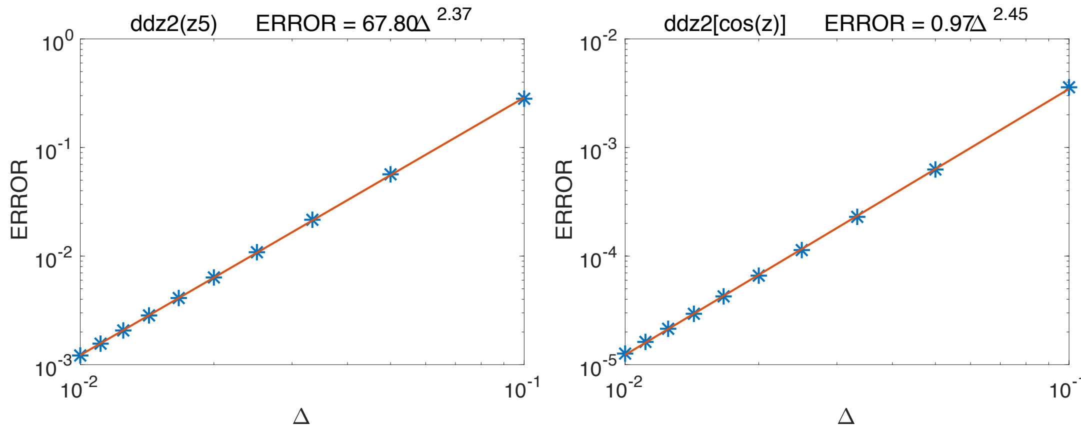

Figure 1: Approximating the second derivative of $f = z^5$ and $f = \cos z$. The error is the absolute difference between the approximation and the true value. Orange line = linear fit.

2: Second derivative matrix

(a) $f_i'' = \frac{f_{i-1}-2f_i+f_{i+1}}{\Delta^2} + \frac{1}{12}\Delta^2 f_1'''' + \ldots$

$f_1'' = \frac{2f_1-5f_2+4f_3-f_4}{\Delta^2} + \frac{11}{12}\Delta^2 f_1'''' + \ldots$ and similar for f_N''.

(b) For the cases shown in figure 1, accuracy is actually a bit better than 2nd order; i.e. the exponent is >2.

Your ddz and ddz2 routines are valuable; you'll be using them constantly. It's worthwhile to keep them organized and to upgrade them if you see an opportunity. Be sure to include comments! One possibility for the function ddz2.m is as follows.

```
function d=ddz2(z)
% Second derivative matrix for independent variable z.
% 2nd order centered differences, with 1-sided derivatives at the boundaries.
% z is assumed to be equally spaced.

% check for equal spacing
if abs(std(diff(z))/mean(diff(z)))>.000001
    disp(['ddz2: values not evenly spaced!'])
    d=NaN;
    return
end

del=z(2)-z(1);N=length(z);

d=zeros(N,N);
for n=2:N-1
    d(n,n-1)=1.;
    d(n,n)=-2.;
    d(n,n+1)=1.;
end
d(1,1)=2;d(1,2)=-5;d(1,3)=4;d(1,4)=-1;
```

```
d(N,N)=2;d(N,N-1)=-5;d(N,N-2)=4;d(N,N-3)=-1;
d=d/del^2;
return
end
```