})	955	955	1225
L (J kg^{-1} K^{-1})	3.5 x 10^5	3.5 x 10^5	3.5 x 10^5
$d\phi/dT$ (K^{-1}) min	0.003	0.007	0.000
$d\phi/dT$ (K^{-1}) max	0.01	0.08	0.00
dT/dx (K m^{-1}) min	0.0588	0.0501	0.0389
dT/dx (K m^{-1}) max	0.0386	0.0059	0.0389

Table 5.5: Cooling-limited flow length

NOTE: Use the range of values from Table 5.4 for *dT/dx*

Case:	Basaltic 'a'a lava flow	Basaltic lava channel	Silicic (blocky) lava flow
dT/dx (K km^{-1})	59	50	39
ΔT (K)	150	150	150
Calculated distance (km)	2.55	3.00	3.86
Actual distance (km)	2.5	--	3.75
Difference (km)	0.05	--	0.11
E_r (m^3 s^{-1})	0.1571	--	--
$L = 10^{3.11} E_r^{0.47}$ (m)	540	--	--
Actual distance (m)	2500	--	--
Difference (m)	-1960	--	--