Additional material on Chapter 3

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Correcting the treatment of the scattering problem in Section 3.2

This replaces a previous posting, dated June 2005, which offered an alternative approach in an attempt to improve on the results in [1, Fig. 3.15]. It transpires that the actual problem was a bug in the original code, due to an error in the upper limits of the indices of the E_x and E_y matrices. The code has been corrected, and posted on the website[1]. The point is quite subtle, and worth elucidating. If the H_z field is defined on a grid of N_x by N_y points, because of the offset locations of the field components as in [1, Fig. 3.1], the corresponding dimensions of both E_x and E_y will be $N_x + 1$ by $N_y + 1$. This is because the H fields are located at the cell centres, whereas the E fields are at cell boundaries. This must be explicitly taken into account in both the matrix dimensioning and the update equations.

The correct treatment removes the "glitch" noted in [1, Fig. 3.11] at around 6ns, described in the text as a computational artifact [1, p.88]). Results computed using this corrected formulation are given in the figure below, which can be compared with [1, Fig. 3.15]. One finds that a larger computational region is necessary to obtain good results — the reasonably good results in Fig. 3.15 may have been rather fortuitous — and in the figure shown here, the computational region has been doubled in size relative to that of the computations shown in the book. The results do show rather better agreement with the eigenfunction solution than did those in Fig. 3.15.

If time domain results are compared, one finds that the coarser the mesh, the more the creeping wave is over-estimated, corresponding to the inaccurate results

[1]Comments from K Kaczmarski, Univ Illinois at Chicago, are gratefully acknowledged in this regard.

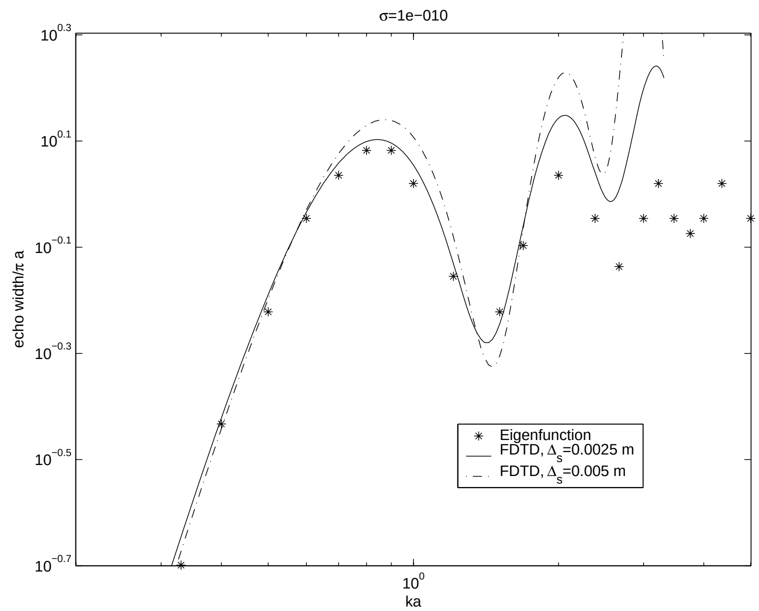

Figure 1: Normalized echo width for the PEC cylinder showing two different FDTD results (computed using the corrected formulation discussed here) compared with the eigenfunction solution.

at higher frequencies for the coarser mesh solutions. This is quite possibly a result of the stair-step approximation of the cylinder.

Extending the treatment of the scattering problem in Section 3.2

It is possible to extend the approach described in [1, Section 3.2]. Instead of zoning the computational region only as in Fig. 3.2 — with fields on the left of the fictitious interface being scattered fields, and those on the right being total — introduce *three* additional such interfaces, one on the right-hand side of the figure, and one each on the upper and lower sides. Hence, there is now a “buffer” region right around the scatterer, consisting entirely of scattered fields, before one reaches the outer boundaries. This permits the incident field to be introduced cleanly for other angles of incidence of the plane wave excitation. One needs to derive additional formulas similar to Eqs. (3.22)–(3.23); the derivation is straightforward. As an example, for the "mirror-image" interface in cell $N_x - L$, at distance $x_R = N_x - L - 1/2$, the update equation for H_z becomes:

$$
\begin{aligned}
&H_z^{\mathrm{tot}}(i_R + \frac{1}{2}, j + \frac{1}{2}, n + \frac{1}{2}) \\
&= \quad H_z^{\mathrm{tot}}(i_R + \frac{1}{2}, j + \frac{1}{2}, n - \frac{1}{2}) \\
&\quad + \frac{\Delta t}{\mu \Delta y} \left[E_x^{\mathrm{tot}}(i_R + \frac{1}{2}, j + 1, n) - E_x^{\mathrm{tot}}(i_R + \frac{1}{2}, j, n) \right] \\
&\quad - \frac{\Delta t}{\mu \Delta x} \left[E_y^{\mathrm{scat}}(i_R + 1, j + \frac{1}{2}, n) + E_y^{\mathrm{inc}}(i_R + 1, j + \frac{1}{2}, n) \right. \\
&\quad \left. - E_y^{\mathrm{tot}}(i_R, j + \frac{1}{2}, n) \right]
\end{aligned} \tag{1}
$$

Again, fields *on* the interface are taken as total fields. Note that, compared to Eq. (3.22), the field on the right of the interface is now the scattered field, and that on the left, the total field.

References

[1] D. B. Davidson, *Computational Electromagnetics for RF and Microwave Engineering*. Cambridge, UK: Cambridge University Press, 2005.