Exercises
Chapter 5

Construction of a lava flow cooling model

Exercise 5.1 – Definition of lava surface temperature

Crisp and Baloga (1990) argued that a lava surface could be represented by a two component thermal model in which the surface was represented a relatively cool crust broken by high temperature cracks. In this two component thermal model, the effective radiation temperature of the flow surface (T_e) can be described by:

$$T_e = [fT_c^4 + (1 - f)\, T_h^4]^{0.25} \quad , \tag{5.1.1}$$

in which f is the surface fraction occupied by the cool (crust) thermal component at temperature T_c (in Kelvin, K = °C + 273.15). The remainder ($1 - f$) of the surface is composed of high temperature cracks at temperature T_h (in K).

Case 1: Basaltic 'a'a flows

The two component model appears valid for basaltic 'a'a flows. For 'a'a flows this two component model describes cool surface rubble (breccia)—this being the crust component—surrounded by incandescent (non-crusted) zones. Crust covering an active 'a'a lava flow at Lonquimay volcano in Chile had an estimated mean temperature (T_c) of 212°C and occupied a surface fraction (f) of 0.99904. On Mt. Etna, incandescent zones in an active 'a'a flow were found to range between 730 and 1040°C, with a mean T_h of 880°C.

Case 2: Basaltic lava channels

For active basaltic lava channels, a four component model may be more appropriate. At a lava channel active on Kilauea the following four thermal components were identified:

(i) Crusted areas at the channel center (T_1 = 870ºC; f_1 = 0.199)
(ii) High temperature areas at the channel center (T_2 = 111ºC; f_2 = 0.101)
(iii) Crusted areas at the channel margin (T_3 = 556ºC; f_3 = 0.550)
(iv) High temperature areas at the channel margin (T_4 = 1113ºC, $1 - f_1 - f_2 - f_3$)

In this case we have a four component model, with the effective radiation temperature of the flow surface (T_e) being described by:

$$T_e = [f_1 T_1^4 + f_2 T_2^4 + f_3 T_3^4 + (1 - f_1 - f_2 - f_3)\, T_4^4]^{0.25} \quad . \tag{5.1.2}$$

Case 3: Silicic (blocky) lava flows

For silicic (blocky) lava flows active at Santiaguito volcano in Guatemala, a one component (crust-dominated) model may be valid. At Santiaguito a very cool crust at between 31 and 150 °C, with a mean T_c of 50°C, was found to occupy 100% of the surface. In this case $T_e = T_c$.

Question 5.1: Calculate the effective radiation temperature for each of the three flow cases given above by filling out the following table.

Table 5.1. Lava flow surface: effective radiation temperature

NOTE: Fractions are in % and temperatures in Kelvin

Case:	**Basaltic 'a'a lava flow**	**Basaltic lava channel**	**Silicic (blocky) lava flow**
f (%)		--	
$1 - f$ (%)		--	
T_c (or T_1, K)			
T_h (or T_2, K)			--
T_3 (K)	--		--
T_4 (K)	--		--
f_1 (%)	--		--
f_2 (%)	--		--
f_3 (%)	--		--
$1 - f_1 - f_2 - f_3$ (%)	--		--
T_e (K)			

Exercise 5.2 – Definition of the convective heat transfer coefficient

Task 1: Free Convection

Heat loss (in W m^{-2}) due to free convection (q_{free}) from a lava flow surface can be written as:

$$q_{free} = h_c\,(T_{surf} - T_a) \ , \qquad (5.2.1)$$

in which h_c is the convective heat transfer coefficient (W m^{-2} K^{-1}), T_{surf} is surface temperature (K) and T_a is the temperature (K) of the overlying fluid (air/water).

