

Partial Differential Equation Analysis in Biomedical Engineering

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Preface

This book is an introduction to the mathematical modeling of biomedical engineering systems. In particular, models based on partial differential equations (PDEs) are presented; chapters include antibody binding kinetics, acid-mediated tumor growth, retinal O_2 transport, hemodialyzer dynamics, epidermal wound healing and polymer matrix drug delivery.

The numerical solution of the model equations is through a single, well-established method for PDEs, the method of lines (MOL) in which the spatial derivatives in the PDEs are replaced with algebraic approximations and a system of ordinary differential equations (ODEs) in an initial value variable, typically time, follows. The spatial approximations are finite differences (FDs) although other approximations could easily be accommodated within the MOL format, e.g., finite elements, finite volumes, spectral methods, Galerkin methods such as collocation, meshfree methods. The final result is a set of routines that numerically integrates the ODEs; the format of these routines is basically the same throughout the book.

To facilitate understanding of the PDE analysis, an introduction to the numerical methods and associated computational routines is presented in the first chapter. Then each application is cross referenced to this introduction in each step where some additional explanation is helpful.

In each example, we follow a combination of the following steps:

- The model PDEs are stated along with the associated auxiliary (initial and boundary) conditions. This introduction to the model contains a reference to the original source and possibly related literature, and includes some discussion of the underlying biophysics, biochemistry and physiology.
- Discussion of the numerical methods (algorithms) for the MOL solution of the model equations, principally by reference to the introduction in Chapter 1.
- Matlab routines based on the MOL numerical solution of the PDEs are listed, then discussed in some detail, typically a few lines of code at a time. This discussion emphasizes how the associated mathematics of the model is programmed.
- Discussion of the numerical solution of the model equations, including the origin of any unusual features of the solution.
- An error analysis to establish if the numerical solution is reliable and has acceptable accuracy; typically techniques such as varying the MOL grid spacing and order of approximation are used to infer convergence of the numerical solution to an acceptable level of accuracy. Also, physical constraints such as conservation of mass energy and energy are used to evaluate the solution.
- Concluding summary and discussion of extensions of the model and the MOL algorithms.

Our intention is not to provide a comprehensive treatise, but rather to provide a set of basic computational procedures that we hope readers can assimilate without becoming deeply involved in the details of numerical methods for PDEs and computer programming so that they can concentrate on the problem of interest with reasonable effort. This might take the form of extending the computer routines provided, or applying and extending the numerical methods that are presented through examples.

In summary, our intention is to provide a methodology for the PDE analysis of biomedical engineering systems. This includes the development of numerical methods and associated computer routines that can be used to study the characteristics and solutions of the model equations. The approach is not theoretical, e.g., limited theorems and no proofs; rather, the presentation is based on detailed example applications. The MOL analysis provides a general framework for the analysis of PDE models that we think can be broadly applied in biomedical engineering, and which can be applied to all of the major geometric classes of PDEs (parabolic, hyperbolic, elliptic). All of the Matlab routines are available (gratis) as a download through a request to wes1@lehigh.edu

We welcome comments from readers concerning this approach and will be pleased to answer questions to the extent possible by e-mail.

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