4: Benard convection

(a) Differentiating the quadratic with respect to $\cos^2\theta$ gives

$$\frac{\partial\sigma}{\partial\cos^2\theta} = \frac{-B_z}{2\sigma + (\nu+\kappa)K^2}.$$

This is positive assuming that (1) $B_z < 0$, as is normally required for convective instability, and (2) σ is real and positive, which is true for convective instability.

Growth is maximized when $\cos^2\theta$ is at its maximum value, 1.

(b) Define $f = (\tilde{k}^{*2} + n^2\pi^2)^3/\tilde{k}^{*2}$ and differentiate with respect to $\tilde{k}^{*2}$:

$$\frac{\partial f}{\partial\tilde{k}^{*2}} = \frac{(\tilde{k}^{*2} + n^2\pi^2)^2}{\tilde{k}^{*4}}\left[2\tilde{k}^{*2} - n^2\pi^2\right],$$

which is zero only if $\tilde{k}^{*2} = n^2\pi^2/2$, or

$$\tilde{k} = n\pi/\sqrt{2}.$$

In this case

$$f = \frac{27}{4}n^4\pi^4 \approx 657.5 \text{ for } n = 1.$$