The heat transfer coefficient can be calculated using the Nusselt number (Nu), where

$$\mathrm{Nu} = h_c H/k_a \ , \qquad (5.2.2)$$

in which H and k_a are the thickness of the layer of hot fluid overlying the lava flow (m) and thermal conductivity of air (W m^{-1} K^{-1}), respectively. For cooling by free convection the Nusselt Number is also given by

$$\mathrm{Nu} = 0.16\mathrm{Ra}^{1/3} \ . \qquad (5.2.3)$$

The Rayleigh (Ra) number considers factors such as the buoyancy, viscosity and thermal conductivity of the fluid, and is the product of the Grashof number (Gr) and the Prandtl number (Pr), i.e.,

$$\mathrm{Ra} = \mathrm{Gr}\ \mathrm{Pr} \ . \qquad (5.2.4)$$

The Grashof number is the ratio of buoyancy force to viscous force, and can be calculated from:

$$\mathrm{Gr} = [\ g\beta(T_{surf} - T_a)H^3\ ] / \nu^2 \ , \qquad (5.2.5)$$

in which g is acceleration due to gravity (9.8 m s^{-2}), $\beta = 1/T$ (K^{-1}), and ν is kinematic viscosity (m^2 s^{-1}). The Prandtl number can be written:

$$\mathrm{Pr} = \nu/\kappa \ , \qquad (5.2.6)$$

in which κ is thermal diffusivity (m^2 s^{-1}).

Question 5.2.1:

(a) Using the effective radiation temperatures calculated for the basaltic channel and silicic flow surfaces in Exercise (5.1), give the convective heat transfer coefficient for free convection at both the channel and silicic flow. The easiest way to do this is to begin with Eq. (5.2.6) and work backwards through the equations (i.e. calculate Pr, then Gr, then Ra and so on). Use the following inputs for your calculations: T_a = 25ºC and H = 1.5 m. All of the parameters for the overlying air are given in Table 5.2. Note that these parameters are temperature dependent, and have been set using a temperature of $(T_{surf} + T_a)/2$ (i.e., the mean temperature of the overlying thermal boundary layer).

(b) By examining the differences in these heat dependant parameters, can you explain the difference between the two h_c values that you have calculated?

(c) Give q_{free} (in W m^{-2}) for the two cases.

Task 2: Forced Convection

Keszthelyi and Denlinger (1996) give heat loss (in W m^{-2}) due to forced convection (q_{force}) as:

$$q_{force} = C_H \, \rho_a \, c_{pa} \, U \, (T_{surf} - T_a) \ . \qquad (5.2.7)$$

This implies that

$$h_c = C_H \, \rho_a \, c_{pa} \, U \ , \qquad (5.2.8)$$

in which C_H is the square of the ratio of wind speed to the slip speed of wind across the ground (0.0036), ρ_a is the density of the air (kg m^{-3}), c_{pa} is the specific heat capacity of the air (J kg^{-1} K^{-1}) and U is mean wind speed (m s^{-1}).

Question 5.2.2:

(a) Using the effective radiation temperatures calculated for the basaltic channel and silicic flow surfaces in Exercise (5.1), give the convective heat transfer coefficient for forced convection at both the channel and silicic flow. Use the following inputs for your calculations: T_a = 25ºC and U = 5 m s^{-1}. All of the parameters for the overlying air are given in Table 5.2. Note that these parameters are temperature dependant, and have been set using a temperature of (T_{surf} + T_a)/2 (i.e., the mean temperature of the overlying thermal boundary layer).

(b) By examining the differences in these heat dependant parameters, can you explain the difference between the two h_c values that you have calculated?

(c) Give q_{force} (in W m^{-2}) for the two cases.

Table 5.2 Calculation of convective heat transfer coefficient

NOTE: all values are taken from tables in Holman (1992) using the appropriate T_{bound}

Parameter	Basaltic channel	Silicic lava flow
T_{surf} (K)		
T_a (K)		
T_{bound} (K) [= $(T_{surf} + T_a)$ /2]		
Pr		
kinematic viscosity, ν (m^2 s^{-1})	5.13 x 10^{-5}	2.08 x 10^{-5}
thermal diffusivity, κ (m^2 s^{-1})	7.51 x 10^{-5}	2.98 x 10^{-5}
Gr		
g (m s^{-2})	9.8	9.8
β (K^{-1})		
$T_{surf} - T_a$ (K)		
kinematic viscosity, ν (m^2 s^{-1})	5.13 x 10^{-5}	2.08 x 10^{-5}
Ra		
Nu		
H (m)	1.5	1.5
K_{air} (W m^{-1} K^{-1})	0.05	0.03
h_c (W m^{-2} K^{-1})		

Exercise 5.3 – Lava flow heat loss model

Heat is lost from an active lava flow by radiation (q_{rad}) and convection (q_{conv}) from the flow surface, plus conduction (q_{cond}) through the flow base.

