Answers to Exercises
Chapter 3

Exercise 3.1

(a) In this case there is a balance between the buoyancy pressure P_b (Eq. 3.4b) and the fracture toughness pressure, P_f (Eq. 3.4d). Equating these expressions, we have

$$\Delta\rho g L = \frac{K_c}{\sqrt{L}} \ ,$$

which rearranges to give the buoyancy length scale, $L = L_b = \left(\frac{K_c}{\Delta\rho g}\right)^{2/3}$.

(b) The first stage of the propagation is at constant aspect ratio, $L \sim B$, so there is a balance the buoyancy pressure, Eq. (3.4b), and viscous pressure, Eq. (3.4c), i.e.,

$$\Delta\rho g L = \frac{4\eta LC}{H^2} = \frac{4\eta L^2}{H^2 t}$$

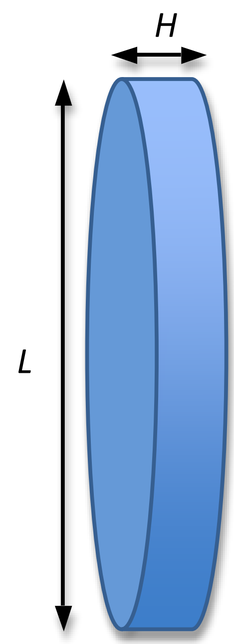
The assumption of constant volume means that we can use $V \sim HBL$, i.e., $H \sim V/L^2$. The means

$$L^5 = \frac{\Delta\rho g V^2}{4\eta} t \quad \text{, i.e., } L \propto t^{1/5}.$$

During the second stage of emplacement, we use the same pressure balance, but with $V \sim HB_f L$, where B_f is a constant. This leads to

$$\Delta\rho g L = \frac{4\eta L^2 L^2 B_f^2}{V^2 t} \ ,$$

which in turn leads to $L \propto t^{1/3}$.



Exercise 3.2

(a) In the first stage of propagation, the fissure is expanding ("the stress intensity factor exceeds the fracture toughness"), which implies a viscous pressure drop, P_v, while at the same time the elastic pressure, P_e, acts to close the fracture. This implies a balance between P_v and P_e (Eqs. 3.4a and 3.4c, where L is the diameter of the penny-shaped crack). The typical velocity for the fluid flow is found from $C \sim L/t$.

Equating Eqs. (3.4a) and (3.4c), substituting for C, and rearranging yields:

$$\left(\frac{L}{H}\right)^3 = \frac{E}{8(1-\nu^2)}t\,.$$

However, the volume of the crack is $V = \pi L^2 H/4$, i.e., $H = 4V/\pi L^2$. Substituting this into the equation above yields

$$L = \left[\frac{8V^3 E}{\pi^3(1-\nu^2)}t\right]^{1/9},$$

i.e., L grows as $t^{1/9}$ (see Table 3.1).

(b) In the second phase, when the fissure reaches the static state, the elastic pressure (Eq. 3.4a) is in equilibrium with the fracturing pressure (Eq. 3.4d), which leads to:

$$\frac{E}{2(1-\nu^2)}\frac{H}{L} = \frac{K_c}{\sqrt{L}}$$

Again, substituting $H = 4V/\pi L^2$ yields

$$L = \left[\frac{2EV}{\pi K_c(1-\nu^2)}\right]^{2/5}$$

Substituting the values given for E, V, K_c, and ν, we obtain $\underline{L \sim 3.7 \text{ km and } H \sim 9 \text{ cm}}$.

Exercise 3.3

(a) To obtain the typical thickness we use the balance between buoyancy (Eq. 3.4b) and viscous pressure drop (Eq. 3.4c) as shown Figure 3.5. This leads to

$$H^2 = \frac{4\eta C}{\Delta\rho g}\quad.$$

Next, to estimate the vertical fluid flow velocity C from the volumetric flux, we first derive the flux per unit length by dividing the volumetric flux by the horizontal extent (breadth) of the dike: $Q_{2D} = Q_{3D}/B$. This flux can also be expressed as the product of the upward velocity and fissure thickness, $Q_{2D} = C\,H$. Therefore, using $C \sim Q_{3D}/(HB)$ in the equation above leads to

$$H = \left(\frac{4\eta Q_{3D}}{\Delta\rho g B}\right)^{1/3}\quad.$$

The expression for width of the dike tail given in Eq. (3.3a) is

$$h_\infty = \left(\frac{3\eta Q_{2\mathrm{D}}}{2\Delta\rho \mathrm{g}}\right)^{1/3}.$$

This means that H is a factor $(4/(3/2))^{1/3}$ greater than h, when considering three-dimensional, rather than two-dimensional behavior.

(b) For the typical length, we use the balance between elastic pressure (Eq. 3.4a) and buoyancy (3.4b). This leads to the expression

$$L = \left(\frac{EH}{2(1-\nu^2)\Delta\rho g}\right)^{1/2}$$

(c) Evaluating H and L using the expression in (a) and (b), and the values given in part (c) leads to $H \sim 4^{1/3} \sim 1.6$ m and $L \sim 3.3$ km.

In deriving these expressions the student should ensure that the equations are dimensionally correct, and be critical about the numerical values obtained. For example, if the student inadvertently uses a value Young's modulus of 10 Pa (rather than 10 GPa), the length scale L would be only 9 cm. This is clearly unreasonable, and so reassessment of the approach to problem is necessary.