Symbol Table
for Manifolds, Tensors, and Forms

Paul Renteln

Department of Physics
California State University
San Bernardino, CA 92407
and
Department of Mathematics
California Institute of Technology
Pasadena, CA 91125
prenteln@csusb.edu

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1

List of Symbols

1.1 Rings, Fields, and Spaces

Symbol	Description	Page
$\mathbb{N}$	natural numbers	264
$\mathbb{Z}$	integers	265
$\mathbb{F}$	an arbitrary field	1
$\mathbb{R}$	real field or real line	1
$\mathbb{R}^n$	(real) n space	1
$\mathbb{RP}^n$	real projective n space	68
$\mathbb{H}^n$	(real) upper half n-space	167
$\mathbb{C}$	complex plane	15
$\mathbb{C}^n$	(complex) n space	15

1.2 Unary operations

Symbol	Description	Page
$\bar{a}$	complex conjugate	14
$\overline{X}$	set complement	263
$\|x\|$	absolute value	16
$\|X\|$	cardinality of set	264
$\|\|x\|\|$	length of vector	57
$[x]$	equivalence class	264

$f^{-1}(y)$	inverse image of y under f	263
f^{-1}	inverse map	264
$(-1)^\sigma$	sign of permutation σ	266
$\star$	hodge dual	45
∇	(ordinary) gradient operator	73
∇_X	covariant derivative in direction X	182
d	exterior derivative	89
δ	coboundary operator (on cohomology)	127
δ	co-differential operator	222
Δ	Hodge-de Rham Laplacian	223
∇^2	Laplace-Beltrami operator	242
f^*	pullback map	95
f_*	pushforward map	97
f_*	induced map on simplices	161
i_X	interior product	93
$\mathcal{L}_X$	Lie derivative	102
ΣX	suspension	119
∂	partial derivative	59
∂	boundary operator	143
∂^*	coboundary operator (on cochains)	170
$[S]$	simplex generated by set S	141
D	vector bundle connection	182
$\mathrm{ind}(X, p)$	index of vector field X at p	260
$I(f; p)$	index of f at p	250

1.3 More unary operations

Symbol	Description	Page
Ad	(big) Ad	109
ad	(little) ad	109
alt	alternating map	42
Ann	annihilator	9
Aut	automorphism group	4
cl	closure	56

coker	cokernel	21
deg	degree	251
Deg	degree	251
dim	vector space dimension	2
dim	manifold dimension	63
eu	Euler class	262
im	image	3
ker	kernel	3
mod n	modulo n	264
pf	Pfaffian	261
rk	rank	3
supp	support	129

1.4 Binary relations

Symbol	Description	Page
$\in$	element of	263
$\subseteq$	subset of	263
$\cong$	isomorphic	3
$\approx$	homeomorphic	58
$\sim$	equivalent	264
$\sim$	homotopic	118

1.5 Binary operations

Symbol	Description	Page
$\cup$	union	263
$\cap$	intersection	263
$\bigoplus$	direct sum	2
$\circ$	composition	264
$\langle x, y \rangle$	dual pairing	8
$g(u, v)$	inner product	14
$g(u, v)$	(smooth) metric	193
$\otimes$	tensor product (of tensors)	30

$\otimes$	tensor product (of maps)	49
$v \wedge w$	wedge product	38
$\odot$	symmetric tensor product	51
(X, Y)	Killing form	110
$X \vee Y$	wedge sum	138
$X \amalg Y$	disjoint union	138
$[X, Y]$	Lie bracket of X and Y	77
$\{f, g\}$	Poisson bracket	114
$\binom{n}{k}$	binomial coefficient n choose k	39
$K * L$	join of simplicial complexes	152

1.6 Vector spaces and vector fields

Symbol	Description	Page
V/U	quotient space	6
V^*	dual space	8
$L^2(\mathbb{R})$	square integrable functions on $\mathbb{R}$	27
T_pM	tangent space to M at p	75
X_p	tangent vector at a point p	75
X_p	vector field X evaluated at p	77
T_p^*M	cotangent space to M at p	79
X_f	Hamiltonian vector field	114
Δ	distribution	284

1.7 Matrices and maps

Symbol	Description	Page
$\boldsymbol{A}^T$	transpose matrix	10
$\boldsymbol{A}^{-1}$	inverse matrix	8
$W^\perp$	orthogonal complement of W	17
$\det \boldsymbol{T}$	determinant of matrix $\boldsymbol{T}$	17
$\det T$	determinant of map T	43
A^*	dual map	20
$A^\dagger$	adjoint map	20

$\boldsymbol{A}^\dagger$	adjoint matrix	20
$\boldsymbol{A}(i\|j)$	matrix $\boldsymbol{A}$ with row i and column j removed	22
$\widetilde{A}_{ij}$	cofactor of A_{ij}	22
adj $\boldsymbol{A}$	adjugate matrix	22
tr $\boldsymbol{A}$	trace	24