Heat flux (in W m^{-2}) due to radiation can be calculated using the Stefan-Boltzmann equation:

$$q_{rad} = \varepsilon\,\sigma\,(T_{surf}^4 - T_a^4) \ , \qquad (5.3.1)$$

in which ε is emissivity (~0.95 for lava), σ is the Stefan-Boltzmann constant (5.6697 × 10^{-8} W m^{-2} K^{-4}) and $T_{ambient}$ is the temperature of the environment into which radiation is being emitted (i.e., atmospheric temperature).

Convective heat flux (in W m^{-2}) can be estimated using the convective heat transfer coefficient (h_c) in:

$$q_{conv} = h_c\,(T_{surf} - T_a) \ . \qquad (5.3.2)$$

Theoretically, h_c for the atmosphere over an active lava channel has been calculated to have a value of ~10 W m^{-1} K^{-1} (Table 5.2), with measured values giving ~50 W m^{-1} K^{-1} (Keszthelyi *et al.*, 2003).

Heat loss due to conduction across the thermal boundary layer at the flow base can be constrained using Fourier's Law:

$$q_{cond} = -\,k\,\frac{dT}{dy} \ , \qquad (5.3.3)$$

in which k is the lava thermal conductivity, dT is the temperature difference between the top and the bottom of the boundary layer (= $T_{core} - T_{base}$) and dy is the thickness of the layer.

The thickness of the thermal boundary layer can be calculated as a function of time from:

$$dy = \sqrt{\kappa \pi t} \ , \qquad (5.3.4a)$$

in which κ is thermal diffusivity (in m^2 s^{-1}) and t is time (in seconds).

Thermal diffusivity can, in turn, be calculated from the lava thermal conductivity (k in W m^{-1} K^{-1}), density (ρ in kg m^{-3}) and specific heat capacity (c_p in J kg^{-1} K^{-1}) in

$$\kappa = k\,/\,\rho\,c_p \ . \qquad (5.3.4b)$$

For basalt, thermal conductivity has been shown to vary with temperature (T) and can be estimated from

$$k = 0.848 + 0.0011(T) \ , \tag{5.3.5}$$

in which T is in Kelvin.

Question 5.3: Using the effective radiation temperatures calculated in Exercise 5.1, complete Tables 5.3a–5.3c to obtain the heat fluxes for each of the three cases.

Assume the following values:

T_a = 25°C
h_c = 30 W m^{-1} K^{-1}
T_{base} = 25°C
t = 1 day for the two basaltic cases
= 6 months for the silicic case
c_p = 1225 J kg^{-1} K^{-1}
ρ = 2800 kg m^{-3} for the two basaltic cases
= 2350 kg m^{-3} for the silicic case

In Eq. (5.3.5) use the typical temperature for the boundary layer (T_{bound}) for T, i.e., $T = T_{bound}$, where $T_{bound} = (T_{core} – T_{base}) / 2$

First, the parameters in Table 5.3a need to be calculated before the lava flow heat loss can be estimated.

Table 5.3a: Lava flow heat loss: input parameters

Case:	Basaltic 'a'a lava flow	Basaltic lava channel	Silicic (blocky) lava flow
T_{surf} (= T_e, K)			
T_a (K)			
T_{core} (°C)	1080	1150	830
T_{core} (K)			
T_{base} (K)			
T_{bound} (K)			
dT (K)			
k (W m^{-1} K^{-1})			
κ (m^2 s^{-1})			
dy (m)			
dT/dy (K m^{-1})			

Now, the parameters from Table 5.3b can be used to calculate the lava flow heat loss. Calculate the absolute heat loss values in Table 5.3b, then give the percent contribution of each heat flux term to the total flux (q_{tot}) in Table 5.3c. Finally, rank each case by q_{tot}, i.e., which case has the highest and which has the lowest q_{tot}? (Note that $q_{tot} = q_{rad} + q_{conv} + q_{cond}$)

Table 5.3b: Lava flow heat loss – absolute contributions

Case:	Basaltic 'a'a lava flow	Basaltic lava channel	Silicic (blocky) lava flow
q_{rad} (W m^{-2})			
q_{conv} (W m^{-2})			
q_{cond} (W m^{-2})			
q_{tot} (W m^{-2})			

Table 5.3c: Lava flow heat loss – relative contributions

Case:	Basaltic 'a'a lava flow	Basaltic lava channel	Silicic lava flow
q_{rad} (% contribution)			
q_{conv} (% contribution)			
q_{cond} (% contribution)			
q_{tot} (rank)			

Exercise 5.4 – Lava flow cooling rate model

To estimate the lava flow cooling rate, we now need to set up a lava flow heat balance model. This model can be applied as follows:

If all heat flowing into the lava flow system (q_{in}) then flows out of the system (q_{out}), we have a heat balance whereby:

$$q_{in} = q_{out} \quad . \tag{5.4.1a}$$

Heat flowing out of the system is represented by q_{rad}, q_{conv} and q_{cond}. Heat flowing into the system is represented by advected heat (q_{adv}) and heat of crystallization (q_{cryst}). As a result, Eq. (5.4.1a) can be expanded to:

$$q_{adv} + q_{cryst} = q_{rad} + q_{conv} + q_{cond} \quad . \tag{5.4.1b}$$

The advected heat flux (in W m^{-2}) can be written:

$$q_{adv} = u\, d\, \rho\, c_p \frac{dT}{dx} \quad , \tag{5.4.1c}$$

in which u is flow velocity, d is flow depth and dT/dx is flow cooling rate (in K m^{-1}).
Heat generated by crystallization (in W m^{-2}) can be described similarly:

$$q_{cryst} = u\, d\, \rho\, L \frac{d\phi}{dT} \frac{dT}{dx} \tag{5.4.1d}$$

in which L is latent heat of crystallization (3.5×10^5 J kg^{-1}) and $d\phi/dT$ is the volume fraction of crystals grown per degree cooling (K^{-1}).

Question 5.4:
Insert Eqs. (5.4.1d) and (5.4.1c) into (5.4.1b).

Next re-arrange the result to derive an equation that allows the lava flow cooling rate per unit advance *(dT/dx)* to be calculated from the total heat loss (q_{tot}), i.e., $q_{rad} + q_{conv} + q_{cond}$.

Exercise 5.5 – Lava flow cooling rate

We can now estimate the cooling rate for the three flows. For the three cases we will input the following values:

(1) Channel-fed 'a'a flow:

Flow velocity = 0.05 m s^{-1}
Flow depth = 1.0 m
Bulk ρ = 2180 kg m^{-3}
Bulk c_p = 955 J kg^{-1} K^{-1}
$d\phi/dT$ = 0.003 to 0.006 K^{-1}

(2) A channel fed flow:

Flow velocity = 0.2 m s^{-1}
Flow depth = 1.5 m
Bulk ρ = 2180 kg m^{-3}
Bulk c_p = 955 J kg^{-1} K^{-1}
$d\phi/dT$ = 0.007 to 0.08 K^{-1}

(3) A channel fed silicic flow:

Flow velocity = 12.5 m per day
Flow depth = 70 m
Bulk ρ = 2350 kg m^{-3}
Bulk c_p = 1225 J kg^{-1} K^{-1}
$d\phi/dT$ = 0.000 K^{-1} (i.e., no crystallization)

Question 5.5:
Using the cooling rate equation derived in Exercise 5.4, estimate the cooling rate for each case by filling out Table 4.

<u>**Note:**</u>

- Values are for (1) the Kilauea channel at which Flynn and Mouginis-Mark (1994) made their measurements, (2) the lava channel active on Etna visited by Harris *et al*. (2005), and (3) a silicic lava flow active at Santiaguito (Guatemala) in January 2000 (Harris *et al*., 2002).
- Dense rock values have been corrected for vesicularity in the two basaltic cases by multiplying the dense rock density and specific heat capacity by 1 – ω, in which ω is vesicularity (a value of ω = 0.22 was used).

Table 5.4: Lava flow cooling rate

Case:	Basaltic 'a'a lava flow	Basaltic lava channel	Silicic (blocky) lava flow
q_{tot} (W m^{-2})			
u (m s^{-1})			
d (m)			
ρ (kg m^{-3})			
c_p (J kg^{-1} K^{-1})			
L (J kg^{-1} K^{-1})			
$d\phi/dT$ (K^{-1}) min			
$d\phi/dT$ (K^{-1}) max			
dT/dx (K m^{-1}) min			
dT/dx (K m^{-1}) max			

Exercise 5.6 – Lava flow run-out distance

If we assume that a lava stops flowing when it has cooled by ΔT degrees, we can use the cooling rate to estimate the cooling-limited distance that a flow can extend (L) from

$$L = \frac{\Delta T}{dT/dx} \quad , \qquad (5.6.1)$$

Question 5.6a:
Assuming ΔT of 150 K, use the cooling rates from Table 5.4 to estimate how far each of the flow cases can extend by completing Table 5.5.