1.8 Tensor components and tensor spaces

Symbol	Description	Page
δ_{ij}	Kronecker delta	8
η_{ij}	Minkowski metric	19
F_{ij}	field strength	192
g_{ij}	generic metric	195
$\Gamma^a{}_{bc}$	Christoffel symbols (not a tensor)	202
$R^a{}_{bcd}$	Riemann curvature tensor	204
R_{ij}	Ricci curvature tensor	206
R	Ricci curvature scalar	207
G_{ij}	Einstein tensor	207
$T_{(i_1\ldots i_p)}$	symmetric part of T	36
$T_{[i_1\ldots i_p]}$	antisymmetric part of T	36
$a^{(i_1,\ldots,i_p)}$	ascending indices	40
a^I	multi-index	40
$T^{i_1\ldots i_r}{}_{j_1\ldots j_s;k}$	semicolon notation	225
$\varepsilon_{i_1\ldots i_n}$	epsilon or Levi-Civita symbol	239
$\epsilon_{i_1\ldots i_n}$	epsilon or Levi-Civita (pseudo-)tensor	240
$\delta^{i_1\ldots i_k}_{j_1\ldots j_k}$	generalized Kronecker symbol	240
$\omega^i{}_j$	connection 1-forms	183
$\Omega^i{}_j$	curvature 2-forms	184
$\Omega(X,Y)$	curvature matrix	186
$R(X,Y)$	curvature operator	186
$K(\Pi)$	sectional curvature	232
ds^2	line element	197
τ	torsion form/tensor	199
$\mathcal{T}^r$	tensors of order r	31

$\mathcal{T}^r_s$	tensors of type (r, s)	33
$\widetilde{\mathcal{T}}^r_s$	multilinear maps of type (r, s)	35
$\mathrm{Sym}^p V$	symmetric tensors of order p on V	37
$\mathrm{Alt}^p V$	antisymmetric tensors of order p on V	37
$\bigwedge^p V$	p-vectors on V	38
$\bigwedge V$	exterior algebra of V	41

1.9 Manifolds and bundles

Symbol	Description	Page
S^2	two sphere	54
D^2	two disk = two ball	148
S^n	n-sphere	133
B^n	n-ball	133
T^2	two torus	54
TM	tangent bundle of M	176
T^*M	cotangent bundle of M	176
$\Gamma(E)$	sections of bundle E	178
M^k_p	functions on M vanishing to order $(k-1)$ at p	81
(E, M, Y, π)	vector bundle	176
g_{UV}	vector bundle transition functions	178

1.10 Lie groups and Lie algebras

Symbol	Description	Page
$GL(n, \mathbb{R})$	general linear group (over the real field)	105
$SL(n\mathbb{R})$	special linear group (over the real field)	235
$GL(n, \mathbb{C})$	general linear group (over the complex field)	108
$PGL(n, \mathbb{C})$	projective general linear group (over the complex field)	235
$M_n(\mathbb{R})$	set of (real) $n \times n$ matrices	105
$M^+_n(\mathbb{R})$	set of (real) symmetric $n \times n$ matrices	105
$O(n)$	orthogonal group	23
Exp	exponential map (on Lie group)	107
Exp	exponential map (on Riemannian manifold)	276

$\mathfrak{g}l(n,\mathbb{R})$	Lie algebra of $GL(n,\mathbb{R})$	107
$\mathfrak{g}l(n,\mathbb{C})$	Lie algebra of $GL(n,\mathbb{C})$	108
$SO(n)$	special orthogonal group	106
$\mathfrak{s}o(n)$	Lie algebra of $SO(n)$	108
$U(n)$	unitary group	108
$SU(n)$	special unitary group	108
$\mathfrak{s}u(n)$	Lie algebra of $SU(n)$	108

1.11 Homotopy, homology, cohomology, and holonomy

Symbol	Description	Page
$\pi_1(X,p)$	fundamental group of X based at p	136
$\Omega^k(M)$	space of k-forms on M	87
$Z^k(M)$	space of closed k-forms on M	122
$B^k(M)$	space of exact k forms on M	122
$H^k_{\mathrm{dR}}(M)$	k^{th} de Rham cohomology group on M	123
$\bar{\sigma}, \langle p_0, p_1, \ldots \rangle$	oriented simplex	142
$C_m(K)$	m-chains of simplicial complex K	143
$Z_m(K)$	m-cycles of simplicial complex K	145
$B_m(K)$	m-boundaries of simplicial complex K	145
$H_m(K)$	m^{th} simplicial homology group of K	145
β_m	m^{th} Betti number	145
$S_m(X)$	singular m-chains of space X	146
$Z_m(X)$	singular m-cycles of space X	147
$B_m(X)$	singular m-boundaries of space X	147
$H_m(X)$	singular m^{th} homology group of space X	147
$\chi(X)$	Euler characteristic of space X	149
$\sigma, (\bar{\sigma}, U, \phi)$	smooth singular simplex	158
$C^\infty_m(M)$	smooth singular m-chains on M	158
$Z^\infty_m(M)$	smooth singular m-cycles on M	159
$B^\infty_m(M)$	smooth singular m-boundaries on M	159
$H^\infty_m(M)$	m^{th} smooth singular homology group of M	159
$C^m(M)$	smooth singular m-cochains of M	170
$Z^m(M)$	smooth singular m-cocycles of M	170

$B^m(M)$	smooth singular m-coboundaries of M	171
$H^m(M)$	smooth singular m^{th} cohomology group of M	171
ϑ_t	parallel transport map	213
$\mathcal{P}$	path ordering operator	218
$H(\nabla; p)$	holonomy group of ∇ based at p	219
$\widetilde{H}(\nabla; p)$	restricted holonomy group of ∇ based at p	219