During May 2001 a lava flow with similar properties to the basaltic 'a'a case of Table 5.4 extended 2.5 km on Etna. Between 1999 and 2001 a lava flow with similar properties to the silicic case extended 3.75 km on Santiaguito.

Question 5.6b:
How do the calculated lengths in Table 5.5 compare with these actual lengths?

Calvari and Pinkerton (1998) used data for 17 lava flows erupted on Mount Etna between 1974 and 1993 to obtain the following empirical relationship:

$$L = 10^{3.11}\, E_r^{\,0.47} \qquad r^2 = 0.86 \quad , \qquad (5.6.2)$$

in which L is in meters and E_r is in $m^3\ s^{-1}$.

For the basaltic 'a'a (Etna) case the effusion rate (E_r) may be estimated from

$$E_r = \pi\, d^2\, u \quad , \qquad (5.6.3)$$

in which d is flow depth and u is mean velocity

Question 5.6c
What effusion rate does this give?

Question 5.6d
What distance does this effusion rate give when used in the empirical relationship (Eq. 5.6.2)?

Question 5.6e
How does this length compare with the actual length for the Etna case?

Table 5.5: Cooling-limited flow length

NOTE: Use the range of values from Table 5.4 for *dT/dx*

Case:	Basaltic 'a'a lava flow	Basaltic lava channel	Silicic (blocky) lava flow
dT/dx (K km^{-1})			
ΔT (K)			
Calculated distance (km)			
Actual distance (km)		--	
Difference (km)		--	
E_r (m^3 s^{-1})		--	--
$L = 10^{3.11} E_r^{0.47}$ (m)		--	--
Actual distance (m)		--	--
Difference (m)		--	--

Exercise References

Calvari, S. and Pinkerton, H. (1998). Formation of lava tubes and extensive flow field during the 1991-1993 eruption of Mount Etna. *Journal of Geophysical Research,* **103**(B11), 27291-27301.

Crisp J. and Baloga, S. (1990). A model for lava flows with two thermal components. *Journal of Geophysical Research,* **95**(B2), 1255-1270.

Flynn, L. P. and Mouginis-Mark, P. J. (1994). Temperature of an active lava channel from spectral measurements, Kilauea Volcano, Hawaii. *Bulletin of Volcanology,* **56,** 297-301.

Harris, A. J. L, Flynn, L. P., Matías, O. and Rose, W. I. (2002). The thermal stealth flows of Santiaguito: implications for the cooling and emplacement of dacitic block lava flows. *Geological Society of America Bulletin,* **114**(5), 533-546.

Harris, A. J. L., Flynn, L. P., Matias, O., Rose, W. I. and Cornejo, J. (2004). The evolution of an active silicic lava flow field: An ETM+ perspective. *Journal of Volcanology and Geothermal Research,* **135**, 147-168.

Harris, A., Bailey, J., Calvari, S. and Dehn, J. (2005). Heat loss measured at a lava channel and its implications for down-channel cooling and rheology. *Geological Society of America Special Paper*, **396**, 125-146.

Holman, J. P. (1992). *Heat Transfer*. London: McGraw-Hill, 713 p.

Keszthelyi, L. and Denlinger, R. (1996). The initial cooling of pahoehoe flow lobes. *Bulletin of Volcanology,* **58**, 5-18.

Keszthelyi, L. and Self, S. (1998). Some physical requirements of the emplacement of long basaltic lava flows. *Journal of Geophysical Research*, **103**(B11), 27447-27464.

Keszthelyi, L., Harris, A. J. L. and Dehn, J. (2003). Observations of the effect of wind on the cooling of active lava flows. *Journal of Geophysical Research,* **30**(19), 4-1–4-4. doi:10.1029/2003GL017994.

Oppenheimer, C. (1991). Lava flow cooling estimated from Landsat Thematic Mapper infrared data: the Lonquimay eruption (Chile, 1989). *Journal of Geophysical Research*, **96**(B13), 21865-21878.